(Please write your Exam Roll No.) Exam Roll No. 06014202013

END TERM EXAMINATION

FIRST SEMESTER [BCA] DECEMBER 2013

Paper Code: BCA101	Subject: Mathematics-I (2011 onwards)

Time : 3 Hours **Maximum Marks :75**

Note: Attempt five questions including Q.no.1 which is compulsory. Select one question from each unit.

Q-1) (10 x 2.5=25)

a) If A= $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$ then what are the the eigen values of A^{-1}.

b) Prove that $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

c) Evaluate: $\lim_{x\to\infty} \frac{(3x^2+5x+7)}{(4x^2+4x+9)}$

d) At what points is the function $\frac{x}{(x-1)(x-2)}$ continuous?

e) Using chain rule, differentiate log $\{\sin(x^2+1)\}$

f) Find dy/dx if $y = \tan^{-1}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$

g) Evaluate $\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$

h) State Rolle's theorem

i) Integrate $\int \log x dx$

j) Evaluate the gamma function $\Gamma(7/2)$

UNIT-I

Q-2)

a) Find the eigen values and eigen vectors of A= $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ (6)

b) Examine the system of vectors for linear dependence, if so, find the relation between them. X_1= (1,-1, 1), X_2= (2, 1, 1), X_3= (3, 0, 2) (6.5)

Q-3)

a). Prove that the matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ satisfies its characteristic equation. Hence find A^{-1}. (6)

b) Test the consistency, of the given system of equations, using rank. Also, find the solution, if any

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x + 4y + 7z = 30$$

(6.5)

P.T.O.

UNIT-II

Q-4)

a) Evaluate the following limits:

(a) $\lim_{x\to 0}\left(\frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}\right)$ (b) $\lim_{x\to 0}\frac{(\sin 3x+7x)}{(4x+\sin 2x)}$ **(6)**

b) Discuss the type of discontinuities, if any, of the given function f(x) defined on [0, 1] at the points x= 0, ½, 1:

$f(0) = 0$
$f(x) = x+\frac{1}{2}, \; 0<x<\frac{1}{2}$
$f(\frac{1}{2}) = \frac{1}{2}$
$f(x) = 3x-\frac{1}{2}, \; \frac{1}{2}<x<1$
$f(1) = 1$ **(6.5)**

Q-5)

a) If $\lim_{x\to\infty}\left\{\frac{x^2+1}{x+1}-(ax+b)\right\}=2$, find a,b. **(6)**

b) Show that f(x) is discontinous at x=0, where

$$f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & x\neq 0 \\ 1, & x=0 \end{cases}$$ **(6.5)**

Also locate the type of discontinuity.

UNIT-III

Q-6)

a) Differentiate: x^{x^2} with respect to x. **(6)**

b) Find the n[th] derivative of sin(ax+b) and hence using Leibnitz theorem find the n[th] derivative of e^x sinx. **(6.5)**

Q-7)

a) Evaluate: (a) $\lim_{x\to 0}\frac{e^x-1-\log(1+x)}{x^2}$ (b) $\lim_{x\to 0}\frac{\log(\sin 3x)}{\log(\sin x)}$ **(6)**

b) Determine the maximum value of f(x) = sinx + cos x in (0,π/2) **(6.5)**

UNIT-IV

Q-8)

a) Integrate (i) $\int\frac{3x+2}{(x-1)(2x+3)}dx$ (ii) $\int\frac{x^2+1}{x^4+1}dx$ **(6)**

b) State and prove the relationship between beta and gamma functions. **(6.5)**

Q-9)

a) Express $\int_0^1 x^5(1-x^3)^{3/2}\,dx$ in terms of beta function. **(6)**

b) Obtain the reduction formula for $\int\sin^n x\,dx$ .Hence integrate $\int\sin^5 x\,dx$ **(6.5)**

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