(Please write your Exam Roll No.) Exam Roll No. 01914202012

(18)

END TERM EXAMINATION

FIRST SEMESTER [BCA] DECEMBER-2012

Paper Code: BCA101	**Subject: Mathematics-I**
Time : 3 Hours	**Maximum Marks :75**

Note: Attempt all questions. Internal choice is indicated.

Q1 (a) Define Symmetric and Skew Symmetric Matrix. Give an example.

(b) Find the Eigen values and Eigen vectors for the matrix $A=\begin{bmatrix}-5 & 2\\ 2 & -2\end{bmatrix}$.

(c) Evaluate the nth derivative of $\log(ax+b)$.

(d) Examine the following system of vectors for linearly dependence, if dependent, find the relation between them $X_1=(1,2,3)$; $X_2=(2,-2,6)$.

(e) Evaluate $\lim\limits_{x\to\theta}\dfrac{\sin^2 x-\sin^2\theta}{x^2-\theta^2}$.

(f) State Leibnitz's theorem.

(g) Find the reduction formula for $\int \tan^3 x\,dx$.

(h) Integrate $\int\dfrac{dx}{\sqrt{x}+\sqrt[3]{x}}$.

(i) Show that $f(x)=|x|+|x-1|$ is continuous at x=0 and x=1.

(j) Find the maximum and minimum values, if any of the given function, $f(x)=-|x-1|+5$ for all x∈R. **(2.5x10=25)**

UNIT-I

Q2 (a) If x-2y=4 and -3x+5y=-7 then solve using Cramer's Rule. **(5)**

(b) For what values of a and b do the equations- x+2y+3z=6, x+3y+5z=9, 2x+5y+az=b have (i) No solution (ii) A unique solution (iii)More than one solution. **(7.5)**

OR

(a) Reduce the matrix given below into normal form and find its Rank $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$. **(5)**

(b) Verify Cayley-Hamilton Theorem for the matrix $A=\begin{bmatrix}0 & 0 & 1\\ 3 & 1 & 0\\ -2 & 1 & 4\end{bmatrix}$, hence find A^{-1}. **(7.5)**

UNIT-II

Q3 (a) Evaluate $\lim\limits_{x\to 0}\dfrac{e^x-1}{\sqrt{1-\cos x}}=1$ and show that $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=1$ **(5)**

(b) Determine value of a, b, c if the function

$$f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x}, & if\ x<0\\ C, & if\ x=\text{[illegible]},\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & if\ x>\text{[illegible]}\end{cases}$$

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is continuous at x=0. **(7.5)**

OR

(a) Prove that $f(x)=\sin\frac{1}{x}$ is not continuous at x=0. Also, name the kind of discontinuity it has. **(5)**

(b) Find the value of 'a' if $f(x)=\begin{cases} 2x-1 & ; \quad x<2 \\ a & ; \quad x=2, \\ x+1 & ; \quad x>2 \end{cases}$

is continuous at x=2. **(7.5)**

UNIT-III

Q4 (a) For what choice of a and b, the function $f(x)=\begin{cases} x^2 & ; \quad x\le C \\ ax+b & ; \quad x>C \end{cases}$ is differentiable at x=c. **(5)**

(b) If $I_n=\int_0^{\pi/4}\tan^n x\,dx$, prove that $I_{n+1}+I_{n-1}=\frac{1}{n}$. Deduce the value of I_5. **(7.5)**

OR

(a) Given $y=x^x+(\sin x)^{\log_e x}$, find $\frac{dy}{dx}$. **(5)**

(b) Find the asymptotes of the curve $2y^3-2x^2y-4xy^2+4x^3+14xy+6y^2+4x^2+6y+1=0$. **(7.5)**

UNIT-IV

Q5 (a) Show that $\int_0^{\pi/2}\sin^p\theta\cos^q\theta\,d\theta=\frac{\Gamma(p+1)/2 . \Gamma\frac{(q+1)}{2}}{\Gamma(p+q+2)/2}$ where Γ denote gamma function. **(5)**

(b) Integrate (i) $\int\sec^3 x\,dx$ (ii) $\int\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$ (iii) $\int_0^a\frac{x^7}{\sqrt{a^2-x^2}}dx$. **(7.5)**

OR

(a) If $y=a\cos(\log x)+b\sin(\log x)$, then show that $x^2y_{n+2}+(2n+1)y_{n+1}x+(n^2+1)y_n=0$. **(5)**

(b) Prove Legendre's duplication formula $\lambda\Gamma(2m)=2^{2m-1}\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)$ where $\Gamma(n)=\int_0^\infty e^{-x}x^{n-1}dx$ denote the gamma function. **(7.5)**

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