BE.

Seventh Semester Examination, Dec-2007

O P F P ATTONS R F S F A R C H

Note : Attempt any five question.

Q. What is the role of decision-making in O.R.? Define scientific decision-making and explain how it affects O.R. decisions?

Ans. Role of decision making in orperation research :

Decision making is an essential and dominating part of the management process. Although authorities sometimes differ in their definitions of the basic functions of management, every body agrees that one is not a manager unless he has some authority to plan, organize and control the activities of an enterprise and behaviour of the others. Within this context, decision-making may be viewed as the power to determine what plans will be made and how activities of a enterprise and behaviour of the others. Within this context, decision-making may be viewed as the power to determine what plans will be made and how activities will be organized and controlled. The right to make decisions is an integral part of right of authority upon which the entire concept of management rests. Essentially then, decision making pervades the activities of every business manager. Further, since to carry out the key managerial functional of planning, organizing, directing and controlling, the management is engaged in a continuous process of decision-making pertaining to each of them, we can go to the extent to saying that management may be regarded as equivalent of decision making.

Traditionally, decision making has been considered purely as an art, a talent which is acquired over a period of time through experience. It has been considered so because a variety of individual styles can be traced in handling and successfully solving similar type of managerial problems in actual business. However, the environment in which the management has to operate nowadays is complex and fast changing. There is a greater need for supplementing the art of decision making by systematic and scientific methods. A systematic approach to decision making is necessary because today's business and the environment in which it functions are for more complex than in the past and the cost of making errors is becoming graver with time.

The various types of decision-making situations that a manager might encounter can be listed as follows :

1. Decision under certainty where all facts are known fully and for sure or uncertainty where the event that would actually occur is not known but probabilities can be assigned to various possible occurrence.

2. Decision for one time-period only called static decision, or a sequence of interrelated decisions made either simultaneously or over several time periods called dynamic decisions.

3. Decisions where the opponent is nature or a rational opponent.

The elements of any decisions are :

(a) A decision-makes who could be an individual, group, organisation or society.

(b) A set of possible states that might occur.

(c) A set of consequences associated with various combinations of courses of action.

(d) The relationship between the pay-offs and the values of the decision-maker.

(e) A set of possible actions that may be taken to solve the decision problem.

Q. 2. Three grades of coal A, B and C contain phosphorus and ash as impurities. In a particular industrial process, fuel upto 100 ton (maximum) is required which should contain ash not more than 3% and phosphorus not more than 0–0.3%. It is desired to maximize the profit while satisfying these conditions. There is an unlimited supply of each grade. The percentage of impurities and the profits of grades are given below :

Coal	Phosphorus (%)	Ash (%)	Profit in rupees/ton
A	0.02	2	12
B	0.04	3	15
C	0.03	5	14

Find, using the simplex method, the proportions in which the three grades be used to maximize the total profit.

Ans. Max. $Z = 12x_1 + 15x_2 + 14x_3$

Subjected to

$$0.02x_1 + 0.04x_2 + 0.03x_3 \le 0.3$$

$$2x_1 + 3x_2 + 5x_3 \le 3$$

$$x_1, x_2, x_3 \ge 0$$

Or

$$2x_1 + 4x_2 + 3x_3 \le 30$$

$$2x_1 + 3x_2 + 5x_3 \le 3$$

$$x_1, x_2, x_3 \ge 0$$

Integrating the slack variables, we get

$$\text{Max.}, Z = 12x_1 + 15x_2 + 14x_3 + 0.s_1 + 0.s_2$$

Subjected to

$$2x_1 + 4x_2 + 3x_3 + s_1 = 30$$

$$2x_1 + 3x_2 + 5x_3 + s_2 = 3$$

		12	15	14	0	0		
C_i	B	x_1	x_2	x_3	s_1	s_2	b	
0	s_1	2	4	3	1	0	30	$\frac{30}{4} = 7.5$
0	s_2	2	3	5	0	1	3	= 13
		12	15	14	0	0		
			↑					

		12	15	14	0	0		
C_i	B	x_1	x_2	x_3	s_1	s_2	b	
0	s_1	$-\frac{2}{3}$	0	$-\frac{11}{3}$	1	−4	18	−12
15	x_2	$\frac{2}{3}$	1	$\frac{5}{3}$	0	$\frac{1}{3}$	1	$\frac{3}{2}$ →
		$\frac{112}{9}$		−1	0	−5		
		↑						

C_i	B	x_1	x_2	s_1	s_2	b	
0	s_1	0	1	−2	1	$-\frac{11}{3}$	19
12	x_1	1	$\frac{3}{2}$	$\frac{5}{2}$	0	$\frac{1}{2}$	$\frac{3}{2}$
		0	−3	−6	0	−6	−18

$$x_1 = \frac{3}{2}$$

$$x_2 = 0$$

$$x_3 = 0$$

$$\text{Max., } Z = 12 \times \frac{3}{2} = \text{Rs.18}$$

Profit on 1 ton = Rs. 18

Profit on 100 ton = Rs. 1800

Q. 3. A small garment making unit has five tailors stitching five different types of garments. All the five tailors are capable of stitching all the five types of garments. The output per day per tailor and the profit (Rs.) for each type of garment are given below :

Tailors	Garments				
	1	2	3	4	5
A	7	9	4	8	6
B	4	9	2	7	8
C	8	5	2	9	8
D	6	5	8	10	10
E	7	8	10	9	9

(i) Find optimal assignments of garments to tailors.

(ii) If tailor D is abset for a specified period and no other tailor is available, what should the optimal assignments?

Ans.

	Garments				
Tailors	1	2	3	4	5
A	7	9	4	8	6
B	4	9	2	7	8
C	8	5	2	9	8
D	6	5	8	10	10

	E	**7**	**8**	**10**	**9**
Profit (Rs).		**2**	**3**	**2**	**3**
per garment					

The above problem is an minimisation problem to make it maximise problem we have to multiply the respective row with their respective profit. We get,

	1	2	3	4	5
A	14	27	824	24	
B	8	27	4	21	32
C	16	15	4	27	32
D	12	15	16	30	40
E	14	24	20	27	36

Using Hungerian method subtruct the each element with the maximum number, we get

	1	2	3	4	5
A	26	13	32	16	16
B	32	13	36	19	8
C	24	25	36	13	8
D	28	25	24	10	0
E	26	16	20	13	4

Subtract each row with the maximum element

	1	2	3	4	5
A	6	19	0	16	16
B	4	23	0	17	28
C	12	11	0	23	28
D	0	3	4	18	0

E	0	10		13	22

Subtract each column with the maximum element

	1	2	3	4	5
A	6	4	0	7	16
B	14	0	0	6	28
C	12	12	0	0	28
D	0	20	4	5	0
E	0	13	6	10	22

Optimal assignment of garments to tailors

$$= 4+9+9+10+7 = 39$$

(b) When tailor D is absent

	1	2	3	4	5
A	14	27	8	24	24
B	8	27	4	21	32
C	16	15	4	27	32
D	0	0	0	0	0
E	14	24	20	27	36

Using Hungerian method subtract the each element with the maximum number, we get,

	1	2	3	4	5
A	22	9	28	12	12
B	28	9	32	15	4
C	20	21	32	9	14
D	0	0	0	0	0
E	22	12	16	9	0

Subtruct each row with the maximum element,

	1	2	3	4	5
A	6	19	0	16	16

B	4	23	0	17	28
C	12	11	0	23	28
D	0	0	0^+	0	0
E	0	10	6	13	0

Using Hungerian method since 10 is smallest number

	1	2	3	4	5
A	6	9	0	6	16
B	4	0	0	7	28
C	12	1	0	13	0
D	10	0	10	0	10
E	0	0	6	3	0

Optimal assignment of garment to tailors

$$= 4 + 9 + 8 + 10 + 7$$

$$= 38.$$

Q. 4. (a) Write short notes on :

(i) Shadow prices of resources.

(ii) Economic interpretation of dual variables.

Ans. (i) Shadow prices of resources :

The individual rental rates, y_1 and y_2, the dual variables corresponding to the slack variables, are called the shadow prices or imputed prices, indicating the worth of the resources. These prices, assigned to the services of the two resources, materials and labour hours, are imputed from the profit obtained from utilizing their services and bear no relationship with the original cost of the two.

(ii) Economic interpretation of dual variables :

Not only the primal and the dual related mathematically, they are also related in economic sense. We shall now consider the economic interpretation of the dual-first or a maximisation and then for a minimization problem.

The maximization problem,

$$\text{Maximize } Z = 40x_1 + 35x_2 \text{ profit}$$

Subject to,

$$2x_1 + 3x_2 \leq 60 \text{ Raw mtl. constraint}$$

$$4x_1 + 3x_2 \leq 96 \text{ Labour hours constant}$$

The optimal solution of this problem indicates that producing 18 units of A and 8 units of B per week would yield the maximize profit equal to Rs. 1000.

Q. 4. (b) A company wants to produce three products A, B and C. The unit profits on these products are Rs. 4, Rs. 6 and Rs. 2 respectively. These products require two types of resources manpower and material. The following L.P. model is formulated for determining the optimal product mix :

$$\textbf{Maximize } Z = 34x_1 + 6x_2 + 2x_3$$

$$\textbf{Subject to } x_1 + x_2 + x_3 \leq 3 \textbf{ (man-power)}$$

$$x_1 + 4x_2 + 7x_3 \leq 9 \textbf{ (material)}$$

$$x_1, x_2, X_3 \geq 0$$

where x_1, x_2 and x_3 are the number of products A, B and C produced.

(i) Find the optimal product mix and the corresponding profit.

(ii) What happens if profit of product C is increased to Rs. 12? What is the new product mix in this case?

Ans. Maximum $Z = 4x_1 + 6x_2 + 2x_3$

Subjected to

$$x_1 + x_2 + x_3 \leq 3 \text{ (man-power)}$$

$$x_1 + 4x_2 + 7x_3 \leq 9 \text{ (material)}$$

$$x_1, x_2, x_3 \geq 0$$

By duality we have,

$$\text{Maximum } Z = 3y_1 + 9y_2$$

$$\text{S.T. } y_1 + y_2 \leq 4$$

$$y_1 + 4y_2 \leq 6,$$

$$y_1 + 7y_2 \leq 2$$

$$y_1, y_2 \geq 0.$$

Q. 5. (a) Discuss basic elements of waiting line situations.

Ans. Basic elements of waiting line situations :

1. Arrival process :

The arrivals from the input population may be classified on different bases as follows :

(a) According to source

(b) According to numbers

(c) According to time.

2. Service system :

There are two aspects of a service system : (a) Structure of the service system and -(a) Structure of the service system and

(b) The speed of service.

(a) Structure of the service system : There are several possibilities. For example there may be

(i) A single service facility.

(ii) Multiple, parallel facilities with single queue.

(iii) Mutliple, parallel facilities with multiple queues.

(iv) Service facilities in a series.

(b) Speed of service :

In a queuing system, the speed with which service is provided can be expressed in either of two ways as service is provided can be expressed in either of two ways as service rate and as service time.

3 Queue structure :

There are number of possibilities are :

(a) First come first served.

(b) Last come first served.

(c) Service in random order

(d) Priority service

Q. 5. (b) In the central railway station 15 computerized reservation counters are available. A customer can book his/her ticket in any train on any day in any one of these counters. The average time spent per customer by each clerk is 5 minutes. Average arrivals per hour during three types of activity periods have been calculated and customers have been surveyed to determine how long they are willing to wait during each type of period.

Type of period	Arrivals/hour	Customer's acceptable waiting time (minutes)
Peak	110	15
Normal	60	10
Low	30	5

Making suitable assumptions on this queuing process, determine how many counters should be kept open during each type of activity period?

Ans. C, Number of counters = ?

$$\mu = \frac{1}{5} \times 60 = 12 \text{ per hour}$$

During peak load,

$$\lambda = 110$$

$$\mu = 12$$

$\frac{\lambda}{C\mu} < 1$, that is, the mean arrival rate must be less than maximum potential service rate of the system.

$$\frac{9.166}{C} < 1$$

$$\boxed{C = 10}$$

Minimum number of counters required during peaks load are 10.

During Normal Load :

$$\frac{\lambda}{\mu} = \frac{60}{12} = 5$$

$$\frac{5}{c} < 1$$

$$\boxed{C = 6}$$

Minimum number of counters required during normal load are 6.

During load : $\frac{\lambda}{\mu} = \frac{30}{12} = 2.5$

$$\frac{2.5}{c} < 1$$

$$\boxed{C = 3}$$

Minimum number of counters required during low load are 3.

$$\lambda(\text{mean}) = \frac{110 + 60 + 30}{3} = 66.67$$

$$\frac{\lambda}{\mu} = \frac{66.67}{12} = 5.55$$

Considering the mean load the number of counters required are.

$$\frac{5.55}{C} < 1$$

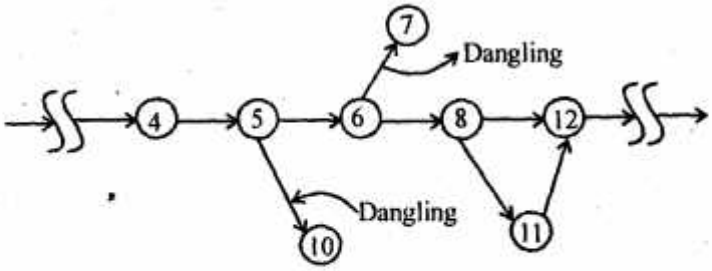
$$\boxed{c = 6}.$$

Q. 6. (a) Discuss the various defects in networks.

Ans. Three types of errors are most commonly observed in drawing network diagrams.

1. Dangling :

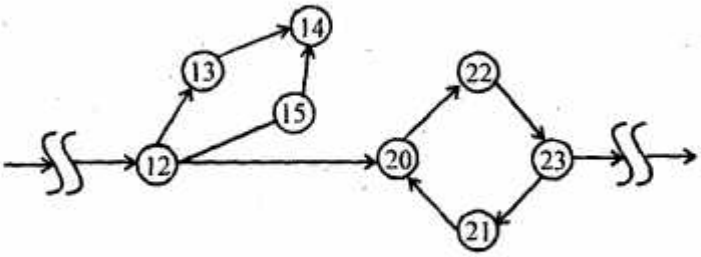
To disconnect an activity before the completion of all activities in a network diagram is known as dangling. As shown in figure, activities (5–10) & (6–7) are not +ve last activities in the network. So the diagram is working and indicates the errors of dangling.



Dangling error diagram.

2. Looping (or cycling) :

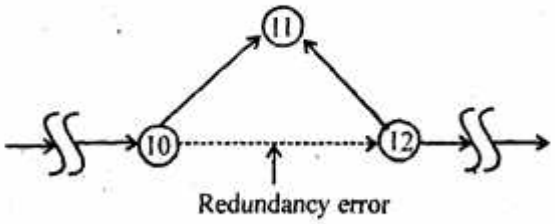
Looping error is also known as cycling error in a network diagram. Drawing an endless loop in a network is known as an error of looping as shown in the following figure :



Loop in (or cycling) error diagram

3. Redundancy :

Unnecessarily inserting the dumming activity in a network logic is known as the error of redundancy as shown in the following diagram.



Redundancy error

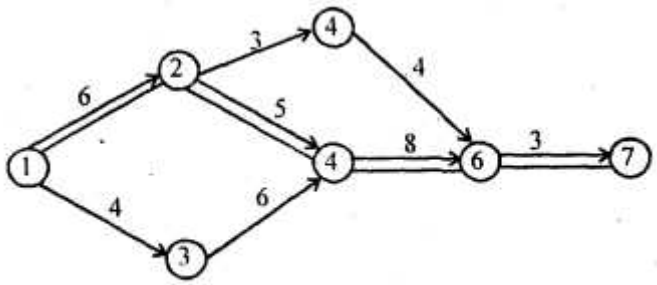
Q. 6. (b) The following table gives data on normal time & cost and crash time & cost for a project :

Activity	Normal Time (days)	Crash Cost (Rs.)	Time (days)	Cost (Rs.)
1–2	6	60	4	100
1–3	4	60	2	200
2–4	5	50	3	150
2–5	3	45	1	65
3–4	6	90	4	200
4–6	8	80	4	300
5–6	4	40	2	100
6–7	3	45	2	80

The indirect cost per day is Rs. 10. Determine the minimum total time and corresponding cost.

Ans.

Activity	Normal Time (days)	Crash Cost (Rs.)	Time (days)	Cost (Rs.)
1–2	6	60	4	100
1–3	4	60	2	200
2–4	5	50	3	150
2–5	3	45	1	65
3–4	6	90	4	200
4–6	8	80	4	300
5–6	4	40	2	100
6–7	3	45	2	80



The critical path is 1–2–4–6–7

The normal cost project duration is 22 days

s it is associated cost as follows :

Direct cost of the project = Rs. 1195

Indirect cost $= 10 \times 22 = 220$ $= 10 \times 22 =$ Rs.220

Total project cost = Rs. 1415

Minimum total time = 22 days.

Q. 7. (a) What is simulation? Describe the simulation process. What are the reasons for using simulation?

Ans. Simulation :

Using simulation, an analyst can introduce the constants and variables related to the problem, setup the possible courses of action and establish criteria which act as measures of effectiveness. The benefit of simulation from the viewpoint of the analyst stems from the fact that the results of taking a particular course of action can be estimated prior to its implementation in the real world. Instead of using hunches and intuition to determine what may happen, the analyst using simulation can test and evaluate various alternatives and select the one that gives the best results.

Process of simulation :

Broadly, there are four phases of the simulation process.

They are :

(a) Definition of the problem and statement of objectives.

(b) Construction of an appropriate model.

(c) Experimentation with the model constructed and

(d) Evaluation of the results of simulation.

The last step in the process of simulation is to analyse and interpret the results of the runs. The interpretaion of results is, in a large measure dependent on the extent to which the simulation model port rays the reality.

Obviously, closer the approximation of the real system by the simulation model, lesser will be the need for adjusting the result and also lesser will be the risk inherent in applying the results.

Q. 7. (b) A dentist schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following table shows the relevant data :

Category	Time required (minutes)	Probability of Category
Filling	45	0.40
Crown	60	0.15
Cleaning	15	0.15
Extraction	45	0.10
Checkup	15	0.20

Simulate the dentist's clinic for 4 hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival times, starting at 8 A.M. Use the following random numbers for handling the above problem :

40, 82, 11, 34, 25, 66, 17 and 79.

Ans. If the numbers 00–99 are allocated in proportion to the probabilities associated with each categories of work, hen various kinds of dental work can be sampled, using random number table :

Category :

Filling	Grain	Cleaning	Extraction	Checkup	
Probability	0.40	0.15	0.15	0.10	0.20
Cum. Probability	0.40	0.55	0.70	0.80	1.00
Random nos :	00–39	40–54	55–69	70–79	80–99

Using the given random numbers, a worksheet can now be completed as follows :

Patient	Scheduled arrival	Random no.	Category	Service time (min)
1	8.00 A.M.	40	Grain	60
2.	8 : 30 A.M.	82	Checkup	15
3.	9 :00 A.M.	11	Filling	45
4.	9 : 30 A.M.	34	Filling	45
5.	10 : 00 A.M.	25	Filling	45
6.	10:30 A.M.	66	Cleaning	15
7.	11 : 00 A.M.	17	Filling	45
8.	11 : 30 A.M.	79	Extraction	45

Now let us simulate the dentist's clinic for four knows starting from 9 ; 00 A.M.

Time	Event (Arrival/Departure)	Patient No. (service time)	Patients
8 : 00 A.M.	Ist Arrives	Ist (60 min.)	–
8 : 30 A.M.	2nd arrives	Ist (30 min)	2nd
9 : 00 A.M.	Ist departs, 3rd arrives	2nd (15 min)	3rd
9 : 15 A.M.	2nd departs	3rd (45 min)	–
9 : 30 A.M.	4th arrives	3rd (30 min.)	4th
10 : 00 A.M.	3rd departs, 5th arrives	4th (45 min.)	5th
10 : 30 A.M.	6th arrives	4th (15 min)	5th & 6th
10 : 45 A.M.	4th departs	5th (45 min.)	6th
11 : 00 A.M.	7th arrives	5th (30 min.)	6th & 7th
11 : 30 A.M.	5 departs, 8th arrives	6th (15 min.)	7th & 8th
11 : 45 A.M.	6th departs	7th (45 min.)	8th

12 : 00 P.M.	End	7th (30 min.)	8th
12 ; 30 P.M.		8th (45 min.)	

This table shown that the dentist wear not idle during the entire stimulated period.

The waiting times for the patients were as follows :

Patient :	1	2	3	4	5	6	7	8
Arrival :	8.00	8.30	9.00	9.30	10.00	10.30	11.00	11.30
Service starts :	8.00	9.00	9.15	10.00	10.45	11/30	11/45	12.30
Waiting (min.)	0	30	15	30	45	60	45	60
								Total 280

The average waiting time of a patient was $\frac{285}{8} = 35.625$ min.

Q. 8. (a) What is decision theory? Describe some methods which are useful in decision-making under uncertainty.

Ans. Decision theory : The decision theory, also called the decision analysis, is used to determine optimal strategies where a decision maker is faced with several decision alternatives and an uncertain, or risky, pattern of future events. To recapitulate, all decision making situations are characterized by the fact that two or more alternate courses of action are available to be the decision maker to choose from. Further, a decision may be defined as the selection by the decision maker of a act, considered to be the best according to some preesignated standard, from among the available options.

Decision making under uncertainty :

(a) Laplace principle : The Laplace principle is based on the simple philosophy that if we are uncertain about the various events then we may treat them as equally probable.

(b) Maximin or minimax principle : This principle is adopted by pessimotic decisions makers who are conservative in their approach.

(c) Maximax or minimin principle : The maximax principle is optimists principle of choice. It suggests that for each strategy, the maximum profit should be considered and the strategy with which the highest of these values is associated should be choosen.

(d) Herwicz principle : The Hururiz principle of decision making stipulates that a decision maker's view may fall somewhere between the extreme pessimism of the maximin principle and the extreme optimize of the

Q. 8. (b) Under an employment promotion programme, it is proposed to allow sale of newspapers on the buses during off-peak hours. The vendor can purchase the newspapers at a special concenssional rate of 25 paise per copy against the selling price of 40 paise. Any unsold copies are however, a dead loss. The vendor has estimated the following probability distribution for the number of copies demanded :

Number of copies :	**15**	**16**	**17**	**18**	**19**	**20**
Probability :	**0.04**	**0.19**	**0.33**	**0.26**	**0.11**	**0.07**

(i) How many copies should he order so that this expected profit will be maximum?

Ans.

Number of copies :	15	16	17	18	19	20
Probability :	0.04	0.19	0.33	0.26	0.11	0.07

$$IP = 0.40 - 0.25$$

$$= 0.15$$

$$IL = 0.25 - 0 = 0.25$$

$$P = \frac{IL}{IP + IL} = \frac{0.15}{0.15 + 0.25} = \frac{0.15}{0.40} = 0.375$$

It implies that in order to justify the stacking of an additional unit there must be atleast 0.375 cumulative probability of selling that unit. See the result as shown below :

Numberof copies	Probability	Cumulative probability
15	0.04	1.00
16	0.19	0.96
17	0.33	0.77
18	0.26	0.44
19	0.11	0.18
20	0.07	0.07

Therefore, the optimal decision in to order 18 copies.

Q. 8. (b) (ii) Compute EPPI.

Ans. Conditional profit table ;

Possible action

Number of copies			Probability	15	16	17	18	19	20
15	0.04	15×0.15	15×0.15	15×0.15	15×0.15	15×0.15	15×0.15		
16	0.19	15×0.15	16×0.15	16×0.15	16×0.15	16×0.15	16×0.15		
		−0.25×1							
17	0.33	15×0.15	16×0.15	17×0.15	17×0.15	17×0.15	17×0.15		
		−0.25×2	−0.25×2						
18	0.26	15×0.15	16×0.15	17×0.15	18×0.15	18×0.15	18×0.15		
		−0.25×3	−0.25×2	−0.25×1					
19	0.11	15×0.15	16×0.15	17×0.15	18×0.15	19×0.15	19×0.15		
		−0.2×4	−0.25×3	−0.25×2	−0.25×1				
20	0.07	15×0.15	16×0.15	17×0.15	18×0.15	19×0.15	20×0.15		
		0.25×5	0.25×4	0.25×3	0.25×2	0.25×4			

Number of copies	Probability	Possible action (i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
		15	16	17	18	19	20	
15	0.04	2.25	2.25	2.25	2.25	2.25	2.25	
16	0.19	2	2.4	2.4	2.4	2.4	2.4	
17	0.33	1.75	2.15	2.55	2.55	2.55	2.55	
18	0.26	1.5	1.9	2.3	2.7	2.7	2.7	
19	0.11	1.25	1.65	2.05	2.45	2.45	2.85	2.85

20	0.07	1.00	1.40	1.80	2.20	2.60	2.60	3.00

Expected profit table :

Number of copies	Probability	(i) & (ii)	(i) & (iii)	(i) & (iv)	(i) & (v)	(1) & (vi)	(i) & (vii)
15	0.04	0.09	0.09	0.09	0.09	0.09	0.09
16	0.19	0.38	0.456	0.456	0.456	0.456	0.456
17	0.33	0.577	0.709	0.841	0.841	0.841	0.841
18	0.26	0.39	0.494	0.598	0.702	0.702	0.702
19	0.11	0.137	0.181	0.225	0.269	0.313	0.313
20	0.07	0.07	0.098	0.126	0.156	0.182	0.21
		1.644	2.028	2.336	2.512	2.584	2.612
						↑	

Maximum EMV = 2.612

Number of copies		Conditional profit under certainity	Probability	EPPI
15	2.25	0.04	0.09	
16	2.40	0.19	0.456	
17	2.55	0.33	0.8413	
18	2.70	0.26	0.702	
19	2.85	0.11	0.31345	
20	3.00	0.07	0.22	
			2.613	

EPPI = Rs. 2.613

Q. 8. (b) (iii) The vendor is thinking of spending on a small market survey to obtain additional information regarding the demand levels. How much should he be willing to spend on such a survey?

Ans. EVPI = 2.613–2.612

= Rs. 0.001