B.E.

Seventh Semester Examination, Dec-2008

OPERATIONS RESEARCH

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. Define Operation Research and discuss its usefulness in decision making process.

Ans. While it is difficult to mark the beginning of the operations research/management science, the scientific approach to management can be traced back to the area of industrial revolution and even to periods before that. But operations research, as it exists today, was born during the second world war when the British military management called upon a group of scientists to examine the strategies and tactics of various military operations with the intention of efficient allocation of scarce resources for the was effort. The name operational research was derived directly from the content in which it was used-research activity on operational area of the armed forces. British scientists spursed the American military management to similar research activities. Among the investigations carried out by them were the determination of (i) optimum convoy size to minimize losses from submarine attacks, (*ii*) the optimal way to deploy radar units in order to maximize potential coverage against possible enemy attacks and (iii) the invention of new flight patterns, and the determination of correct colour of the aircraft in order to minimize the chance of detection by the sub-marines.

After the war, operations research was adopted by the industry and some of the techniques that had been applied to the complex problems of war were successfully transferred and assimilated for use in the industrialised sector.

The dramatic development and refinement of the techniques of operations research and the advent of digital computers are the two prime factors that have contributed to the growth and application of OR in the post war period. In the 1950s OR was mainly used to handle management problems that were clear cut, well-structured and repetitive in nature. Typically, they were of a tactical and operational nature such as inventory control, resource allocation, scheduling of construction projects, etc.

Decision Making :

Primarily, OR is addressed to managerial decision-making or problem solving. A major premise of OR is that decision-making, irrespective of the situation involved, can be considered as a general systematic process that consists of the following steps :

(a) Select the alternative courses of action for consideration.

(b) Determine the model to be used and the values of the parameters of the process.

(c) Evaluate the alternatives and choose the one which is optimum.

(d) Define the problem and establish the criterion which will be used. The criterion may be the maximisation of profits, utility and minimisation of costs, etc.

Q. 2. (a) Solve the following LPP by graphical method

Maximize $X_1 + 3X_2$

Subject to $-X_1 + X_2 \geq 1$

$X_1 + X_2 \geq 2$; $X_1 \geq 0$, $X_2 \geq 0$.

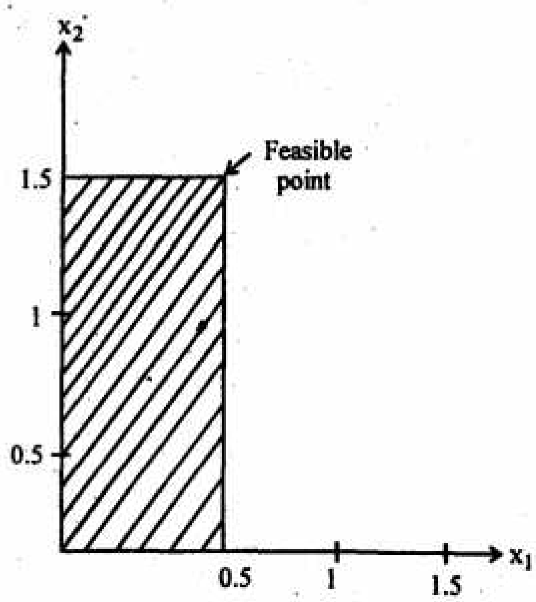
Ans. Maximize $x_1 + 3x_2$ ...(1)

Subject to

$-x_1 + x_2 \geq 1$ ...(2)

$x_1 + x_2 \geq 2$ ...(3)

$x_1 \geq 0$, $x_2 \geq 0$



To get the feasible points we have to solve equations (2) & (3) for x_1 & x_2 we get

$-x_1 + x_2 = 1 \quad \alpha - 1$

$x_1 + x_2 = 2 \quad \alpha 1$

We get after making the coefficient of x_1 similar

$$x_1 - x_2 = -1$$

$$x_1 + x_2 = 2$$

$$- \quad - \quad -$$

$$+2x_2 = 13$$

$$x_2 = \frac{3}{2} = 1.5$$

$$1.5 + x_1 = 2$$

$$x_1 = 2 - 1.5$$

$$x_1 = 0.5$$

Now put the values of x_1 & x_2 in equation (1) to get the maximize equation,

$$z = x_1 + 3x_2$$

$$z = 0.5 + 3 \times 1.5$$

$$z = 5.$$

Q. 2. (b) Write a note on duality in LPP. Write the dual of the above problem.

Ans. Corresponding to every given linear programming problem, there is another linear programming problem. The given problem is called the primal and the other, the related problem is known as the dual. The two problems are replicas of each other. When the primal problem is of the maximisation type, the dual would be of the minimisation type, and vice-versa. Thus, suppose the primal problem is,

Maximise subject to $\quad Z = cx$

$$ax \le b$$

$$x \ge 0$$

Where,

c = row matrix containing the coefficient in the objective function.

x = column matrix containing the decision variables.

a = matrix containing the coefficients in the constraints.

b = column matrix containing the RHS values of the constraints

The dual corresponding to this problem is defined as minimise subject to $G = b'y$

$$a'y \geq c'$$

$$y \geq 0$$

Where,

b' = transpose of the b matrix of the primal problem,

a' = transpose of the coefficients matrix of the primal problem,

c = column matrix of the objective function coefficients of the primal problem.

y = matrix of the dual variables.

Q. 3. (a) Solve by Dual Simplex method.

Maximize $-3X_1 - X_2$

Subject to $X_1 + X_2 \geq 1$

$2X_1 + 3X_2 \geq 2$

$X_1, X_2 \geq 0$

Ans. Maximize $-3x_1 - x_2$

Subject to $x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2 \qquad x_1; x_2 = 0$

Primal	Dual
Maximize $Z = -3x_1 - x_2$	Minimize $G = 1y_1 + 2y_2$
Subject to	Subject to
$x_1 + x_2 \geq 1$	$y_1 + 2y_2 \geq -3$
$2x_1 + 3x_2 \geq 2$	$y_1 + 3y_2 \geq -1$
$x_1, x_2 \geq 0$	$y_1, y_2 \geq 0$

The last two constraints can be merged to form an equation so that the dual can be expressed as given below corresponding to the primal given as,

Maximize $Z = -3x_1 - x_2$

Subject to $x_1 + x_2 \geq 1$

$$2x_1 + 3x_2 \geq 2 \qquad x_1, x_2 \geq 0$$

Now when we convert the primal part into the dual which is written as

Minimize $\qquad G = 1y_1 + 2y_2$

Subject to

$$y_1 + 2y_2 \geq -3$$

$$y_1 + 3y_2 \geq -1$$

$$y_1, y_2 \geq 0.$$

Q. 3. (b) Discuss the use of sensitivity analysis for post optimal problems.

Ans. Sensitivity :

Equally important as obtaining the optimal solution to a linear programming problem is the equation as to how the solution would be affected if the parameters b_i, c_j or a_{ij} of the problem change. This question is answered by the post-optimally analysis. Of course, we may directly substitute the changed values in a given situation and re-solve the problem to determine the effect on the optimal solution. It will be appreciated that it would be a lot advantageous if we could obtain this information directly from the tableau containing to final solution to the problem, without being required to carry out the whole exercise again. Let as to how can we get the information of this type. We shall illustrate by once again considering the example, whose optimal solution is given maximize subject to

$Z = 5x_1 + 10x_2 + 8x_3$	Profit
$3x_1 + 5x_2 + 2x_3 \leq 60$	Fabrication hours
$4x_1 + 4x_2 + 4x_3 \leq 72$	Finishing hours
$2x_1 + 4x_2 + 5x_3 \leq 100$	Packaging hours
$x_1, x_2, x_3 \geq 0$	

Q. 4. (a) Discuss the concept of MODI method.

Ans. Before we discuss the MODI method for testing the optimability of a transportation solution, a few words on the tracing of a closed loop follow.

Tracking a loop when a closed loop is to be traced, start with the empty cell which is to be evaluated. Then, moving clockwise, draw an arrow from this cell to an occupied cell in the same row or column, as the case may be. After that, more vertically or horizontally to other occupied cells before returning to the original empty cell. In the process of moving from one occupied cell to another (a) Move only horizontally or vertically, but never diagonally and (b) step over empty or occupied cells, if the need be, without changing them. Thus, a loop would always have right angled turns with corners only on the occupied cells.

Having traced the path, place plus and minus signs alternately in the cells on each turn of the loop, beginning with a plus(+) sign in the empty cell. An important restriction is that there must be exactly one cell with a plus sign and one cell with a minus sign in any row or column in which the loop takes a turn.

Q. 4. (b) Solve the assignment problem for optimal solution. Figures in the matrix indicate profits :

	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

Ans.

30	37	40	28	40
40	24	27	21	36
40	32	33	30	35
25	38	40	36	36
29	62	41	34	39

Subtract all rows and columns by min. no. which is 21.

9	16	19	7	19
19	3	6	0	15
19	11	12	9	14
4	17	19	15	15
8	41	20	13	18

Subtract column 1 by 4, columns 2 by 3, column 3 by 6 & column 5 by 14.

5	13	13	7	5
15	0	0	0	1
15	8	6	9	0
0	14	13	15	1
4	38	14	13	4

Subtract row 1 by 5 & row 5 by 4.

0	8	8	2	0
15	∞	∞	∞	1
15	8	6	9	∞
∞	14	13	15	1
0	34	10	9	0

The solution to above assignment problem is given by

$$25+24+27+21+35=132$$

Q. 5. Based on the following data, draw the network and determine the following :

(i) Critical path

(ii) Project completion time

(iii) Total float.

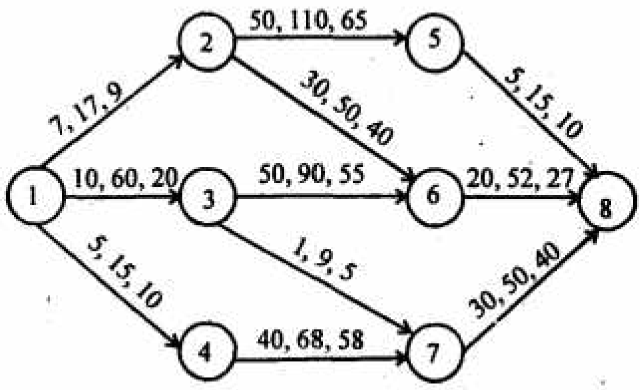
Activity	1–2	1–3	1–4	2–5	2–6	3–6	3–7	4–7	5–8	6–8	7–8
t_o	7	10	5	50	30	50	1	40	5	20	30
t_p	17	60	15	110	50	90	9	68	15	52	50
t_m	9	20	10	65	40	55	5	48	10	27	40

Ans.

	Time					
Activity	ta	tp	tm	$t_{e_i}=\frac{a+4m+b}{6}$	$\sigma_i=\frac{b-a}{6}$	σ_i^2
1–2	7	17	9	14	1.67	2.77
1–3	10	60	20	14	1.67	2.77
1–4	5	15	10	45	0.83	0.69
2–5	50	110	65	92.5	2.5	6.25
2–6	30	50	40	45	1.67	2.77
3–6	50	90	55	77.5	0.83	0.69
3–7	1	9	5	42	0.67	0.45

4–7	40	68	48	60	1.34	1.79
5–8	5	15	10	12.5	0.83	0.69
6–8	20	52	27	42.5	1.16	1.36
7–8	30	50	40	45	1.67	2.77

Network is drawn below :



(i) Using the expected times of activity duration, we obtain the critical path as $1-4-7-8$.

Thus, we have the expected project length,

$$T_e = 15+68+50$$
$$= 133.$$

(ii) And the variance of the project length,

$$V_T = 0.69+1.79+2.77$$
$$= 5.25$$

Now, the project duration being normally distributed with the mean

$$(T_e) = 133.$$

and the standard derivation.

$$\sigma = \left(\sqrt{V_T}\right)$$
$$= \sqrt{5.25}$$

$= 2.29$

(iii) Total float

t_o	t_p	Float
7	17	10
10	60	50
5	15	5
50	110	60
30	50	20
50	90	40
1	9	8
40	68	28
5	15	10
20	52	32
30	50	20

Q. 6. (a) Discuss Kendall's notations.

Ans. Kendall's notations :

In many real-world business applications, the linear programming problems of larger magnitude, involving a large number of decision variables and constraints, the task proves forbidding because of the computational effort needed. In many real-world business applications, the linear programming problems involved are indeed very complex and call for the use of computers for their solutions. A number of computer programmes have been developed to analyse such problems and even thousands of variables may be handled on a modern computer. It is interesting to observe that recently a sensational and revolutionary mathematical method for solving complex linear programming problems, the karmarkar algorithm, has been developed and applied on an international scale. The method has been evolved while working at the Bell laboratories. The karmarkar algorithm presents a revolutionary break through and is for superior, both in terms of the number of variables than can be handled and the computer time needed, to the simplex algorithm. A discussion of the karmarkar method, however, is beyond the scope of this book. When we compare this solution tableau with the optimal one of the original problem, we observe that the two are identical except that for the column headed x_1.

Q. 6. (b) At a bank counter, the customers arrive according to the Poisson process with mean 3 per hour. The time for servicing each customer is exponentially distributed with mean 4 per hour. The cashier can handle only one customer at a time. On the basis of this information compute the following :

(a) Probability that the facility will be idle.

(b) Waiting time in the system.

(c) Waiting time in the queue.

(d) Utilization of the service.

Ans. (a)
$$P(n) = e^{-m} \times \frac{m^n}{n!}$$

Where $m = \lambda T$ and $e = 2.7183$

With $\lambda = 3$, we have $m = 3 \times \frac{1}{4} = 0.75$ (i) $m = 3 \times \frac{1}{2} = 1.5$ for (ii)

n	$T = \frac{1}{4} h_r$	$T = \frac{1}{2} h_r$
	P(n)	P(n)
0	0.47	0.22
1	0.35	0.33
2	0.13	0.25
3	0.033	0.12
4	0.0098	0.046
5	0.0092	0.014

(ii) Waiting time in the system

$$W_s = \frac{1}{\mu - \lambda}$$
$$= \frac{1}{4-3}$$
$$= \frac{1}{1} = 1 \text{ hr}$$

(iii) Waiting time in the queue,

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$
$$= \frac{3}{4(4-3)}$$

$$= \frac{3}{4}$$

$$= 0.75$$

(iv) Utilization of the service

$$\rho = \frac{\lambda}{\mu}$$

$$= \frac{3}{4}$$

$$= 0.75.$$

Q. 7. (a) Discuss the methods of Monte Carlo simulation.

Ans. Monte Carlo simulation :

Although simulation can be a many types, our discussion will focus on the probabilistic simulation using the Monte Carlo method. Also called computer simulation, it can be described as a numerical technique that involves modelling a stochastic system with the objective of predicting the system's behaviour. The chance element is a very significant feature of Monte Carlo simulation and this approach can be used when the given process has a random or chance, component. In using the Monte Carlo method, a given problem is solved by simulating the original data with random numbers generators. Basically, its use requires two things. First, as mentioned earlier, we must have a model, that represents an image of the reality of the situation. Here the model refers to the probability distribution of the variable in question. What is significant here is that the variable may not be known to explicitly follow any of the theoretical distributions like Poisson. Normal, etc. The distribution may be obtained by direct observation or from past records.

Q. 7. (b) Discuss the types of decision making environment.

Ans. The decision situations where there is no way in which the decision maker can assess the probabilities of the various states of nature are called decisions under uncertainty. In such situations, the decision maker has no idea at all as to which of the possible states of nature would occur nor has he a reason to believe why a given state is more, or less, likely to occur as another. With probabilities of the various outcomes unknown, the actual decisions are based on specific criteria. The several principles which may be employed for taking decisions in such conditions are discussed below :

(i) Laplace principle :

The Laplace principle is based on the simple philosophy that if we are uncertain about the various events then we may treat them as equally probable.

(ii) Maximum or Minimax Principle :

This principle is adopted by pessimistic decision makers who are conservative in their approach.

(iii) Maximum or minimum principle :

The maximax principle is optimists principle of choice. It suggests that for each strategy, the maximum profit should be considered.

(iv) Hurwicz principle :

The Hurwicz principle of decision making stipulates that a decision maker's view may full somewhere between the extreme pessimism of the maximum principle.

(v) Savage principle :

The savage principle is based on the concept of regret and calls for selecting the course of action that minimizes the maximum regret.

Q. 8. Write notes on :

(a) SIMON Model

(b) Applications of simulation

(c) Travelling salesman problem.

Ans. (a) SIMON Model :

Once the problem is defined, the next step is to build a suitable model. As has already been mentioned, the concepts of models and model building lie at the very heart of the operations research approach to problem solving. A model is a theoretical abstraction of a real life problem. In fact many real life situations tends to be very complex because there are literally innumerable inherent factors in any given situation. Thus, the decision maker has to abstract from the empirical situation those factors which are most relevant to the problem. Having selected the critical factors, he combines them in some logical manner so that they form a counterpart or a model of the actual problem. Having selected the critical factors, he combines them in some logical manner so that they form a counterpart of or a model of the actual problem. Thus, a model is a simplified representation of a real world situation that ideally, strips a natural phenomenon of its bewildering complexity.

(b) Applications of simulation :

Inspite of its limitations, simulation is a very potent, flexible and therefore, a commonly employed quantitative tool for solving decision problems. It has been applied successfully to a broad spectrum of problems. To count a few, its use extends to areas like police dispatching and beat design, location of emergency vehicles like ambulances, making inventory policy decisions, evaluation of operatives alternatives at airports. In financial planning-both portfolio selection and capital budgeting, scheduling the production processes; large scale military battles as well as individual weapons systems for aiding in the designing of both the weapon systems and of the strategic and tactial operations. Simulation is of paramount importance where the experimentation with the real situation is riskly. The use of computer simulation for studying the likely behaviour of the nuclear reactors during accidents for studying.

(c) Travelling salesman problem :

The travelling salesman problem is like this. A salesman is assigned n cities to visit. He is given distances between all pairs of cities and instructed to visit each of the cities once in a continuous trip and return to the origin, using the shortest route. In this context, we define the visit to the cities in a sequence, ending at the same city where it begins including a visit to each of the cities only once, as a tour. Since a complete cycle is involved, it makes no difference general algorithm available for its solution. There are a large number of solving methods but all are enumerative to some degree or another and hence become impractical for problems of large size.