(Please write your Exam Roll No.) Exam Roll No. 01314223007

65

END TERM EXAMINATION

FIRST SEMESTER [BCA] DECEMBER 2007

Paper Code: BCA101, Paper Id-201279 (Batch: 2005-2007)	Subject: Mathematics-I

Time : 3 Hours Maximum Marks :75

Note: Q.1 is compulsory. In the remaining paper, attempt one question from each unit.

Q1 (a) What is the difference between minor and co-factor? **(2.5x10=25)**

(b) State Cramers Rule.

(c) What is mean value theorem?

(d) Evaluate the limit $\lim_{x\to 0} \frac{(\log x)}{\cot x}$

(e) Write the various types of discontinuities.

(f) State Liebnitz theorem.

(g) Write the reduction formula for $\int x^n e^{ax} dx$.

(h) State the condition under which two vectors are parallel.

(i) Define Gamma and Beta functions.

(j) Write the partial fractions of $\frac{1}{(x+1)(2x+1)}$.

UNIT-I

Q2 (a) Show that $\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$. **(5)**

(b) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, find A^{-1}, and show that $A^3=I$ the unit matrix. **(6+1.5)**

Q3 (a) Solve x+2y+z=7, x+3z=11, 2x-3y=1, using Cramers rule. **(5)**

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ **(5+2.5)**

UNIT-II

Q4 (a) (i) Find the limit of the following $\lim_{x\to\sqrt{2}} \frac{\sqrt{3+2x}-(\sqrt{2}+1)}{x^2-2}$. **(4)**

(ii) $\lim_{x\to 0} \frac{\sqrt{4+x}-\sqrt{4-x}}{\sin^{-1} x}$ **(4)**

(b) Find the values of a and b, so that $f(x)=\begin{cases} x^2+ax+b & if \quad 0 \le x < 2 \\ 3x+2 & if \quad 2 \le x \le 4 \\ 2ax+5b & if \quad 4 < x \le 8 \end{cases}$

is continuous in [0,8]. **(4.5)**

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27

Q5 (a) Examine the continuity of the following (i) $f(x)=\begin{cases}3x-2, & x\le 0\\ x+1, & x>0\end{cases}$

at x=0. (3)

(ii) Show that $|x|+|(x-1)|$ is a continuous function. (3)

(b) Evaluate the limit of (i) $\lim\limits_{x\to 0}\dfrac{\tan^{-1}2x}{\sin 3x}$ (ii) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1-x}}$. (3+3.5)

UNIT-III

Q6 (a) (i) Find $\dfrac{d^2y}{dx^2}$, if $x=\dfrac{2at^2}{1+t}$, $y=\dfrac{3at}{1+t}$. (3)

(ii) Find $\dfrac{dy}{dx}$, if y=$(\sin x)^{\cos x}+(\cos x)^{Sinx}$. (3)

(b) Show that the rectangle of maximum area that can be inscribed in a circle of radius r is a square of side $\sqrt{2}$r. (6.5)

Q7 (a) Sketch the curve y=(x-1)(x-2)(x-3). (6.5)

(b) Find a point on y=x^2-6x+1 where the tangent is parallel to the chord joining (1,-4) and (3,-8). (6)

UNIT-IV

Q8 (a) Find the integral of the following: (i) $\int x\cos^{-1}x\,dx$, (ii) $\int\dfrac{x^2}{(1+x^3)(2+x^3)}dx$ (3+3)

(b) (i) Evaluate the following as beta function- $\int_0^1\dfrac{x^4}{\sqrt{1-x^2}}dx$ (3.5)

(ii) Prove that $[\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}]=[\vec{a},\ \vec{b},\ \vec{c}]^2$ (3)

Q9 (a) (i) Integrate as limit of sum, of the following $\int_0^4 e^x\,dx.$ (3.5)

(ii) Integrate the following- $I=\int\dfrac{x^2\,dx}{x^2+6x+12}$ (3)

(b) (i)Using vector analysis show that diagonals of a rhombus are perpendicular. (3)

(ii) using reduction formulae find, $\int\sin^7 x\,dx.$ (3)

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END TERM EXAMINATION

FIRST SEMESTER [BCA] DECEMBER-2008

Paper Code: BCA101	Subject: Mathematics-I
Paper Id: 20101	(Batch: 2005-2008)
Time : 3 Hours	Maximum Marks :75

Note: Q.1 is compulsory. Attempt one question from each unit.

Q1 (a) Define minor and cofactors in a square matrix.

(b) Evaluate the cofactor of 'a' in the determinant $\begin{vmatrix} 3 & -4 & -3 \\ 2 & 7 & a \\ 5 & -9 & 2 \end{vmatrix}$

(c) Evaluate 3A-4B where A=$\begin{bmatrix} 3 & -4 & 6 \\ 5 & 1 & 7 \end{bmatrix}$ and B=$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \end{bmatrix}$.

(d) Find the limit of the following $\lim_{x \to 0} \frac{\sin 2x + \sin 3x}{2x + \sin 3x}$.

(e) State Leibnitz's theorem.

(f) Define Beta and Gamma functions.

(g) Evaluate $\int_0^{\pi/2} Cos^6 x dx$.

(h) What is the difference between scalar and vector product of two vectors?

(i) Differentiate the following w.r.t. x.

$y=x^x$.

(j) Write partial fractions of $\frac{x-1}{(x+1)(x-2)}$. (2.5x10=25)

UNIT-I

Q2 (a) If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$, in which a,b,c are different, show that abc=1. (6)

(b) Determine the rank of the following matrix, $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$. (6.5)

Q3 (a) Verify the Cayley Hamilton theorem for the matrix A=$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse. Also express $A^5-4A^4-7A^3+11A^2-A-10I$ as a linear polynomial in A. (5)

(b) Find the eigen values and eigen vectors of the matrix A=$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ (5+2.5=7.5)

UNIT-II

Q4 (a) (i) Evaluate $\lim_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$. (ii) If $\lim_{x \to -a} \frac{x^9 + a^9}{x+a} = 9$, find value of a. (3+3.5)

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18

(b) Discuss the continuity of the function $f(x)=\begin{cases}2-x, & x<2\\ 2+x, & x\geq 2\end{cases}$ at $x=2$ **(6)**

Q5 (a) Show that $f(x)=\begin{cases}5x-2, & when\ 0<x\leq 1\\ 4x^3-3x, & when\ 1<x<2\end{cases}$ is discontinuous at x=1. **(5)**

(b) Find the value of a so that the function $f(x)=\begin{cases}\dfrac{\sin^2 ax}{x^2}, & x\neq 0\\ 1 & x=0\end{cases}$, may be continuous in R. **(7.5)**

UNIT-III

Q6 (a) (i) If $y=1+\dfrac{x}{1!}+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+......$ show that $\dfrac{dy}{dx}=y$. **(3)**

(ii) $y=e^x\sin x+x^x\cos x,\ w.r.t.\ x.$ **(3)**

(b) Find the altitude and the semi vertical angle of a cone of least volume which can be circumscribed to a sphere of radius a. **(6.5)**

Q7 (a) If $y=(\sin^{-1}x)^2$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$. **(5)**

(b) Find the asymptotes of the curve

$$y^3-2xy^2-x^2y+2x^3+3y^2-7xy+2x^2+2y+2x+1=0.$$ **(7.5)**

UNIT-IV

Q8 (a) Evaluate $\int_0^{\pi/2}\log\sin x\,dx$. **(6)**

(b) (i) Evaluate $\int_0^{\pi/4}\cos^4 3\theta\sin^3 6\theta\,d\theta$ **(3.5)**

(ii) Show that $\beta(p,q)=\int_0^\infty \dfrac{y^{q-1}}{(1+y)^{p+q}}dy=\int_0^1\dfrac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}dx$ **(3)**

Q9 (a) (i) In a triangle ABC, D,E,F are the midpoints of the sides BC, CA, AB, prove that $\Delta DEF=\Delta FCE=\dfrac{1}{2}\Delta ABC$, using vector analysis. **(3.5)**

(ii) Calculate area of triangle whose vertices are A(1,0,-1), B(2,1,,5) and C(0,1,2). **(3)**

(b) (i) Show that the points

$-6\hat{i}+3\hat{j}+2\hat{k},\ 3\hat{i}-2\hat{j}+4\hat{k},\ 5\hat{i}+7\hat{j}+3\hat{k}\ and\ -13\hat{i}+17\hat{j}-\hat{k}$ are coplanar. **(3)**

(ii) Using vector analysis find volume of the tetrahedron formed by the points, (1,1,1), (2,1,3), (3,2,2) and (3,3,4). **(3)**

a.(b×c)

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END TERM EXAMINATION

FIRST SEMESTER [BCA] DECEMBER-2009

Paper Code: BCA101	Subject: Mathematics-I
Paper Id-20101	(Batch: 2006-2009)
Time : 3 Hours	Maximum Marks :75

Note: Q.1 is compulsory. Attempt one question from each unit.

Q1 (a) Define Hermitian and Skew Hermitian matrix.

(b) Find minor and co factors of each element in the determinant $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

(c) Show that $\lim_{x\to 0} x\sin\frac{1}{x} = 0$.

(d) Find the volume of the parallelopiped whose edges are represented by A=2i-3j+4k, B=i+2j-k, C=3i-j+2k.

(e) State MeanValue Theorem.

(f) If A=$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then show that A^2-5A-2I=0, where I is the 2x2, identity matrix.

(g) Evaluate $\int_0^{\pi/2} \sin^8 x\cos^7 x dx$.

(h) If $x = t\log t$, $y = (\log t)/t$. Find dy/dx when t=1.

(i) Evaluate $\int \frac{(x+2)^3}{x^6} dx$

(j) State Cayley-Hamilton theorem. **(2.5x10=25)**

UNIT-I

Q2 (a) Solve the following sysem of equations using Cramer's rule.

$$x+y+z = 1$$
$$ax+by+cz = k$$
$$a^2x+b^2y+c^2z = k^2$$

(b) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ verify Cayley Hamilon theorem for A and find its inverse. **(6+6.5=12.5)**

Q3 (a) Define linear dependence and independence of vectors. Examine for linear dependence [1,0,2,1], [3,1,2,1], [4,6,2,-4], [-6,0,-3,-4] and find the relation between them, if possible. **(6.5+6=12.5)**

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

UNIT-II

Q4 (a) (i) Find $\lim_{x \to 0} \frac{e^{1/x}-1}{e^{1/x}+1}$, if it exist. (ii) Find $\lim_{x \to 0} \frac{e^x - 1}{\log(1+x)}$, *if* it exist.

(b) A function is defined in (0,3) in the following way:-
$f(x)=x^2$ when $0<x<1$, $f(x)=x$ when $1\le x<2$, $f(x)=(1/4)x^3$ when $2\le x<3$
Show that f(x) is continuous at x=1 and x=2. **(6.5+6=12.5)**

Q5 (a) Show that $f(x)=|x|+|x-1|$ is continuous at x=0 and x=1. **(6+6.5=12.5)**

(b) Show that the function f defined as $f(x)=\begin{cases} \frac{e^{1/x}}{e^{1/x}+1} & when\ x\neq 0 \\ 0 & when\ x=0 \end{cases}$ is not continuous at x=0.

UNIT-III

Q6 (a) If $x^m y^n=(x+y)^{m+n}$, prove that $\frac{dy}{dx}=\frac{y}{x}$. **(6+6.5=12.5)**

(b) If $x=\frac{2t}{1+t^2}, y=\frac{1-t^2}{1+t^2}$, prove that $\frac{dy}{dx}+\frac{x}{y}=0$.

Q7 (a) Trace the curve $r^2=a^2\cos 2\theta$. **(6+6.5=12.5)**
(b) Find all the asymptotes of the curve $r(\pi+\theta)=ae^{\theta}$.

UNIT-IV

Q8 (a) If $I_n=\int_0^{\pi/2}\tan^n x dx$, show that for n>1, $I_n+I_{n-2}=\frac{1}{n-1}$. Deduce the value of I_5. **(6+6.5=12.5)**

(b) (i) Evaluate $\int\frac{dx}{\sqrt{x}+\sqrt[3]{x}}$, (ii) $\int\left[\log\log x+\frac{1}{(\log x)^2}\right]dx$

Q9 (a) If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, then show that (i) $\nabla\left(\frac{1}{|\vec{r}|}\right)=-\frac{\vec{r}}{|\vec{r}|^3}$ (ii) $\nabla|\vec{r}|^n=n|\vec{r}|^{n-2}\vec{r}$ **(3+3.5)**

(b) (i) Find the tangent plane and normal line to the cone $z=\sqrt{x^2+y^2}$ at the point A $(1,1,\sqrt{2})$.
(ii) Find the directional derivative of $\phi=xy+yz+zx$ at (1,2,0) in the direction of $\hat{i}+2\hat{j}+2\hat{k}$. **(3+3)**

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