Note : Attempt any five questions.

Part-A

Q. 1. (a) Define the following terms :

Lissajous figure, Engine order, Campbell diagram, Harmonic motion and Beats.

Ans. (i) Lissajous Figure :

When output of a system is measured and is continuously compared with the required value, then it is known as closed-loop or monitored system. In this system, the output is measured and through a feedback transducer, it is sent to an error detectors which detects any error in the output from the required value thus adjusting the input in a way to get the required output.

(ii) Engine order :

A multi-fuel engine is one which would operate satisfactorily with substantially unchanged performance and efficiency, on a wide variety of fuels ranging from diesel oil, crude oil, JP-4 to lighter fuels like gasoline, and even normal lubricating oil.

(iii) Campbell diagram :

Almost all diesel engines are capable of burning a wide variety of fuels which include heavy diesel oil, crude oil, JP-4 and kerosene.

(iv) Harmonic motion :

Let a harmonic force $F = F_0 \sin wt$ is acting on a vibrating body having motion

$x = x_0 \sin(wt - \phi)$. The work done by the force during small displacement dx is Fdx.

(v) Beats :

When two harmonic motions pass through some point in a medium simultaneously, the resultant displacement at that point is the vector sum of the displacement due to two component motions.

Q. 1. (b) Show that two simple harmonic motions with frequency p and 2p when added will result in a periodic function of frequency p. Generalize the above for a number of harmonic motions with frequencies p, 2p, -np et.

Ans. Let $A\sin(wt+\phi)$ extra motion be given to the body.

Then $$A\sin(wt+\phi) + x_1 + x_2 = 0$$

Replace t by p we get

$$A\sin(wtp+\phi)+x_1+x_2=0$$

Expanding various terms, we get

$$A\sin wp\cos\phi + A\cos wp\sin\phi$$

$$A\cos pt\sin\phi$$

$$A\cos 2pt\sin\phi$$

$$A\cos(-np)t\sin\phi$$

Since the coefficients of $\sin wt$ and cos wt are same.

Then the frequencies will not be affected by the change of the amplitudes.

Hence the final frequencies are

P

2P

–np etc.

Q. 2. (a) Discuss the various ways to derive equation of motion for SDF systems.

Ans. Various ways to derive equation of motion :

1. Energy method :

The equation of motion can also be derived assuming the system to be a conservative one. In a conservative system the total sum of the energy is constant.

2. Rayleigh's method :

In deriving the expression for motion, it is assumed that the maximum kinetic energy at the mean position is equal to the maximum potential energy at the extreme position.

3. Torsional vibrations :

If a system having a rotor of mass moment of inertia I connect to a shaft of torsional vibration stiffness K_T, is twisted by an angle θ.

4. Equivalent siffness of spring combinations :

Certain systems have more than one spring. The springs are joined in series or parallel or both. They can be replaced by a single spring of the same stiffness as they all show the same stiffness jointly.

5. The compound pendulum :

The system which is suspended vertically and oscillates with a small amplitude under the action of the force of gravity, is known as compound pendulum.

6. Transverse vibrations of beam :

Beams are widely used for structural elements such as floor support, parts of chassis etc.

7. Beam with several masses :

In this type of beam consider a weightless beam carrying several masses on itself.

8. Trifilar suspension :

In trifilar suspension there is a disc. Usually circular or triangular type suspended by three wires of equal length.

Q. 2. (b) A spring-mass system has spring stiffness of kN/m and a mass of M kg. It has natural frequency of vibration as 12 Hz. An extra 2 kg mass is coupled to M and the natural frequency reduces by 2Hz. Find K and M.

Ans. Given :

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{K}{w/g}} = 12$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{\frac{w+2}{g}}} = 12 - 2 = 10$$

$$\frac{Kg}{W} = (12)^2 .4\pi^2$$

$$\frac{Kg}{w+2} = 100.4\pi^2$$

$$\frac{w+2}{w} = \frac{144}{100} = 1.44$$

$$w = \frac{2}{0.44} = 4.54\,\text{kg}$$

$$\frac{kg}{w} = (12)^2 \times 4\pi^2$$

$$K = \frac{(12)^2 \times 4\pi^2 \times 4.54}{981}$$

$$= 26.28\,\text{kg/cm}.$$

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Q. 3. (a) Show that the mass of a system having over-damping will never pas through the equilibrium position if it is given :

(i) An initial displacement only.

(ii) An initial velocity only.

Ans. When the value of damping ratio ε is more than one i.e. $\varepsilon > 1$, the system is known as over-damped one. This motion is called a periodic. When t = 0 the displacement is the sum of A_1 and A_2 i.e.

$$x = A_1 + A_2$$

The value of constants A_1 and A_2 can be determined from initial conditions. For example at t = 0, if the displacement and velocity are x_0 and $\dot{x}_0$ respectively.

$$x_0 = A_1 + A_2$$

$$\dot{x}_0 = \left(\frac{dx}{dt}\right)_{t=0} = A_1\left[-\varepsilon + \sqrt{\varepsilon^2 - 1}\right]w + A_2\left[-\varepsilon - \sqrt{\varepsilon^2 - 1}\right]w$$

In case of critically damped system. The system is said to be critically damped when $\varepsilon = 1$. i.e,

$$\frac{2}{2m} = \sqrt{\frac{k}{m}}$$

The two rooms of equation u_1 and u_2 are equal to each other.

$$u_1 = u_2 = -\varepsilon w = -w$$

So, the approximate solution of equation may be written as,

$$x = A_1 e^{-wt} + A_2 + e^{-wt}$$

$$= (A_1 + A_2 t)e^{-wt}.$$

Q. 3. (b) A mas of 1 kg is to be supported on a spring having a stiffness of 9800 N/m. The damping coefficient is 4-9 N-sec/m. Determine the natural frequency of the system. Find also the logarithmic decrement and the amplitude after three cycles if the initial displacement is 0.30 cm.

Ans. Critical damping is determined as,

$$C_c = 2\sqrt{km} = 2\sqrt{9800 \times 1} = 197.98 \text{ N-sec/m}$$

$$\varepsilon = \frac{c}{c_c} = \frac{4.9}{197.98} = 0.02474$$

Natural frequency of Damped vibration

$$w_0 = w\sqrt{1-\varepsilon^2}$$

$$= \sqrt{\frac{9800}{1}}(0.9994)$$

$$= 98.96 \text{ rad / sec.}$$

$$f_d = \frac{w_d}{2\pi} = \frac{98.96}{2\pi} = 15.74 \text{ Hz}$$

Logarithmic Decrement is given as,

$$\delta = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$= \frac{2\pi \times 0.02474}{\sqrt{1-(0.02474)^2}}$$

$$= 0.15549$$

$$\delta = \frac{1}{n}\ln\frac{x_1}{x_n}$$

$$0.15549 = \frac{1}{3}\ln\frac{0.30}{x_n}$$

$$x_n = 0.1025$$

Q. 4. (a) Show that the maximum velocity of vibration of the mass of a spring-mass-dashpot system occurs at resonance and is independent of damping.

Ans. The amplitude is given as,

$$A = \frac{F/K}{\sqrt{\left[1-(w/w_n)^2\right]^2 + \left[(2\varepsilon w/w_n)^2\right]}}$$

Let us assume that $\frac{F}{K} = X_S$ where X_S is called zero frequency deflection. The above equation can be put as,

$$A = \frac{X_s}{\sqrt{\left[1-(w/w_n)^2\right] + \left[(2\varepsilon w/wn)^2\right]}}$$

$$\frac{A}{X_S} = \frac{1}{\sqrt{\left[1-(w/wn)^2\right]^2 + \left[(2\varepsilon w/wn)^2\right]}}$$

The non-dimensional quantity $\frac{A}{X_S}$ is known as magnification factor or amplitude ratio. So, the particular solution of equation can be written as,

$$x_p = X_S \frac{\sin(wt-\phi)}{\sqrt{\left[\left(1-(w/wn)^2\right)\right]^2 + \left[(2\varepsilon w/wn)^2\right]}}$$

The complete solution can be written as

$$x = x_c + x_p$$

The value of x_c can be taken from as So x can be written as,

$$x = A_1 e^{-\varepsilon w_n t} \cos\left(\sqrt{1-\varepsilon^2}\, w_n t + \phi_1\right)$$

$$+ \frac{X_s \sin(wt-\phi)}{\sqrt{\left[1-(w/w_n)^2\right]^2 + \left[(2\varepsilon w/w_n)^2\right]}}$$

The value of constant A_1 and ϕ_1 can be determined from initial conditions.

The frequency at which maximum amplitude occurs can be obtained by differentiation w.r.to w/w_n which leads to

$$\frac{w_{max}}{w_n} = \sqrt{1-2\xi^2}$$

Q. 4. (b) Accelerometer is used to measure the motion of a structure which vibrates at 15 r.p.m. The static deflection of the Seismic mass of the accelerometer is to 30 cm. Determine the amplitude of the structure if the reading of instrument is 0.6 cm.

Ans. The controlling equation is given as

$$\frac{Z}{B}=\frac{r^2}{\sqrt{\left(1-r^2\right)^2+(2\xi r)^2}}$$

$$=\frac{r^2}{\left(1-r^2\right)} \quad \text{(When } \xi=0.0\text{)}$$

$$f_n = 6\,\text{Hz}$$

$$f=\frac{N}{60}=\frac{15}{60}=0.25$$

$$r=\frac{2}{6}=0.33$$

z= 0.05m = reading of instrument

Using the above equation, we get

$$\frac{0.05}{B}=\frac{(0.33)^2}{1-(0.33)^2}=0.122$$

$$B = 0.409 \text{ mm.}$$

Part-B

Q. 5. (a) Describe principle and working of a centrifugal pendulum vibration absorber.

Ans. Centrifugal Pendulum Vibration Absorber :

As discussed earlier, the undamped dynamic vibration absorber is fully effective only at a particular frequency for which it has been designed. In case of torsional system, it is possible to use a dynamic vibration absorber of the pendulum type that is effective at all speeds of rotation of the system. This is the pendulum or centrifugal pendulum type of absorber.

The centrifugal force is given as,

$$F = mrw^2$$

The differential equation of motion for oscillation of pendulum is given

$$I\ddot{\theta} = -F\sin\alpha \times L \qquad \text{...(1)}$$

$$ML^2\ddot{\theta} = -\left(mrw^2 \sin\alpha\right)\times L$$

From equation (2), we get

$$\ddot{\theta} + \frac{r}{L} w^2 \sin\alpha = 0$$

Applying the law of since to the triangle ΔOPB, we get

$$\frac{R}{\sin\alpha} = \frac{r}{\sin(180° - \theta)}$$

$$r\sin\alpha = R\sin\theta .$$

Q. 5. (b) In a two mass torsional system two wheels are mounted 0.15 m apart on a shaft 40 mm diameter. If the moment of inertia of the two wheels are $I_1 = 1.2\,kg - m^2$ and $I_2 = 2.0\,kg - m^2$, find the position of the node and the frequency of free torsional vibrations.

Ans. Given

$$I_1 = 1.2\,kg - m^2$$

$$I_2 = 2\,kg - m^2$$

$$l = 0.15m$$

$$d = 490\text{ mm}$$

$$d = 40 \times 10^{-3} m$$

$$d = 0.04\,m$$

As,
$$K = \frac{GJ_l}{l_l}$$

$$K = \frac{9 \times 10^{11} \times \pi \times (0.04)^4}{0.15 \times 32}$$

$$= 1.51 \times 10^6 \text{ N-m/rad}$$

Where I = Mass Moment of inertia

J = Polar moment of Inertia.

The natural frequency is given as,

$$W = \sqrt{\frac{K(I_1 + I_2)}{I_1 I_2}}$$

$$= \sqrt{\frac{1.51 \times 10^6 (1.2 + 2)}{1.2 \times 2}}$$

$$= 1175.71 \text{ rad / sec.}$$

Q. 6. (a) Explain Rayleigh method to find out the natural frequency of a system when transverse loads are acting.

Ans. Rayleigh Method : The maximum kinetic energy is equated to maximum potential energy of the system to determine the natural frequency.

The maximum potential energy of the system given as :

$$\text{P.E.} = \frac{1}{2} p_1 y_1 + \frac{1}{2} p_2 y_2 + \frac{1}{2} p_3 y_3 + \frac{1}{2} p_4 y_4$$

$$\text{P.E.} = \frac{1}{2} \Sigma Py$$

The maximum K.E. of the system is given as,

$$\text{K.E.} = \frac{1}{2g} p_1 (wy_1)^2 + \frac{1}{2g} p_2 (wy_2)^2 + \frac{1}{2g} p_3 (wy_3)^2 + \frac{1}{2g} p_4 (wy_4)^2$$

$$= \frac{w^2}{2g} \Sigma py^2$$

Where w = Natural frequency of vibration equating the maximum K.E. to maximum P.E., we have

$$\frac{w^2}{2g} \Sigma py = \frac{1}{2} \Sigma py$$

$$w = \sqrt{\frac{g \Sigma py}{\Sigma py^2}}$$

The above equation can be written in a more generalised way by including the distributed mass of the beams. If m is the mass of the beam per unit length and y is the assumed deflection curve, the maximum potential energy of beam of length l is expressed as,

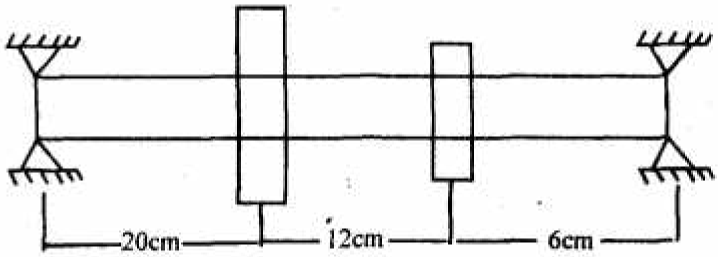
$$\text{P.E.} = \frac{1}{2} \int_0^l M d\theta$$

Where M = Bending Moment

$d\theta$ = Change in slope over a distance dx.

Q. 6. (b) Using rayleigh's method, estimate the fundamental frequency of the system shown in fig.

Take $E = 2.0 \times 1011 N/m^2$, $I = 10^{-6} m^4$, **and** $g = 10 m/s^2$.



Ans. With the help of the given equation, we can find the natural frequency as,

$$\omega = \sqrt{\frac{g\Sigma py}{\Sigma py^2}}$$

The static deflections at two points are given as,

$$y_1 = M_1 g a_{11} + M_2 g a_{12}$$

$$y_2 = M_1 g a_{21} + M_2 g a_{22}$$

$$a_{11} = \frac{l^3}{3EI} = \frac{(0.20)^3}{3 \times 2.0 \times 10^{11} \times 10^{-6}}$$

$$= 5.46 \times 10^{-8}$$

$$a_{12} = a_{21} = \frac{\theta^2(3l - \theta)}{6EI} = \frac{(0.20)^2(3 \times 0.38 - 0.20)}{6 \times 2.0 \times 10^{11} \times 10^{-6}}$$

$$= 3.13 \times 10^{-8}$$

So,

$$y_1 = M_1 g a_{11} + M_2 g a_{12}$$

$$= 100 \times 9.8 \times 1.33 \times 10^{-8} + 50 \times 9.8 \times 3.13 \times 10^{-8}$$

$$= 1.3034 \times 10^{-5} + 1.53 \times 10^{-5}$$

$$= 2.83 \times 10^{-5} m$$

$$y_2 = M_1 g a_{21} + M_2 g_1 a_{22}$$

$$= 100 \times 9.8 \times 3.13 \times 10^{-8} + 50 \times 9.8 \times 5.46 \times 10^{-8}$$

$$= 3.06 \times 10^{-5} + 2.67 \times 10^{-5}$$

$$= 5.74 \times 10^{-5}$$

Thus, natural frequency,

$$w = \sqrt{\frac{9.8(p_1y_1 + p_2y_2)}{(p_1y_1^2 + p_2y_2^2)}}$$

$$= \sqrt{\frac{9.8(100 \times 2.83 \times 10^{-5} + 50 \times 5.74 \times 10^{-5})}{100 \times (2.83 \times 10^{-5})^2 + 50 \times (5.74 \times 10^{-5})^2}}$$

$$= \sqrt{\frac{0.05586}{8.01 \times 10^{-8} + 1.65 \times 10^{-7}}}$$

$$= 477.65 \text{ rad / sec.}$$

Q. 7. (a) Explain briefly a vibration analyser and ways to predict the fault.

Ans. Ways to predict faults in vibration analyser are :

1. Command : The result of the act of adjustment i.e. closing a valve, moving a lever, pressing a button etc., is known as command.

2. Response : The subsequent result of the system to the command is known as response.

3. Process control : The automatic control of variables i.e. change in pressure, temperature or speed etc., in machine is termed as process control.

4. Process controller : The device which controls a process is called a process controllers.

5. Regulator : The device used to keep the variables at a constant desired value is called as regulator.

6. Kinetic control : The automatic control of the displacement or velocity or acceleration of a member of a machine is called a kinetic control.

7. Feedback : It is defined as measuring the output of the machine for comparison with the input to the machine.

8. Error detector : A differential device used to measure the actual controlled quantity.

Q. 7. (b) Explain vibration pickups briefly.

Ans. Vibration Pickups : The automatic control of system is a very accurate and effective means to perform desired function by the system is which the human operator is replaced by a device thereby relieving the human operator from the job thus saving physical strength. The automatic control systems are also called as self-activated systems. The centrifugally actuated ball governor which controls the turottle valve to maintain the constant speed of an engine is an example of an automatically controlled system.

Q. 7. (c) Explain predictive and preventive maintenance with examples.

Ans. Predictive and preventive maintenance : When the input to a system is independent of the output from the system, then the system is called an open-loop control system/ It is also called as a calibrated system. Most measuring instruments are open-loop control systems, as for the same input signal, the readings will depend upon things like ambient temperature and pressure.

Q. 8. Write short notes on any three of he following :

(a) Vibration exciters, **(b) Whirling of shafts,**

(c) Dunkerley method, **(d) Gyroscopic effect on rotating shafts.**

Ans. (a) Vibration exciters : The structures designed to support the high speed engines and turbines are subjected to vibration. Due to faulty design and poor manufacture there is unbalance in the engines which causes excessive and unpleasant stresses in the rotating system because of vibration. The vibration causes rapid wear of machine parts such as bearings and gears. Unwanted vibrations may cause loosening of parts from the machine. Because of improper design or material distribution, the wheels of locomotive can leave the track due to excessive vibration which fall because of vibration.

(b) Whirling of shafts : We have already discussed that when the natural frequency of the system coincides with the external forcing frequency, it is called resonance. The speeds at which resonance occurs are known as the critical speeds. These speeds are also termed as whirling speeds or whipping. At these speeds the amplitude of vibration of rotors is excessively large and the large amount of force is transmitted to the foundations or bearings. In the region of critical speeds the system may fail because of violent nature of vibrations in the transverse direction. Therefore, it is very important to find the natural frequency of the shaft to avoid the occurrence.

(c) Dunkerley method : Natural frequencies of structures are evaluated by this method. This method is used to find the natural frequency of transverse vibrations. The load of the system is uniformly distributed. Dunkerley's equation can be written as,

$$\frac{1}{w^2} = \frac{1}{w_1^2} + \frac{1}{w_2^2} + \ldots\ldots \frac{1}{w_g^2}$$

Where, w = natural frequency of transverse vibration of shaft for many point loads. w_1, w_2, w_3, etc. = natural frequency of individual point loads.

w_g = natural frequency of transverse vibration because of the weight of shaft.

(d) Gyroscopic effect on rotating shafts :

Consider a disc spinning with an angular velocity w rad/s about the axis of spin OX, in anticlockwise direction. When seen from the front. Since the plane in which the disc is rotating is parallel to the plane YOZ, therefore it is called plane of spinning. The plane XOZ is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis OY. In other words, the axis of spin is said to be rotating or processing about on axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY at an angular velocity w_p rad/s.