7. (a) If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$, find value of

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

(b) Expand $\sin xy$ in powers of $(x-1)$ and $\left(y-\frac{\pi}{2}\right)$.

SECTION – D

8. (a) Find the area lying inside the circle $r = a\sin\theta$ and outside the cardioid $r = a(1-\cos\theta)$.

(b) By changing the order of integration, evaluate :

$$\int_0^a \int_{x^2/a}^{2a-x} xy\, dy dx .$$

9. (a) Derive the relation between Beta and Gamma functions.

(b) Find the volume of the solid formed by the revolution about the x–axis by the curve $y^2 = x^2(a-x)$.

Roll No. .....

24002

B. Tech. Ist Semester

Examination – December, 2015

MATHEMATICS - I

Paper : Math - 101-F

Time : Three Hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in total selecting at least ***one*** question from each Section. Question No. **1** is ***compulsory***.

1. (a) Examine the convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1} - \sqrt{n^3}\right)$.

(b) Define the rank of matrix and hence find rank of

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}.$$

(c) If $x = r\cos\theta$, $y = r\sin\theta$, find value of $\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}$.

(d) Evaluate $\iint_R y\,dx\,dy$, where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

SECTION – A

2. (a) Examine the convergence of

$$1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+\ldots\ldots+\frac{2^{n+1}-2}{2^{n+1}+1}x^n+\ldots\ldots(x>0).$$

(b) Test the convergence of the series

$$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\frac{4!}{5^4}x^4+\ldots\ldots\ldots\infty$$

3. (a) Test the convergence of the series

$$\sum_{n=1}^{\infty}\frac{1}{(n+1)\log(n+1)}$$ by integral test.

(b) For what values of x is the series convergent

$$x-\frac{x^2}{2^2}+\frac{x^3}{3^2}-\frac{x^4}{4^2}+\ldots\ldots\ldots\ldots$$

SECTION – B

4. (a) For the matrix

$$A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}.$$

find non-singular matrices P and Q such that PAQ is in the normal form.

(b) If A and B are orthogonal matrices, prove that AB is also orthogonal.

5. (a) Verify Cayley - Hamilton theorem for the matrix

$$A=\begin{bmatrix}7 & -1 & 3\\ 6 & 1 & 4\\ 2 & 4 & 8\end{bmatrix}.$$

(b) Diagonalise the matrix $A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$.

SECTION – C

6. (a) Find the n^{th} derivative of $\frac{x^4}{(x-1)(x-2)}$.

(b) Find the asymptotes of the curve r = a (sec θ + tan θ)