(Please write your Exam Roll No.) *Exam Roll No.* 0552140 209

END TERM EXAMINATION

FIRST SEMESTER [BCA]– DECEMBER 2010

Paper Code: BCA101	***Subject: Mathematics-I***
Paper ID: 20101	

Time : 3 Hours ***Maximum Marks : 75***

Note: Q.1 is compulsory. Attempt one question from each unit.

Q1 (a) Define an orthogonal matrix. Give an example.

(b) Solve the matrix equation $\begin{pmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{pmatrix} = \begin{pmatrix} 0 & -7 \\ 3 & 2a \end{pmatrix}$ for x, y, z and a.

(c) Evaluate $\lim\limits_{x \to 0^-} \left(\frac{|x|}{x} \right)$.

(d) $\vec{a}$ and $\vec{b}$ are two non zero vectors such that $|\vec{a}| = |\vec{b}|$. Let $\vec{p} = \vec{a} + 2\vec{b}$ and $\vec{q} = 5\vec{a} - 4\vec{b}$. If $\vec{p}.\vec{q} = 0$, determine the angle between the vectors $\vec{a}$ and $\vec{b}$.

(e) Show that the function $f(x) = x|x|$ is derivable at x=0.

(f) If $y = \log_e \left[x + \sqrt{x^2 + a^2} \right]$, prove that $\frac{dy}{dx} = \frac{1}{\sqrt{x^2 + a^2}}$.

(g) Find the points of inflexion of the curve $(a^2 + x^2)y = a^2 x$.

(h) Using the substitution $\sqrt{x} = t$ and the definition $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$ of Gamma function of n, show that $\int_0^\infty x^{\frac{1}{4}} e^{-\sqrt{x}} dx = \frac{3}{2}\sqrt{\pi}$.

(i) Given vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, show that $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

(j) Prove that $x < \sin^{-1} x$ whenever 0<x<1. **(10x2.5=25)**

UNIT-I

Q2 (a) Express the matrix $A = \begin{pmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{pmatrix}$ as a sum of a symmetric and a skew-symmetric matrix. **(6..5)**

(b) If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, find the real numbers α and β such that $(\alpha I_2 + \beta A)^2 = A$ where I_2 denotes the identify matrix of order 2. **(6)**

Q3 (a) Find the characteristic equation of the matrix $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{pmatrix}$. Using Cayley-Hamilton Theorem, find A^{-1}. **(6)**

(b) Given the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$, find adj(A), A^{-1} and then solve the

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system of linear equations x+y+z=3, x+2y+3z=4, x+49+9z=6. **(6.5)**

UNIT-II

Q4 (a) Show that $\lim_{x \to 0} \frac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} = 0$. **(6)**

(b) Show that the function f defined on R as $f(x) = \frac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}$ if x≠0

$= 0$ if $x = 0$

is continuous from the right at x=0 and has a discontinuity of the first kind from the left at x=0. **(6.5)**

Q5 (a) State Intermediate Value Theorem. Show by means of an example, that if a function f:[a,b]→R is not continuous on [a,b], then the conclusion of the Intermediate Value Theorem may not hold. **(6.5)**

(b) Discuss the continuity of the function $f(x) = \begin{cases} \frac{x+|x|}{|x|} & if \quad x \neq 0 \\ 0 & if \quad x = 0 \end{cases}$ at x=0. **(6)**

UNIT-III

Q6 (a) Given $y = x^x + (\sin x)^{\log_e x}$, find $\frac{dy}{dx}$. **(6.5)**

(b) If $x = a(\theta - \sin\theta), y = a(1-\cos\theta)$, show that $\frac{d^2y}{dx^2} = -\frac{1}{(1-\cos\theta)^2}$. **(6)**

Q7 (a) Determine the values of p and q for which $\lim_{x \to 0}\left(\frac{x(1+p\cos x) - q\sin x}{x^3}\right)$ exists and equals 1. **(6.5)**

(b) Find the asymptotes of the curve $2y^3-2x^2y-4xy^2+4x^3-14xy+6y^2+4x^2+6y+1=0$. **(6)**

UNIT-IV

Q8 (a) (i) Show that $\int_0^\infty \frac{dx}{(1+x)\sqrt{1+x+x^2}} = \log_e 3$. **(3)**

(ii) Prove that $\int_0^{\pi/2} \log_e(\sin x)dx = -\frac{\pi}{2}\log_e 2$. **(3.5)**

(b) If $I_n = \int_0^{\pi/4} \tan^n x dx$, prove that $I_{n-1} + I_{n+1} = \frac{1}{n}$. Deduce the value of I_5. **(6)**

Q9 (a) Prove Legendre's Duplication Formula: $\sqrt{\pi}\,\Gamma(2m) = 2^{2m-1}\,\Gamma(m)\,\Gamma(m+\frac{1}{2})$

where $\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}dx$ denotes the Gamma function of n. **(6.5)**

(b) Let $\vec{A} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ $\vec{B} = 6\hat{i} + 3\hat{j} + \hat{k}$ $\vec{C} = 5\hat{i} + 7\hat{j} + 3\hat{k}$ $\vec{D} = 2\hat{i} + 2\hat{j} + 6\hat{k}$. Determine $\overrightarrow{AB}.(\overrightarrow{AC}x\overrightarrow{AD})$. **(6)**

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