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END TERM EXAMINATION

SECOND SEMESTER [BCA]-MAY 2007

Paper Code: BCA-102 **Subject: Mathematics-II (2005)**

Time : 3 Hours **Maximum Marks : 75**

Note: Q.1 is compulsory and carries 25 marks. Attempt four questions selecting one from each Unit.

Q1. (a) If $A \subseteq C$ and $B \subseteq D$, then show that $AXB \subseteq CXD$ (2)

(b) Determine whether the relation R on the set A is an equivalence relation. A is the set of positive numbers, and a relation R is defined as aRb if $a=b^K$, $(a=b^K)$ where k is some positive integer. (1.5)

(c) R be a reflexive relation on a set A. Show that R is an equivalence relation if and only if (a,b) and (a,c) $\in$R implies that (b,c) $\in$ R. (2)

i.e. (a,b) & (a,c) $\in$R $\Rightarrow$(b,c) $\in$R

(d) Let C denote the set of complex numbers and let R denote the set of real numbers. Prove that the mapping $f: C \rightarrow R$ given by $f(x+iy)=|x+iy|$, where x and y are real ,is neither one to one nor onto. (3)

(e) Find the distance of the point (1,-2,3) from the plane x-y+z=5 measured parallel to the line (2)

$$\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$$

(f) If the three thermodynamic variables P,V,T are connected by a relation f(P,V,T)=0, show that (3)

$$\left(\frac{\partial P}{\partial T}\right)_v \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T = -1$$

(g) Show, by double integration, the area between the parabolas $y^2=4ax$ & $x^2=4ay$ is $\frac{16}{3}a^2$. (2.5)

(h) Let S be any non empty set and P(S) be the power set of S. If '$\subseteq$' (a supset of) is a relation defined on P(S), then show that $(P(S),\subseteq)$ is a poset. (3)

(i) If Z is a homogeneous function of degree n in x &y, show that (3)

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial z^2}=n(n-1)z$$

(j) For any set A and B, answer the following question: (3)

(a) Is the set $AX\phi$ well defined?

(b) If $AXB=\phi$ what can you say about the sets A and B?

(c) Is it possible that $AXA=\phi$, for some set A?

UNIT-I

Q2 (a) (i) Show that the transitive closure of a symmetric relation is symmetric. (3)

(ii) Let R be a transitive and reflexive relation on A. Let T be a relation on A such that (a,b) is in T if and only if both (a,b) and (b,a) are in R. Show that T is an equivalence relation. (3.5)

(b) Let R be a binary relation and S={(a,b)|(a,c) & (c,b) $\in$ R, for some C}.Show that if R is an equivalence relation then S is also an equivalence relation. (6)

OR

Q3. (a) Prove that if R is reflexive and transitive, then $R^n = R$ for all n. (5)

(b) A survey was conducted among 1000 people. 595 of them are democrats wear glasses and 550 like ice cream. 395 democrats wear glasses.350 democrats like ice cream. 400 of the people wear glasses and like ice cream. 250 democrats wear glasses and like ice cream.

Answer the following questions: (4)

(i) How many people are not democrats who do not wear glasses and do not like ice cream.

(ii) How many people are democrats who do not like ice cream and donot wear glasses.

(c) Let R and S be binary relations from A to B. Is it true that $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$. Justify your answer. (3.5)

UNIT-II

Q4. (a) What can you say about the relation R on set A if R is a partial order and an equivalence relation? (3.5)

(b) Determine whether D_n is a finite Boolean algebra, where (5)

(i) n=12 (ii) n=40 (iii) n=75 (iv) n=21 (v) n=70?

(c) Find all the maximal, minimal elements and greatest and least elements (if exist) of the Poset (A, ≤), A ={2,3,4,6,8,24,48} and ≤ is defined as the partial order of divisibility. (4)

OR

Q5. (a) Let A {1,2,3,5,6,10,15,30} be a set and R be a relation of divisibility on A. Show that R is a Partial order on A and draw a Harse diagram of R. Is R a linear order relation? What about if A= {2,4,8,16,32}? (6.5)

(b) Let L be a bounded lattice with at least two elements. Show that no element of L is its own complement? (3)

(c) Find the complement of each element in D_{42}. (3)

UNIT-III

Q6. (a) Show that the plane ax + by + cz + d = 0 touches the surface

$px^2 + qy^2 + 2z = 0$ if $\frac{a^2}{p} + \frac{b^2}{q} + 2cd = 0$ (6)

(b) If f(x,y)=0 show that (6.5)

$$\left(\frac{\partial f}{\partial y}\right)^3 \frac{d^2 y}{dx^2} = 2\left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial^2 f}{\partial x \partial y}\right) - \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial x^2} - \left(\frac{\partial f}{\partial x}\right)^2 \left(\frac{\partial^2 f}{\partial y^2}\right)$$

OR

Q7. (a) In a plane triangle, Find the maximum value of CosA CosB CosC? (4)

(b) Transform the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar coordinates. (4)

(c) The equation $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial t^2}$ refers to the conduction of heat along a bar without radiation. Show that it $u = Ae^{-gx} \sin(nt - gx)$ where A,g, n are positive constants then $g = \sqrt{\frac{n}{2\mu}}$ (4.5)

UNIT -IV

Q8. (a) Find the volume bounded by the paraboloid $x^2 + y^2 = az$ the cylinder $x^2 + y^2 = 2ay$ and a plane z = 0. (6.5)

(b) Evaluate $\iint \frac{r\,dr\,d\theta}{\sqrt{a^2 + r^2}}$ over one loop of the lemniscate $r^2 = a^2 \cos 2\theta$ (6)

OR

Q9 (a) Evaluate $\iint r\sin\theta \, dr\, d\theta$ over the cardiode $r = a(1 - \cos\theta)$ above the initial line. (6.5)

(b) Calculate the area included between the curve $r = a(\sec\theta + \cos\theta)$ and its asymptate. (6)

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END TERM EXAMINATION

SECOND SEMESTER [BCA] MAY-2008

Paper Code: BCA102	Subject: Mathematics-II
Paper Id: 20102	(Batch: 2001-2004)

Time : 3 Hours Maximum Marks :75

Note: Part-A is compulsory. Attempt any five questions from Part-B.

PART-A

Q1 (a) Find the inverse of the following matrix $\begin{pmatrix} 1 & 3 & 2 \\ 2 & 2 & 0 \\ 3 & 1 & 2 \end{pmatrix}$. (3)

(b) Show that the conditional statement $p \to q$ and its contrapositive $\sim q \to \sim p$ are logically equivalent. (3)

(c) Draw Hasse diagram for the set of the factors of 100. (3)

(d) If a/bc and (a,b)=1, then show that a/c. (3)

(e) State Lagrange's Theorem and using this find all subgroups of a group of prime order. (1+2)

PART-B

Q2 (a) Solve the system of linear equations
$$x-2y+3z = 6$$
$$3x+y-4z = -7$$
$$5x-3y+2z = 5$$ (6)

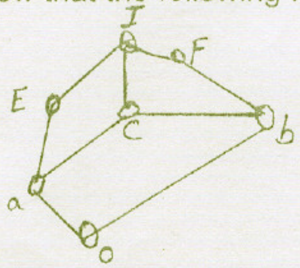
(b) Show that $2^n \neq 0(p(n))$, where p(n) is any polynomial. (6)

Q3 (a) Test the validity of the following argument: (6)
If 18486 is divisible by 18, then 18486 is divisible by 9.
If 18486 is divisible by 9, then the sum of the digits of 18486 is divisible by 9.
$\therefore$ If 18486 is divisible by 18, then the sum of the digits of 18486 is divisible by 9.

(b) If p is a prime number and a is an integer then prove that $a^p \equiv a \pmod p$. (6)

Q4 (a) Prove that a non-empty subset H of a group G is a subgroup of G if and only if $a, b \in H$ implies $ab^{-1} \in H$. (6)

(b) Define a lattice and show that the following figure represents a lattice: (6)



Q5 (a) How many numbers greater than a million can be formed with the digits 0,1,2,2,2,3,3? (6)

(b) Prove that:- (i) $n! = O(n^n)$ (ii) $\dfrac{(n^2+\log n)(n-1)}{(n+n^2)} = O(n)$ (3+3=6)

Q6 (a) For $1 \le r \le n$, prove that:- (3+3=6)

(i) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ (ii) $\dfrac{(2n)!}{n!} = 1.3.5\ldots(2n-1)2^n$.

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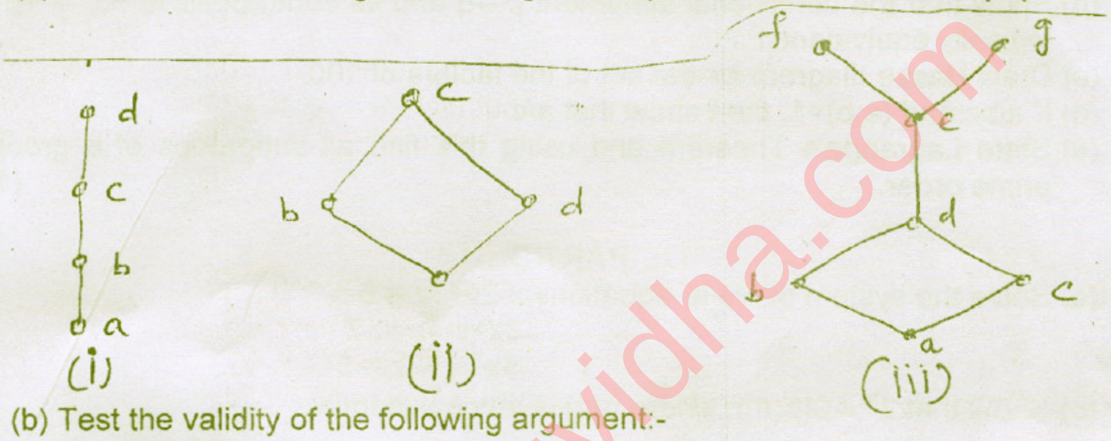
(b) Find the rank of the following matrix: $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & 0 & 3 \end{pmatrix}$. **(6)**

Q7 (a) Prove that intersection of two subgroups of a group G is also a subgroup of G. Is the union of two subgroups of a group G is also a subgroup of G. Justify your answer. **(3+3=6)**

(b) If $a|c$, $b|c$ and $(a,b)=1$, then prove that $ab|c$. Also show that the conclusion is false if $(a,b)\neq 1$. **(4+2=6)**

Q8 (a) Which of the following Hasse diagrams represents lattices? Justify. **(2+2+2=6)**



(b) Test the validity of the following argument:- **(6)**

$p \rightarrow (q \vee \sim r)$

$q \rightarrow (p \wedge r)$

$\therefore p \rightarrow r$

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END TERM EXAMINATION

SECOND SEMESTER [BCA] MAY-2008

Paper Code: BCA102	*Subject: Mathematics-II*
Paper Id: 20102	*(Batch: 2005-2007)*
Time : 3 Hours	*Maximum Marks :75*

Note: Q.1 is compulsory. Attempt one question from each section.

Q1 (a) Give all partitions of S={2,3,4}. **(2)**

(b) Let $f(x)=x^2-2$, $g(x)=3x$ and $h(x)=(x+1)^2$ be functions on R. Find goh, f^2og, g^3. **(3)**

(c) Let A={2,3,7,8}, B={1,3,5}, C={3,5,9,11} find (i) $B\oplus C$ (ii) $(A-B)\cup(B-C)$ (iii) $(AxB)\cap(BxB)$. **(3)**

(d) Give an example of a relation which is (i) neither symmetric nor anti-symmetric (ii) not irreflexive. **(3)**

(e) Give a topological sorting of the poset (D_{24}, |), where D_n denotes the set of all positive divisors of and | denote divides. **(3)**

(f) Give an example of an infinite lattice with finite length. **(2)**

(g) Find the angle between the line $\frac{x-3}{2}=\frac{y-1}{4}=\frac{z-2}{3}$ and the plane x-y+2z=3. **(3)**

(h) What is the shortest distance between two given lines? Also, give the equations of shortest distance. **(3)**

(i) Change the order of integration in $I=\int_0^{2a}\int_{x^2/4a}^{3a-x} f(x,y)dxdy$. **(3)**

SECTION-A

Q2 (a) Using set theory, prove the identity $(AxB)\cap(PxQ)=(A\cap P)x(B\cap Q)$. **(6.5)**

(b) Find whether the function $f:N\to N$ defined by $f(n)=n^2+n+1$ is invertible or not. **(6)**

Q3 (a) Given A={1,2,3,4,5,6}. Let R be a relation on A defined as R={(x,y); x+y is a divisor of 24} **(6.5)**

(i) Determine the matrix of relation R.

(ii) Find the composition R_oR.

(iii) Find the domain and range of R.

(iv) Compute transitive closure of R.

(b) Find the domain and range of the functions (i) $f(x)=\frac{1}{\sqrt{x-2}}$ (ii) $f(x)=\frac{|(x-3)|}{(x-3)}$ **(6)**

SECTION-B

Q4 (a) Consider the poset ({{1}, {2}, {4}, {1,2}, {1,4}, {2,4}, {3,4}, {1,3,4}, {2,3,4}}, $\subseteq$) **(6.5)**

(i) Find all maximal and minimal elements.

(ii) Find the first and last elements.

(iii) Find all the upper bounds of {{2}, {4}} and its supremum, if it exists.

(iv) Find all the lower bounds of {1,3,4} and its infimum, if it exists.

(b) In a distributive lattice, if an element has a complement then this complement is unique. **(6)**

Q5 (a) Consider a relation R on the set Z of all integers as follows $aRb\Leftrightarrow a+b$ is even for all $a, b\in Z$. Is R a partial order relation? Prove or give a counter example. **(4)**

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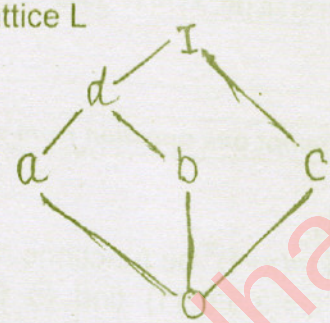
(b) Give an example of a relation, on the set {1,2,3} which is both a partial ordering and an equivalence relation. **(2.5)**

(c) Consider the bounded lattice L **(6)**



(i) Find all join-irreducible elements.
(ii) Find the atoms.
(iii) Is L complemented?
(iv) Is L distributive?

SECTION-C

Q6 (a) If $u = \sin^{-1}\left\{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}\right\}^{1/2}$ then show that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-1}{12}\tan u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(13 + \tan^2 u)$ **(6.5)**

(b) Find the equations of the spheres through the circle $x^2+y^2+z^2=5$, $x+2y+3z=3$ and touching the plane $4x+3y=15$. **(6)**

Q7 (a) Find the maxima and minima of the function $f(x,y)=x^3+y^3-63(x+y)+12xy$. **(6.5)**

(b) Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $4x-3y+1=0=5x-3z+2$ are coplanar. Also find their point of intersection. **(6)**

SECTION-D

Q8 By changing to polar co-ordinates evaluate $\iint_R (x^2+y^2)\,dxdy$ where R is the region in xy-plane bounded by $x^2+y^2=4$ and $x^2+y^2=9$. **(12.5)**

Q9 Find the volume of the sphere $x^2+y^2+z^2=a^2$. **(12.5)**

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END TERM EXAMINATION

SECOND SEMESTER [BCA] MAY-2010

Paper Code: BCA 102	Subject: Mathematics-II
Paper ID: 20102	

Time : 3 Hours Maximum Marks : 75

Note: The symbol R denotes the set of all real numbers.
Question 1 is compulsory. Attempt one question from each unit.

Q1. (a) Show that the equation of the tangent plane to the surface $2x^2 + y^2 + 2z - 3 = 0$ at the point (2, -1, -3) is $4x + y + z - 6 = 0$.

(b) Determine the projection of the vector $\hat{i} - 2\hat{j} + \hat{k}$ on $4\hat{i} - 4\hat{j} + 7\hat{k}$.

(c) If A= {1,2,3}, B={4, 5}, find AXB, BXA and (AXB)$\cap$ (BXA).

(d) Find the area enclosed by the pair of curves y= 2-x and y^2=2(2-x).

(e) Consider the functions of f: R$\to$R and g:R$\to$R defined as f(x)=x+1 and g(x)=x^2 for all x in R. Find g(f(x)) and f(g(x)) for all x in R.

(f) Give an example of an equivalence relation.

(g) Consider the function f:R$\to$R defined as f(x)=2x+1 for all x in R. Find a function g:R$\to$R such that g(f(x))=x for all x in R.

(h) Write the dual of the statement:
(a$\wedge$b) vc=(b$\vee$ c) $\wedge$ (c$\vee$ a).

(i) Let Q denote the set of all rational numbers with the usual order $\leq$. Prove that no element in Q has an immediate successor or an immediate predecessor.

(j) If x= r cos θ, y= r sin θ, find $\left[\frac{\partial r}{\partial x}\right]_y$, the partial derivative of r with respect to x,, treating y as constant. **(10x2.5=25)**

Unit-I

Q2. (a) If A = { 2, 3, 4}, B= {3, 4, 5} and C= {4, 5, 6}, find (A$\cap$B) X (B$\cap$C) and (AXB) $\cap$ (BXC). **(6)**

(b) Consider the function f: **R** $\to$**R** defined as f(x)=2x for all x in R. Prove
(i) f is one-to-one
(ii) f is onto

Also, find f(f(x)) and f(f(x^2)) for all x in R. **(6.5)**

Q3. (a) Let A = { 4, 5, 6, 7, 8} and B= {7, 8, 9, 10}. Verify that
A$\cup$ B= (A –B) $\cup$ (A$\cap$B) $\cup$ (B -A). **(6.5)**

Note: The notation E – F means the set of all the elements of E which do not belong to F.

(b) Define the cardinality of a set.
Let n(E) denote the cardinality of the set E.
Let A={1,5,6,9,11}, B={3,9,11} verify
n(A $\cup$ B)= n(A) + n(B) – n(A $\cap$B). **(6)**

Unit-II

Q4. (a) Let Z denote the set of all integers. Let a $\hat{R}$ b mean that b= a^r for some positive integer r. Show that $\hat{R}$ is a partial ordering of Z. **(4.5)**

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(b) Define the Hasse diagram of a finite partially ordered set.
Let A = { 1, 2, 3, 4, 6, 8, 9, 12, 18, 24} be ordered by the relation " x divides y". Draw the Hasse diagram of A. Determine a chain in A. **(2+4+2=8)**

Q5. (a) Let D_{36} denote the set of all divisors of 36 ordered by divisibility. Draw the Hasse diagram of D_{36}. **(4)**

(b) Let (L, ∧, ∨) be a lattice. Prove the following:
(i) $a \wedge b = a$ if and only if $a \vee b = b$ **(4)**
(ii) Let $a \leq b$ mean $a \wedge b = a$ or $a \vee b = b$. Then, ≤ is a partial order on L. **(4.5)**

Unit-III

Q6. (a) Verify Euler's theorem for the function **(6.5)**
$$\mu(x, y) = Sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right).$$

(b) Using the condition for the coplanarity of two lines, determine the values of λ and μ such that the points (-1, 3, 2), (-4, 2, -2) and (5, λ , μ) lie on a straight line. **(6)**

Q7. (a) If $f(x, y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log_e\left(\frac{x}{y}\right)}{x^2 + y^2}$, prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 2f = 0$. **(4.5)**

(b) If Z= $x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, *prove* that $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{x^2 - y^2}{x^2 + y^2}$. **(4)**

(c) Examine the function.
$z=f(x, y) = x^2 + y^2 + 6x + 12$ for maxima and minima. **(4)**

Unit-IV

Q8. Evaluate $\iiint_s (x^2 + y^2 + z^2)\, dxdydz$ over the region S bounded by the planes x=0, y=0, z=0, and x+y+z =a where a>0. **(12.5)**

Q9. (a) Change the order of integration in the double integral $\int_0^1 \int_{x^2}^{2-x} xy\,dxdy$ and evaluate the resulting double integral. **(7.5)**

(b) Evaluate the double integral $\int_0^{\Pi/2} \int_{a(1-\cos\theta)}^{a} r^2\, dr d\theta$. **(5)**

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