B.E.

Seventh Semester Examination, May-2007

MECHANICAL VIBRATION

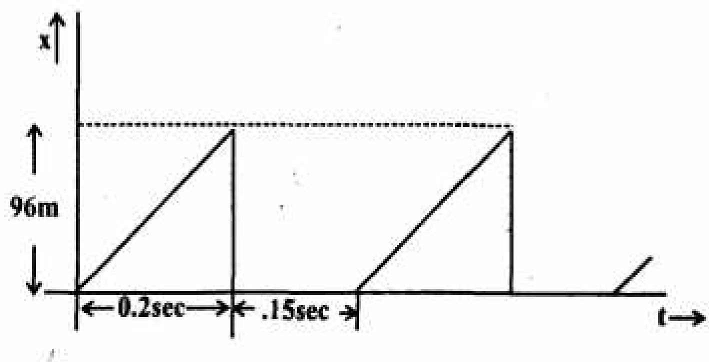
Note : Attempt any five questions.

Q. 1. (a) Represent the following complex numbers in rectangular form :

(i) $10e^{-j1.1}$

(ii) $5e^{j2.1}$ and

Represent the following periodic motion by harmonic series.



Ans. (i) $10e^{-j1.1}$

$$X = 10e^{-j1.1}$$

Here $$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$X = Ae^{+j\theta} = A((\cos\theta + j\sin\theta))$$

And $$x = Ae^{-j\theta} = A(\cos\theta + i\sin\theta)$$

Given $\theta = 1.1$ (radians)

$$= \frac{1.1}{3.14} \times 180^\circ = 63.06^\circ$$

Thus $$X = A(\cos 63.06 - j\sin 63.06)$$

$$= 10[0.45 - j(0.89)]$$

$$= 4.5 - j(8.9)$$

(ii) $5 - e^{j2.1}$

$$x = 10e^{j21}$$

Here $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$X = Ae^{+j\theta} = A(\cos\theta + j\sin\theta)$$

$$X = Ae^{-j\theta} = A(\cos\theta - j\sin\theta)$$

Given $\theta = 2.1$ (radians)

$$= \frac{2.1}{3.14} \times 180 = 120.380^\circ$$

Thus $X = A[\cos 120.38 + j\sin(120.38)]$

$$= 5[-0.506 + 0.86j]$$

$$= -2.53 + 4.3j$$

Harmonic series :

Time period of motion

$$= 0.35 \text{ sec}$$

Frequency $w = \frac{2\pi}{0.35} = 5.11\pi \text{ rad/sec}$

Equation of curve for one cycle

$$x(t) = \frac{90t}{2} \qquad 0 \le t \le 0.2$$

$$= 0 \qquad 0.2 \le t \le 0.35$$

$$a_0 = \frac{w}{2\pi}\int_0^{0.2} x(t)\,dt = \frac{5.71\pi}{2\pi}\int_0^{0.2} \frac{90t}{2}\,dt$$

$$= \frac{5.71}{2} \times \frac{90}{2}\left[\frac{t^2}{2}\right]_0^{0.2} = 2.56$$

$$\frac{a_0}{2} = 1.28$$

$$a_n = \frac{w}{\pi}\int_0^{0.2} x(t)\cos(nwt)\,dt = \frac{5.71\pi}{\pi}\int_0^{0.2}\frac{90t}{2}\cos(4\pi nt)\,dt$$

$$= \frac{90 \times 5.71}{2}\int_0^{0.2} t.\cos(4\pi nt)\,dt = 256.95\left[\frac{t\sin 4\pi nt}{4\pi n} + \frac{\cos(4\pi nt)}{16\pi^2 n^2}\right]_0^{0.2}$$

$$= 256.95\left[\frac{0.2\sin(0.8\pi n)}{4\pi n} + \frac{\cos(0.8\pi n)}{16\pi^2 n^2}\right]$$

$$= 256.95\frac{\sin(0.8\pi n)}{4\pi n} + \frac{256.95}{16\pi^2 n^2}\left[\cos(0.8\pi n) - 1\right]$$

Similarly b_n can also be find out.

As $$b_n = \frac{w}{\pi}\int_0^t x(t)\sin(nwt)\,dt$$

And $$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos(nwt) + b_n\sin(nwt)\right).$$

Q. 1. (b) Show that the Fourier Series expansion for the function $x(t)$ defined in the finite interval $-\pi \le t \le \pi$ by :

$$x(t) = 0 \qquad -\pi \le t \le 0$$

$$= \sin t \qquad 0 \le t \le \pi$$

is given by $x(t) = \frac{1}{\pi} \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nt}{4n^2 - 1} + \frac{1}{2}\sin t$.

Ans.

$$x(t) = 0 \qquad -\pi \le t \le 0$$

$$= \sin t \quad 0 \le t \le \pi$$

$$a_0 = \frac{w}{\pi}\int_{-\pi}^{\pi} x(t)\,dt$$

$$= \frac{w}{\pi}\left[\int_{-\pi}^{0} 0\,dt + \int_{0}^{\pi} \sin t\,dt\right]$$

$$= \frac{w}{\pi}\left[-\cos t\right]_0^{\pi}$$

$$a_n = \int_0^{\pi} \sin t \cos(nwt)\,dt$$

$$= -\frac{2}{\pi}\frac{\cos 2nt}{4n^2 - 1}$$

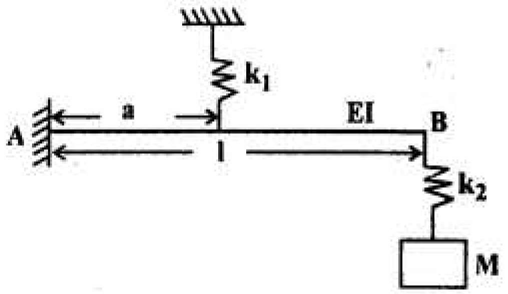
$$b_n = 0$$

Fourier series is given as

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nwt + b_n \sin nwt)$$

$$= \frac{1}{\pi} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos n2t}{4n^2 - 1}.$$

Q. 2. (a) Find the expression for the natural frequency of mass M for the system as shown in fig. 2(a). Net mass of cantilever AB.



Ans. To find the natural frequency of the equivalent spring at A is to be determined. Let us consider a force P at C, so the force at point B can be written as Pl/a

$$F_B \times a = \rho . l$$

$$F_B = \frac{\rho . l}{a}$$

The deflection at point C can be expressed as

$$\delta_B = \frac{F_B}{K_1} = \frac{Pl}{aK_1}$$

$$\rho . \delta_c = F_B . \delta_B$$

$$\delta_c = \frac{F_B \delta_B}{\rho}$$

$$= \rho \left(\frac{l}{a}\right)^2 . \frac{1}{k_1} = \frac{Pl^2}{a^2 k_1}$$

For this value of deflection the corresponding deflection at point B.

$$\frac{P}{K_1}\left(\frac{l}{a}\right)^2 \text{ and stiffness } K_1\left(\frac{a}{l}\right)^2 = K_d$$

K_d and K_2 are connected in series so their equivalence can be written as

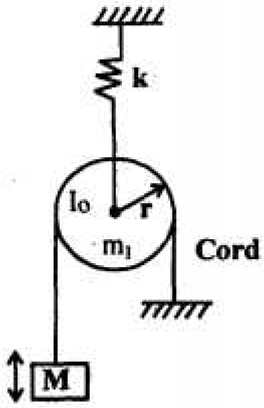
$$K = \frac{K_d . K_2}{K_d + K_2} = \frac{K_1 (a/l)^2 K_2}{K_1\left(\frac{a}{l}\right)^2 + K_2}$$

Natural frequency can be written as

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{K_1 K_2 (a/l)^2}{m\left[K_2 + K_1 (a/l)^2\right]}} Hg.$$

Q. 2. (b) Determine the nat frequency of the system using energy and Newton's method as shown in fig. 2(b).



Ans. Given $J = \frac{1}{2}mr^2$ for pulley

$$T = \text{K.E. of mass} + \text{K.E. of pulley}$$

$$= \frac{1}{2}M_1\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + \frac{1}{2}J\dot{\theta}^2$$

For any small displacement θ

$$y = r\theta$$

(Where x = displacement of mass m at any instant)

and $y = \frac{x}{2}$ (y = vertical displacement of pulley centre)

$$= \frac{1}{2}M_1\dot{x}^2 + \frac{1}{2}m\left(\frac{\dot{x}}{2}\right)^2 + \frac{1}{2}.\frac{1}{2}mr^2.\frac{\dot{y}^2}{r^2}$$

$$= \frac{1}{2}M_1\dot{x}^2 + \frac{3}{16}M_1\dot{x}^2.\frac{\dot{y}^2}{r^2}$$

The potential energy is given by

$$V = \text{P.E.}$$

$$= \frac{1}{2} K y^2$$

$$= \frac{1}{2} K \left(\frac{1}{2} x\right)^2 = \frac{1}{2} K x^2$$

The energy of the system is constant.

So, $$\frac{d}{dt}(T + V) = 0$$

$$M_1 \dot{x}\ddot{x} + \frac{3}{8} M \dot{x}\ddot{x} + \frac{1}{4} K \dot{x}\ddot{x} = 0$$

$$M_1 \ddot{x} + \frac{3}{8} M \ddot{x} + \frac{1}{4} K x = 0$$

$$\left(M_1 + \frac{3}{8} m\right) \ddot{x} + \frac{1}{4} K x = 0$$

So natural frequency is

$$w_n = \sqrt{\frac{1}{4} \frac{K}{\left(M_1 + \frac{3m}{8}\right)}} \text{ rad/sec.}$$

$$= \sqrt{\frac{1}{4M_1 + \frac{3m}{2}}} \text{ rad/sec.}$$

Q. 3. (a) What do you understand by vibration measuring instruments. Explain.

Ans. Vibration measuring instruments :

The instruments which are used to measure the displacement velocity or acceleration of a vibrating body are called vibration measuring instruments, vibration measuring devices having mass, spring, dashpot etc., are known as seismic instruments. The quantities to be measured are display on a screen in the form of electric signal which can be readily amplified and recorded. The output of electric signal of the instrument will be proportional to the quantity which is to be measured. The input is reproduced as output very precisely. Two types of seismic transducers known as vibrometer and acceleratormeter are widely used. A vibrometer and accelerometer are widely used. A vibrometer or a seismometer is a device to measure the displacement of a vibrating body. Similarly, other device known as an accelerator is an instrument to measures the acceleration of a vibrating body. Vibrometer is designed with low natural frequency and accelerometer with higher natural frequency. So, vibrometer is known as low frequency transducer.

Q. 3. (b) Determine the torsional shiftiness of a spring for a torsiograph with a ring having moment of inertia of 2.0×10^{-3} kg – m^2. So that the difference in the relative motion and that of vibrating shaft will not be greater than 3%. When the shaft vibrates with a frequency of 999 cpm or above. Neglect damping. If the shaft amplitude is 0.01 radian, determine the corresponding dynamic torque on the spring.

Ans. Given : $I = 2.0 \times 10^{-3}$ kg – m^2

T.R. = 3%

f = 999 cpm

w = f = 16.65 rad/sec

A = 0.01 radian

Transmissibility is given as

$$\text{T.R.} = \frac{1}{(w/w_n)^2 - 1} \text{ (where } \varepsilon \text{ is negligible)3}$$

$$\frac{1}{3} = \frac{1}{(w/w_n)^2 - 1}$$

$$\left(\frac{w}{w_n}\right)^2 = 3 + 1 = 4$$

$$\left(\frac{w}{w_n}\right)^2 = 4, \ \frac{w}{w_n} = 2$$

$$w_n = \frac{w}{2} = \frac{16.65}{2} = 8.325$$

We know that

$$w_n = \sqrt{\frac{k}{m}}$$

$$w_n = \frac{2\pi N}{60} = \frac{2\pi \times 8.325 \times 60}{60} = 52.31 \text{ rad/sec}$$

As $$\text{T.R.} = \frac{\text{Force transmitted}}{\text{Impressed force}} = \frac{F_{TR}}{F}$$

$$F_{T.R.} = F.(T.R)$$

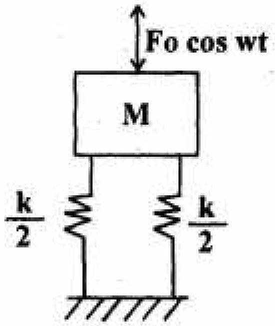
$$= M_1 e w^2 . \frac{1}{3}$$

$$= \frac{1}{9.81} \times \frac{0.05}{100} \times \left(\frac{2\pi 16.65}{60}\right)^2 \times \frac{1}{3} \times 10^5$$

$$= 5N.$$

Q. 4. (a) A spring mass system is shown in fig. 4 (a) and is subject to a harmonic force Fo cos wt Determine the response of the system :

Given : $x(0) = .01m$; $\dot{x}(0) = .04m/sec.$, w = 30 rad/sec, Fo = 1000 N; m = 10kg and k = 500 N/m.



Ans. The differential equation of motion of the system can be written as,

$$m\ddot{x} + kx = F\cos wt$$

Applying Laplace transform, we get

$$mL(\ddot{x}) + KL(x) = FL\cos wt$$

$$m\left\{S^2X(S) - Sx(0) - \dot{x}(0)\right\} + KX(S) = F.\frac{S}{S^2 + w^2}$$

Substituting the values of various terms in the above equation,

$$10\left[S^2X(S) - 0.015 - 0.04\right] + 500X(s) = \frac{1000S}{S^2 + 30^2}$$

Solving above equation for X(S), we get

$$X(S)=\frac{0.1S^2+0.4S^2+1090S+360}{\left(10S^2+500\right)\left(S^2+900\right)} \qquad ...(1)$$

It can be solved by inserting certain constants A, B C and D.

$$X(S)=\frac{AS+B}{10s^2+500}+\frac{S+D}{S^2+900} \qquad ...(2)$$

$$=\frac{(AS+B)\left(S^2+900\right)+(CS+D)\left(10S^2+500\right)}{\left(10S^2+500\right)\left(S^2+900\right)}$$

$$=\frac{AS^3+900AS+BS^2+900B+10CS^3+500CS+10DS^2+500D}{\left(10S^2+500\right)\left(S^2+900\right)}$$

Equating the like powers of S both sides of equation (1)

$$\left.\begin{aligned} A+10C&=0.1\\ B+10D&=0.4\\ 900A+500C&=1090\\ 900B+500D&=360 \end{aligned}\right\} \qquad ...(3)$$

By solving equation (3), we have

$$A=\frac{21.7}{17}=1.276$$

$$B=0.40$$

$$C=-\frac{2}{17}=-0.117$$

$$D=0$$

Equation (2) can be written as

$$X(S)=\frac{1.276S}{10S^2+500}+\frac{0.40}{10S^2+500}-\frac{0.117s}{S^2+900}$$

$$=\frac{1.276S}{10\left(S^2+50\right)}+\frac{0.4}{10\left(S^2+50\right)}-\frac{0.117S}{S^2+(30)^2}$$

$$X(S) = 0.1276\left(\frac{S}{S^2+\left(5\sqrt{2}\right)^2}\right) + \frac{0.4.5\sqrt{2}}{5\sqrt{2}\left(S+\left(5\sqrt{2}\right)^2\right)} - \frac{0.117S}{S^2+(30)^2}$$

Taking Inverse of Laplace transform, we have

$$x(t) = 0.1276\cos 5\sqrt{2}t + \frac{0.04}{5\sqrt{2}}\sin 5\sqrt{2}t - 0.117\cos 30t$$

$$= 0.1276\cos 5\sqrt{2t} + 0.0056\sin 5\sqrt{2t} - 0.117\cos 30t$$

This is the required response of the mass.

Q. 4. (b) A force F(t) is suddenly applied to mass m which is supported by a spring with constant stiffness k. After a short period of time T, the force is suddenly removed. During the time the force is active, it a constant F. Determine the response of the system if t > T. The spring and mass are initially at rest before the force F(t) is applied.

Ans. The equation of motion can be written as

$$m\ddot{x} + kx = F\left[u(t) - u(t-T)\right] \qquad ...(1)$$

Applying Laplace transform to differential equation,

$$m\ddot{x} + kx = F\left[u(t) - u(t-T)\right]$$

$$L[kx] = KX(S)$$

$$LF\left[u(t)\right] = \frac{F}{S}$$

$$LF\left[u(t-T)\right] = \frac{Fe^{-ST}}{S}$$

Initial conditions : $x(0) = 0$, $\dot{x}(0) = 0$

Substituting the values in equation (1), we get

$$mS^2X(s) + KX(s) = \frac{F}{S} - \frac{Fe^{-ST}}{S} = \frac{F}{S}\left(1-e^{-ST}\right)$$

$$\left(mS^2 + K\right)X(S) = \frac{F}{S}\left(1-e^{-ST}\right)$$

$$X(S) = \frac{F}{S}\left(1 - e^{-ST}\right)\left(mS^2 + K\right)$$

Defining $w_n^2 = \frac{k}{m}$ in the above equation

$$X(s) = \frac{F}{m}\left[\frac{1 - e^{-ST}}{S\left(S^2 + W_n^2\right)}\right]$$

From the table of Laplace transforms, the inverse

$$L^{-1}\left[\frac{1}{S\left(S^2 + w_n^2\right)}\right] = \frac{1}{w_n^2}\left(1 - \cos w_n t\right)$$

$$L^{-1}\left[\frac{e^{-ST}}{S\left(S^2 + w_n^2\right)}\right] = \frac{1}{w_n^2}\left[1 - \cos w_n(t - T)\right]u(t - T)$$

For $0 < t < T$, the solution is

$$x(t) = \frac{F}{mw_n^2}\left(1 - \cos w_n t\right)$$

And for $t > T$, the solution is given by

$$x(t) = \frac{F}{mw_n^2}\left[\left(1 - \cos w_n t\right) - 1 - cow_n t(t - T)\right].$$

Q. 5. (a) Distinguish between the normal mode and principal mode of vibration.

Ans. Normal mode and principal mode of vibration :

For a system with three-degree of freedom the orthogonality principle may be written as,

$$\left.\begin{aligned} M_1A_1A_2 + m_2B_1B_2 + m_3C_1C_2 = 0 \\ M_1A_2A_3 + m_2B_2B_3 + m_3C_2C_3 = 0 \\ M_1A_1A_3 + m_2B_1B_3 + m_3C_1C_3 = 0 \end{aligned}\right\} \qquad ...(1)$$

Where M_1, M_2, M_3 are masses.

$A_1, A_2, A_3, B_1, B_2, B_3, C_1, c_2$ and C_3 are the amplitudes of vibration of the system. We will make use of equation (1) in matrix iteration method to find the natural frequencies and mode shapes of the system.

Q. 5. (b) A reciprocating machine weighing 25N running at 6000 rpm after installation has natural frequency very close to the forcing frequency of vibrating system. Design a dynamic absorber of the nearest frequency of the system is to be at least 20% from the excitation frequency.

Ans. Since the natural is very close to the forcing frequency. So, 6000 rpm., may be treated as the natural frequency of the system.

$$w_n = \frac{2\pi \times 6000}{60} = 628\,\text{rad/sec} = w_1$$

But here $w_1 = w_2$

$$\left(\frac{w}{w_2}\right)^2 = \left(1 + \frac{\mu}{2}\right) - \sqrt{\left(\mu + \frac{\mu^4}{4}\right)}$$

Where, μ = Mass ratio.

The resonant frequencies are at least 20% away from the forward frequency of the main system.

So, we have

$$\frac{w}{w_2} = 0.80$$

$$\frac{w}{w_2} = 1.20$$

When $\frac{w}{w_2} = 0.8$, the value of $\mu = 0.2$

and for $\frac{w}{w_2} = 1.2$, the value of $\mu = 0.13$

The larger value of μ is taken for design purpose.

$$\mu = 0.2 = \frac{m}{M}$$

So $\quad m = 0.2 \times 25 = 5\,\text{kg}$ (weight)

Using $\quad w_1 = \sqrt{\frac{k_1 g}{w}}$ and finding $k_1 = w_1^2 \frac{w}{g}$

$$k_1 = w_1^2 \frac{W}{g}$$

$$= (628)^2 \times \frac{25}{981}$$

$$= 10050.5 \text{ kg/cm}$$

and $$K_2 = w_1^2 \times \frac{5}{981}$$

$$= 2010.1 \text{ kg/cm}.$$

Q. 6. (a) Find the torsional oscillations taking into account the inertias of gears fig.

$J_1 = 10\text{kg} - \text{cm sec}^2$

$J_2 = 2.01 J_1$

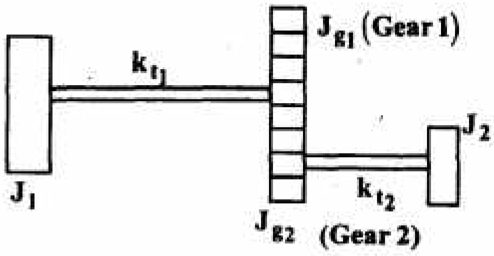
$J_{g_1} = 0.5\text{kg} - \text{cm sec}^2$

$J_{g_1} = 4.J_{g_1}$

$\& \; kt_1 = 3.2 \times 10^5 \text{kg.m.R}$

$kt_2 = kt_1 / 4$

dia of gear 2 = twice the dia of gear 1.



Ans. Let us assume that θ_1 and θ_2 are the angular displacement of gears J_1 and J_2 and n is velocity ratio.

Thus, $$\frac{\ddot{\theta}_2}{\theta_1} = n$$

Or $$\theta_2 = n\theta_1$$

The total kinetic and potential energy of the system remains constant.

$$\left.\begin{aligned} K.E. &= \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}J_2\dot{\theta}_2^2 \\ P.E. &= \frac{1}{2}kt_1\theta_1^2 + \frac{1}{2}kt_2\theta_2^2 \end{aligned}\right\} \quad ...(1)$$

$$\left.\begin{aligned} K.E. &= \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}\left(J_2 n^2\right)\dot{\theta}_1^2 \\ P.E. &= \frac{1}{2}kt_1\theta_1^2 + \frac{1}{2}\left(kt_2 n^2\right)\theta_1^2 \end{aligned}\right\} \quad ...(2)$$

So, J_2 becomes $J_2' = n^2 J_2$

And kt_2 becomes $k_{t_2}' = n^2 R_{t_2}$

The shafts are connected in series, so their equivalent stiffness

$$\frac{1}{k_{t_e}} = \frac{1}{k_{t_1}} + \frac{1}{n^2 k_{t_2}}$$

$$k_{t_e} = \frac{n^2 k_{t_1} k_{t_2}}{k_{t_1} + n^2 k_{t_2}}$$

The relation for frequency is

$$w = \sqrt{\frac{k(J_1 + J_2)}{J_1 J_2}}$$

$$= \sqrt{\frac{k_{t_e}\left(J_1 + n^2 J_2\right)}{J_1 n^2 J_2}}$$

$$= \sqrt{\frac{n^2 k_{t_1} k_{t_2}\left(J_1 + n^2 J_2\right)}{\left(k_{t_1} + n^2 k_{t_2}\right) J_1 J_2 n^2}}$$

$$= \sqrt{\frac{k_{t_1} k_{t_2} \left(J_1 + n^2 J_2\right)}{\left(k_{t_1} + n^2 k_{t_2}\right) J_1 J_2}}$$

We are given

$$J_1^*, J_2^* = J_{g_1} + n^2 J_{g2}, \ J_3 = n^2 J_2, \ k_{t_1} \text{ and } n^2 k_{t_2}.$$

We get equation

$$w^2\left[J_1 J_2 J_3 w^4 - \left\{(J_1 J_2 + J_1 J_3) n^2 k_{t_2} + (J_2 J_3 + J_1 J_3) k_{t_1}\right\} w^2 + k_{t_1} n^2 k_{t_2} (J_1 + J_2 + J_3)\right] = 0$$

$$J_1 = 10$$

$$J_2^* = \left(J_{g_1} + n^2 J_{g2}\right) = J_2 \text{ (say)}$$

$$J_3 = 20$$

$$n^2 k_{t_2} = 4 \times 0.8 \times 10^5 \times 20 = 3.2 \times 10^5$$

$$= 10.24 \times 10^{10}$$

$$J_1 + J_2 + J_3 = 10 + 8.5 + 20 = 38.5$$

$$w^2\left[1700 w^4 - \left\{(85 + 200) 3.2 \times 10^5 + (170 + 200) 3.2 \times 10^5\right\} w^2 + 10.24 \times 10^{10} \times 38.5\right]$$

$$w^4 - 1.2329 \times 10^5 w^2 + 23.2 \times 10^8 = 0$$

$$w^2 = \frac{1.2329 \times 10^5 \pm \sqrt{\left(1.2329 \times 10^5\right)^2 - 4 \times 23.2 \times 10^8}}{2}$$

$$= \frac{(1.2329 \pm 0.76)}{2} \times 10^5$$

$$w_1 = 316.4 \text{ rad / sec.}$$

$$w_2 = 151.4 \text{ rad / sec.}$$

Q. 6. (b) The arrangement of the compressor-turbine and generator in a thermal power plant is shown in fig. Find the natural frequency and mode shape of the system :

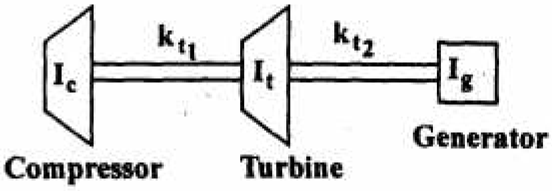
with $kt_1 = 6MN - m/rad$

$kt_2 = kt_{1/2}$

$I_c(MOI) = 18kg - m^2$

$I_t(MOI) = 14kg - m^2$

$I_g(MOI) = 9kg - m^2$



Ans. The equation of motion can be written as

$$I_1\ddot{\theta}_1 + kt_1(\theta_1 - \theta_2) = 0$$

$$I_2\ddot{\theta}_2 + kt_1(\theta_1 - \theta_3) + k_{t_1}(\theta_2 - \theta_1) = 0$$

$$I_3\ddot{\theta}_3 + kt_2(\theta_3 - \theta_2) = 0$$

Put the assumed value of θ_1, θ_2 and θ_3, we get

$$-I_1w^2A_1 + kt_1A_1 - kt_1A_2 = 0$$

$$(-I_1w^2 + kt_1)A_1 - kt_1A_2 = 0$$

$$(-I_2w^2 + kt_2 + kt_1)A_2 - kt_1A_1 - kt_2A_3 = 0$$

$$(-I_3w^2 + kt_2)A_3 - kt_2A_2 = 0$$

The determination of coefficient A_1, A_2 and A_3 equal to zero to get the frequency equation

$$\begin{vmatrix} \left(k_{t_1} - I_1 w^2\right) & -k_{t_1} & 0 \\ -k_{t_1} & k_{t_1} + k_{t_2} - J_2 w^2 & -k_{t_2} \\ 0 & -k_{t_2} & k_{t_2} - J_3 w^2 \end{vmatrix} = 0$$

Expanding the determinant, we get

$$\left(k_{t_1} - I_1 w^2\right)\left[\left(k_{t_1} + k_{t_2} - I_2 w^2\right)\left(k_{t_2} - I_3 w^2\right) - k_{t_2}^2\right]$$

$$+k_{t_1}\left(-k_{t_1}\right)(I_1 + I_2 + I_3) = 0$$

Substituting the values of various terms

$$I_1 I_2 I_3 = 18 \times 14 \times 9 = 2268$$

$$I_1 + I_2 + I_3 = 18 \times 14 + 9 = 41$$

$$I_1 I_2 = 18 \times 14 = 252$$

$$I_1 I_3 = 18 \times 9 = 162$$

$$I_2 I_3 = 14 \times 9 = 126$$

$$k_{t_1} k_{t_2} = 6 \times 10^6 \times 6 \times 10^6 = 36 \times 10^{12}$$

$$w^2\left[2268 w^4 - \left\{(252 + 162) 6 \times 10^6 + (126 + 162) 6 \times 10^6\right\} + w^2 + 6 \times 10^6 \times 6 \times 10^6 (41)\right] = 0$$

$$2268 w^4 - 4.2 \times 10^9 w^2 + 1.476 \times 10^{15}$$

$$w_1 = 1.38 \times 10^6 \text{ rad / sec.}$$

$$w_2 = 4.71 \times 10^5 \text{ rad / sec.}$$

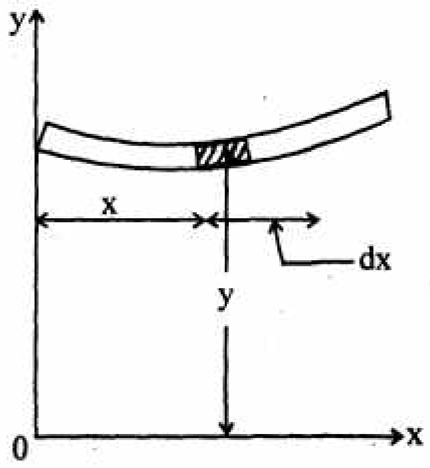
Q. 7. (a) Determine the frequency equation in the transverse vibration for a free-free beam of length l and having a uniform cross-section.

Ans. Net forces acting on the element

$$\theta - \left(\theta + \frac{\partial \theta}{\partial x} dx\right) = dm \text{, acceleration}$$

$$-\frac{\partial\theta}{\partial x}dx = (\rho A dx)\frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial\theta}{\partial x} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \qquad ...(1)$$



Considering the moments about A, we get

$$M - \left(M + \frac{\partial M}{\partial x}dx\right) + \left(\theta + \frac{\partial\theta}{\partial x}dx\right) = 0$$

$$-\frac{\partial M}{\partial x} + \theta + \frac{\partial\theta}{\partial x}dx = 0$$

So, $\theta = \frac{\partial M}{\partial x} = \frac{\partial M}{\partial x}$ higher order derivatives are neglected $\left(\frac{\partial\theta}{\partial x}dx = 0\right)$

$$\frac{\partial\theta}{\partial x} = \frac{\partial^2 M}{\partial x^2} \qquad ...(2)$$

For the above two equation (1) & (2), we get

$$\frac{\partial^2 M}{\partial x^2} = -\rho A\frac{\partial^2 y}{\partial t^2} \qquad ...(3)$$

We know from beam theory that

$$M = -EI\frac{\partial^2 y}{\partial x^2}$$

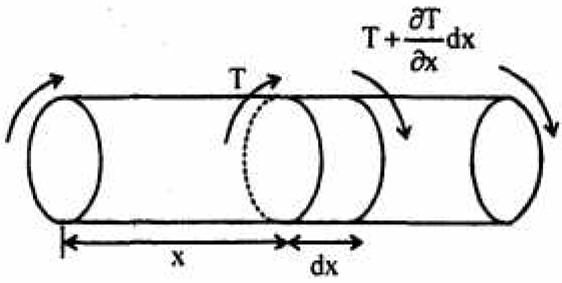
$$\frac{\partial^2 y}{\partial x^2} = -EI\frac{\partial^4 y}{\partial x^4} \qquad ...(4)$$

Combining equation (3) & (4)

$$\frac{\partial^4 y}{\partial x^4} + \left(\frac{\rho A}{EI}\right)\frac{\partial^2 y}{\partial x^2} = 0 \qquad ...(5)$$

Q. 7. (b) Obtain frequency equation for the lateral vibration of a cantilever of uniform section having a length l.

Ans.



According to Newton's second law of rotation

$$T = I.\alpha$$

Net torque can be written as

$$\left(T + \frac{\partial T}{\partial x}dx\right) - T = \frac{Id^2q}{dt^2}$$

$$\frac{\partial T}{\partial x}dx = \frac{Id^2\theta}{dt^2} \qquad ...(1)$$

From strength of Materials, we know that

$$\frac{T}{J} = G\frac{d\theta}{dx}$$

$$T = \frac{GJd\theta}{dx}$$

$$\frac{\partial T}{\partial x}dx = GJ\frac{\partial}{\partial x}\left(\frac{\partial \theta}{\partial x}\right)dx \quad ...(2)$$

From equation (1) & (2), we get

$$GJ\frac{\partial}{\partial x}\left(\frac{d\theta}{dx}\right)dx = I\frac{d^2\theta}{dt^2} \quad ...(3)$$

For a shaft of constant cross-section GJ is constant, and

$$J = \frac{\pi}{32}d^4$$

$$I = \frac{\pi}{32}d^4\rho dx \quad ...(4)$$

Putting the values of I and J from the above equation in (3) equation

$$\frac{G}{\rho}\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2\theta}{\partial t^2}$$

$$\frac{\partial^2\theta}{\partial x^2}(x,t) = \frac{1}{a^2}\frac{\partial^2\theta}{\partial t^2}(x,t) \quad ...(5)$$

Where, $a^2 = \frac{G}{\rho}$

Q. 8. Write the short notes on the following four :

(a) Fourier Series expansion,

(b) Impulse Excitation,

(c) Dunker Lay's Equation,

(d) Logarithmic decrement.

Ans. (a) Fourier series expansion : With the help of the mathematical series known as Fourier series, the vibration results obtained experimentally can be analysed analytically. If $x(t)$ is a periodic function with period T, the Fourier series can be written as

$$x(t) = \frac{a_0}{2} + a_1\cos wt + a_2\cos 2wt + a_3\cos 3wt + ...$$

$$+b_1 \sin wt + b_2 \sin 2wt + b_3 \sin 3wt + \ldots$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nwt + b_n \sin nwt)$$

Where, $w = \frac{2\pi}{T}$ is the fundamental frequency and $a_0, a_1, a_2 \ldots\ldots, b_1, b_2, b_3$ are constant coefficients.

(b) Impulse excitation :

The external excitation is in the form of motion and so produced by one dynamic system to another. Both such systems are connected together rigidly and form one dynamic system having several degrees of freedom. Another excitation is internal and occurs due to unbalance in the system. There are various reasons of unbalance in the system few of them which are listed here

1. Thermal effects.
2. Resonance.
3. Loose or defective mating part.
4. Bent shaft.
5. Magnetic effects.

(c) Dunker lay's equation :

Natural frequencies of structures are evaluated by this method. This method is used to find the natural frequency of transverse vibrations. The load of the system is uniformly distributed. Dunkerley's equation can be written as,

$$\frac{1}{w^2} = \frac{1}{w_1^2} + \frac{1}{w_2^2} + \ldots\ldots + \frac{1}{w_{s4}^2}$$

Where

$w \rightarrow$ Natural frequency of transverse vibration of shaft for many point loads.

w_1, w_2, w_3 etc. $\rightarrow$ Natural frequency of transverse vibration because of the weight of shaft.

(d) Logarithmic decrement : It is defined as the natural logarithm of the ratio of any two successive amplitudes on the same side of the mean line. Let us refer for successive amplitude x_1 and x_2.

As per the definition, logarithmic decrement δ is given as,

$$\delta = \ln \frac{x_1}{x_2}.$$