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B.E.

Third Semester Examination, Dec-2007

DISCRETE STRUCTURE

Note : Attempt any five questions.

Q.1. Out of total of 130 students, 60 are wearing hats to class, 51 are wearing scarves and 30 are wearing both hats and scarves. Of the 54 students who are wearing sweaters, 26 are wearing hats, 21 are wearing scarfs and 12 are wearing both hats and scarves. Everyone wearing neither a hat nor a scarf is wearing gloves.

(a) How many students are wearing gloves?

(b) How many students not wearing a sweater are wearing hats but not scarves?

(c) How many students not wearing a sweaters are wearing neither a hat nor a scarf?

Ans.

Total students	= 130
Wearing hats	= 60
Wearing scarves	= 51
Hat & scarfts	= 30
Sweaters	= 54
Hats	= 26
Scarves	= 21
Hat & scarves	= 12

(a) $= 130 - (60 + 30) = 130 - 90 = 40$

(b) $= 130 - (51 + 30)$

$= 130 - 81 = 49$

(c) $54 = (6 + 21)$

$= 54 - 45$

$= 9$

Q. 2 Consider the following : sales of Houses fall OFF if interest rates rise. Auctioneers are not happy (if sales of houses falls off. Interest rates are rising.). The question is : are these statements results in a tautology?

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Ans. Consider :

P = Sales of house fall off

Q = Interest rates rise

R = Outioneers are not happy

The given arguments can be written is symbolic form as follows :

$P \to Q$

$R \to (P \to Q)$

P	**Q**	**R**	**$P \to Q$**	**$R \to (P \to Q)$**
T	T	T	T	T
T	T	F	T	T
T	F	F	F	F
T	F	T	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	F	F

Hence it is proved it is tautology.

Q. 3. (a) Fifteen basket all players are to be drafted by the three professional teams in Boston, Chicago and New York such that each team will Draft five players. In how many ways can this be done?

(b) Fifteen basketball players are to be divided into three teams of five players each. In how many ways this can be done.

Q 4. Let a_r denotes the number of edges in a complete graph on r vertices :

(a) Drive a recurrence relation for a_r in terms of a_{r-1}.

Ans. Complete graph means that every vertex is connected with all other vertices present in the graph. Now given that in complete graph a_r denotes the number of edges and r denotes the number of vertices.

Now if there is one vertex, there will not be any edge present. Whereas if there are 2 vertices then one edge is present. Similarly for 3 vertices, 2 edges will be present and so on.

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Now the equation according to the recurrence relations will be,

$$a_r = (r-1)$$

Where putting the different values of r we will be getting number of edge present.

Q. 4. (b) Solve the recurrence relation.

Ans. The equation can be re-written as

$$a_r + a_{r-1} + a_{r-2} + (r-1) + (r-2) + (r-3)$$

$$a_r + a_{r-1} + a_{r-2} = 3r - 6 = 3(r-2)$$

Consider L.H.S.

Putting R.H.S. = 0

$$(s^2 + s + 1) = 0$$

The roots of this characteristic equations are imaginary i.e.

$$S = \frac{-1+i\sqrt{3}}{2} \text{ and } \frac{-1-i\sqrt{3}}{2}.$$

Therefore, the homogeneous solution of the equation is

$$a_r(t) = \left[\frac{-1+i\sqrt{3}}{2}\right]^r c_1 + \left(\frac{-1-i\sqrt{3}}{2}\right)^r C_2$$

$$a_r(p) = \frac{3}{2}\left[r^3 - r^2\right]$$

$$a_r = \left[\frac{-1+i\sqrt{3}}{2}\right]^r c_1 + \left[\frac{-1-i\sqrt{3}}{2}\right]^r c_2 + \frac{3}{2}\left[r^3 - r^2\right]$$

Q. 5. Let (A, *) be a semi-group and e be a left identity. Furthermore, for every x in A there exists A such that $\hat{x} * x = e$:

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Show that for any a, b, c in A if $a * b = a * c$ then $b = c$.

Ans. Given that $a, b, c \in A$ also $a * b \Rightarrow a * c$

We have, $$b = eb = (a^{-1}a)b = a^{-1}(ab) = a^{-1}(ac) \quad [\because a * b = a * c]$$

$$= (a^{-1}ac) = ec = c$$

Hence $$a * b = a * c \Rightarrow b = c.$$

Q. 5. (b) Show that (A, *) is a group by showing that e is an identity element.

Ans. Consider that there are two identity elements e and e'

Since $e \in A$ and e' is an identity we have,

$$e'e = ee' = e$$

Also, $e' \in A$ and e is an identity we have,

$$e'e = ee' = e'$$

$$\therefore \quad e = e'$$

Hence e is an identity element.

Q. 6. (a) Show that a regular binary tree has an odd number of vertices.

Ans. This can be proved by Induction. The only node at depth $d = 1$ is root node. Thus the maximum number of nodes on depth $d = 1$ is $2^1 - 1 = 1$.

Now assume that it is true for depth k, $a > k \geq 1$. Therefore, the maximum number of nodes on depth k is $2^k - 1$.

The maximum number of nodes on depth $k - 1$ is $2^{k-1} - 1$. Since, we know that each node in a binary tree maximum number of modes on depth $d = k$ is twice. The maximum number of nodes on depth $k - 1$.

the depth $d = k$, the maximum member of nodes is $(2.2^{k-1}) - 1$

$$= 2^{k-1} - 1 = 2^k - 1$$

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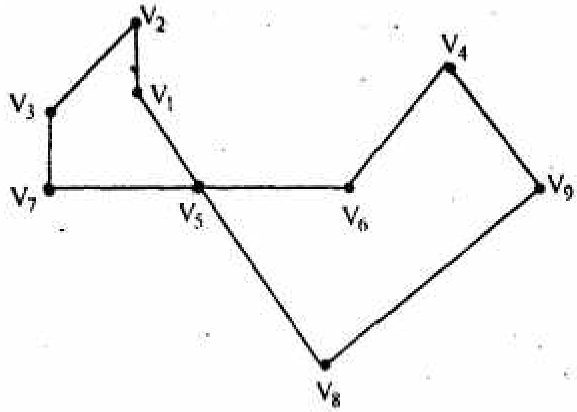
Q.4 (b) Prove that the compliment of a spanning tree does not contain a cut-set and that the compliment of a cut-set does not contain a spanning tree.

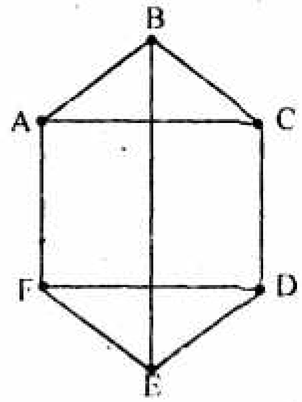
Ans. Consider a connected graph G = (V, E). A cut set for G is smallest set of edges such that removal of he set, disconnects the graph whereas the removal of any proper subset of this set, left a connected subgraph.

In this graph given below the edge set $\{V_1, V_5, (V_7, V_5)\}$ is cutset. After removal of this set, we left with disconnected sub graph while after removal of any of its proper subset we have left with a connected subgraph.



Spanning Tree :

A tree T_1 is called spanning tree of T is T_1 contains all the nodes of T. e.g., given in following diagram.



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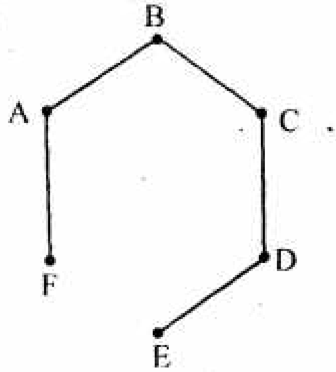
The spanning tree is



Q.7 (a) Suppose G is a finite cycle free graph with at least one edge. Show that G has at least two vertices of degree 1.

Ans. Let G be the graph having n vertices, now of which is isolated and $n-1$ edges where $n \geq 2$. By using induction on n; the case n = 1 is trivial. Thus, G can be connected on $n \geq 2$ vertices. Choose arbitrary vertex V of G and consider graph H=G/V. H is not necessarily connected. Suppose H has connected component Z having n_i vertices that is $n_1 + \ldots\ldots n_k = n-1$ edges. More over, v must be connected in G with each of the components Z_i by atleast one edge. Thus, G contains atleast $(n_1-1)+\ldots\ldots+(n_k-1)+k = n-1$ edge. Hence proved.

Q.7. (b) Show that a connected graph G with n vertices must have at least n-1 edges.

Ans. Suppose G is a graph with n vertices. It is a connected graph. WE prove that (n-1) edges by induction on n. If n = 1 it is tree. Assume that it is free for n = 1, 2, ... (n-1). Since every edge is a bridge, the subgraph G' obtained from G after deleting an edge will have two components G_1 and G_2 with n_1 and n_2 vertices respect. Where $n_1 + n_2 = n$. By induction hypothesis, the number of edges in both the component together is $(n_1-1)+(n_2-1)=(n-2)$. Thus, the number of edges in G will be $(n-2)+1=(n-1)$ which is true.

Q. 8. Write short notes on (any three) :

(a) Eulerian paths and circuits

(b) Classes or sets

(c) Planner graphs and binary trees

(d) Lagrange's theorem.

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Ans. Eulter path :

An Euler path through a graph is a path whose edge list contains each edge of the graph exactly once.

Euler circuit :

An euler circuit is a path through a graph, in which the initial vertex appears second time as the terminal vertex.

Euler graph :

An euler graph is a graph that possesses an euler circuit. An euler circuit uses every edge exactly once but vertices may be repeated.

(b) Planner graphs and binary trees :

A set is defined as a collection of distinct objects of same-type or classes of objects. The objects of a set are called elements or member of sets. The set can be of following types :

(a) Finite Set :

If a set consists of specific number of different elements then that set is called finite sets.

(b) Infinite Set : If a set consists of infinite number of different elements or if the counting of different elements of the set does not come to an end. the set is called infinite set

(c) Disjoint set : Two sets A and B are said to be disjoint if no element of A is in b and no element of B is A e.g.,

$$R = \{a, b, c\},\ S = \{k, p, m\}$$

R and S are disjoint sets.

(d) Null set or empty set :

The set that contains no element is called null set or the empty set and is denoted by of.

(e) Power set :

The power set of any given set A is the set of all subsets of A And is denoted by P(A). If A has n elements then P(A) has 2^n elements.

(c) Planar graph :

A graph is said to be planar if it can be drawn is a place so that number edge cross.

Binary Tree :

If the out degree of every node is less than or equal to 2, in a directed tree than the tree is called binary tree. A tree consisting of no nodes is also a binary tree.

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(d) Lagrange's theorem :

In mathematics of group theory, it states that for any finite group G, the order/number of elements) of every subgroup H of G divides the order of G. This can be proved using the concept of left cosets of H in G. The left cosests are the equivalence classes of a certain equivalence relation on G and therefore from partition of G. If we can show that all cosets of H have same number of elements then we are done, since H itself is a coset of H. Now if aH and bH are two left cosets of H, we can define a map of : $aH \to bH$ by setting $f(X) = ba^{-1}X$.

Thus proof also shows that the quotient of the order $|G|||H|$ is equal to the index $[G:H]$. If we write this statement as,

$$|G| = [G:H].[H]^{-1} \text{ then,}$$

interpreted as a statement about cardinal number, it remains true even for infinite groups G and H.