B. E.

Third Semester Examination, Dec-2006

DISCRETE STRUCTURE

Note : Attempt any five questions.

Q.1 (a) In a class of 100 students, 39 play Tennis, 58 play Cricket, 32 play hockey, 10 play Cricket and Hockey, 11 play Hockey and Tennis and 13 play Tennis and Cricket. Find number of students who play all the three games.

Ans.

$$T = 39 \qquad H + T = 11$$
$$C = 58 \qquad T + C = 13$$
$$H = 32 \qquad T + C + H = 39 + 58 + 32$$
$$C + H = 10$$

Total = 129 students.

$$\frac{H+T}{11} + \frac{T+C}{13} + \frac{C+H}{10} = 2(H+T+C) = \frac{34}{2}$$

Total = 129

Ans = 17 Total no. of students who play all games.

Q.1 (b) If $A = \{a, b, c, d, e\}$ and

$$R = \{(a,a), (a,b), (b,c), (c,e), (c,d), (d,e)\}.$$

Then compute R^2 and matrix of the relation R^2, also obtain digraph of R^2.

Ans.

$$A_1 = a \qquad B_1 = a$$
$$A_2 = b \qquad B_2 = b$$
$$A_3 = c \qquad B_3 = c$$
$$A_4 = d \qquad B_4 = d$$
$$A_5 = c \qquad B_5 = e$$

$$(a,a) = R_{11} = 1$$
$$(a,b) = R_{12} = 1$$
$$(b,c) = R_{23} = 1$$

and

$$(c,e) = R_{35} = 1$$

$$(c,d) = R_{34} = 1$$

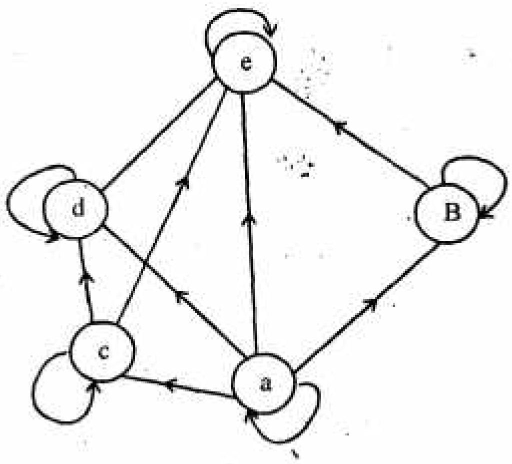
$$(d,c) = R_{45} = 1$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix}$$

$$R^2 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Final R^2 is,

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



Q. 1 (c) Let R be a relation defined on the set of positive integers as : Let a, $b \in Z^*$, we say that a R b iff $a = b^k$ for some $k \in Z^*$. Determine whether R is an equivalence relation.

Ans. A relation r on a set A is called an equivalence relation if and only if it is reflexive, symmetric and transition.

The classical end of an equivalent relation is the relation = on R. In fact, the term equivalence relation is used because those relation which satisfy the definition behave quite like = relation.

Let us take also e.g. $a, b \in z^+$, the relation q on $z * z^+$ defined by,

$$(a, b) q (c, d)$$

If and only if $a = b^k$

$$\boxed{ad = bc}$$

Two order pain are :

$$(a, b) \ \& \ (c, d)$$

Are related if the fractions a/b & c/d are numerically equals.

Q. 2. (a) If $(A, \leq)$ and $(B, \leq)$ are posets, the prove that $(A \times B, \leq)$ is also a poset, with partial order defined as :

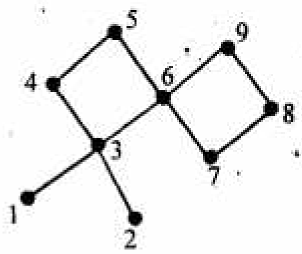
$$(a, b) \leq (a^1, b^1) \text{ in } A \times B \text{ if}$$

$a \leq a^1$ in A and $b \leq b^1$ in B, where $\leq$ denotes the partial order on the respective sets.

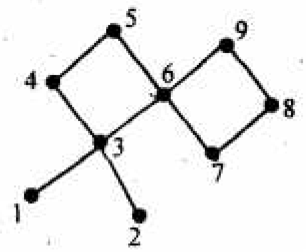
Ans.

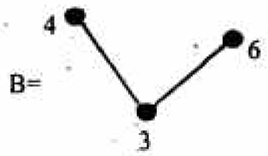
Q. 2. (b) Given the Hasse diagram of a poset, determine all upper bounds, lower bounds, the l.u.b and g.l.b of the subset

$$B = \{3, 4, 6\} \text{ of } A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} :$$



Ans.

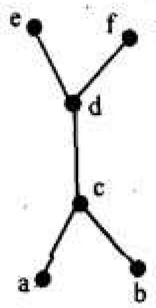




All upper 6 brands, {4, 5, 6, 9, 8}

Are lower bands are : [1, 3, 2, 7].

Q. 2. (c) Determine whether the following Hasse diagram represents a lattice :



Ans.

Let L be a poset under an ordering, Let a, $b \in L$ we define.

$a \vee b$ (read " a join b") as lub of a & 6 and,

$a \wedge b$ (read "a meet b") as geb of a and b,

$\boxed{a \le e_1, b \le e_1 \text{ and } a \le e_2, b \le e_2}$.

Q. 3 (a) Define the terms' Propositional function; universal and existential quantification of a predicate by giving one example in each case.

Ans. Proposition over a universe. Let U be a non empty set. A proposition over U is a sentence that contains a variable that can take on any value in V and which has a definite truth value as a result of any such substitution.

Fog e.g :

(a) $4x^2 - 3x = 0$

$0 \le n \le 5$, k is multiple of 3.

(b) A few propositions over the rational o. are $4x^2 - 3x = 0$, $y^2 = 2$, $(s-1)(s+1) = s^2 - 1$.

(c) A few proposition over the subsets of P are $(A = \phi) \vee (A = P)$, $3 \in A$, $A \cap \{1,2,3\} \neq \phi$.

All of the ones of logic that we listed in section above are valid for proposition over a universe.

For e.g. if p & q are proposition over the integers we can be certain that $p \wedge q \Rightarrow p$ because $(p \wedge q) \rightarrow p$ is a tautology, & is true no mother what values the variables in p & q are given.

Q. 3 (a) Prove that,

$\exists \times (P(x) \Rightarrow Q(x)) \equiv \forall \times P(x) \Rightarrow \exists \times Q(x)$, **where symbols have their usual meanings.**

Ans. The we define $P(x) \Rightarrow Q(x)$, here X is fish and Q (x) is lives in water. We know that the proposition $P(x) \rightarrow Q(x)$ is not always true. In other words.

$$(\forall x)(p(x) \rightarrow Q(x)) \text{ is false.}$$

Another way of stating this fact is that there exists an animal that lives in the water and is not a fish.

i.e. $$\sim (\forall x)(p(x) \Rightarrow Q(x))$$

$$\Leftrightarrow (\exists x) \Leftrightarrow (\sim (p(x) \Rightarrow Q(x)))$$

$$(\exists x)(p(x) \wedge Q(x)).$$

Q. 4. (a) Solve the following recurrence relation :

$$a_r - 6a_{r-1} + 9a_{r-2} = r.3^r$$

Ans. $$a_r - 6a_{r-1} + 9a_{r-2} = r.3^r$$

It like as :

$$a_n = A_{an^{-1}} + B_{an^{-2}}$$

Suppose we have solution $a_n = r^n$,

$$r^n = Ar^{n-1} + Br^{n-2}$$

Dividing through by r^{n-2} we get,

$$r^2 = Ar + B$$

$$r^2 - Ar - B = 0$$

It called characteristics equation of recurrence relation.

Solve for r to obtain the two roots λ_1, λ_2 & if these roots are distinct. We have solutions,

$$a_n = \left(\lambda_1^n + D\lambda_2^n\right).$$

Q. 4. (b) Solve the following recurrence relation using generator functions :

$$\mathbf{a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r,\ r \geq 2,\ a_0 = 1 = a_1.}$$

Ans.

$$a_r = 5a_{r-1} + 6a_{r-2}$$

$$= 2^r + r,\ r \geq 2,\ a_0 = 1 = a_1$$

$$a_{n+1} = 2a_n + 3^n + 5n$$

$$b_{n+1} = 2b_n$$

$$b_n = c_1 2^n$$

$$a_n = c_2 3^n + c_3 n + c_4$$

$$c_2^{n+1} + c_3^{(n+1)} + c_4$$

$$= 2\left(c_2 3^n + c_3^n + c_4 + 3^n + 5n\right)$$

$$+3^n + 5n$$

$$3c_2 = 2c_2 + 1$$

$$c_2 = 1$$

Now coefficient of n is, $= c_3 = 2c_3 + 5$

$$c_3 = -5$$

So generation solution $= a_n = c_1 2^n + 3^n - 5n - 5$.

Q. 5. (a) How many people among 2,00,00 people are born at the same time (i.e., same hour, minute and second)?

Ans. Suppose that for some $k \geq 2$ are of integers 2,3, ... k have a prime decomposition. Consider $k+1$.

Either $k+1$ is prime or is not. If $k+1$ is a prime, if is already described into primes.

If not, then $k+1$ has a divisor, d, other hand 1 and $k+1$. Hence $k+1 = cd$, where c & d are between 2 and k. By the induction hypothesis c & d have prime decomposition $c_1 c_2 .....c_n$ and $d_1 d_2 .....d_n$ respective. Therefore $k+1$, has prime decrease

Q. 5. (b) Five fair coins are tossed and the results are recorded. Find

(i) How many different sequences of heads and tails are possible?

(ii) How many of the sequences in part (i) have exactly three heads recorded?

Ans. $q(0)$ states that $p(0,0)$ = the no. of ways that no element can be selected from ϕ and.

If arranged in order $= 0!/0!$

$= 1$, which is true.

A general law in combinations is that there is exactly one way of doing nothing.

Total no. of combinations are given as : $\frac{\text{heads}}{1} + \frac{\text{tails}}{1} = 2$

Total 8 combination and possible.

Q. 6 ~~(a)~~ Let H be a subgroup of a finite group G. Then prove that $G = \bigcup_{a \in G} Ha$.

Ans. (a) H is closed under * i.e., a, b in H implies that a * b is also in H.

(b) H contains the identify element for *; and

(c) H contains the inverse of every one of its elements if a is in H, that a^{-1} is also in H.

For every group there is at least 2 elements and there are at least 2 subgroups. They are the where group and {e}, since there [2] are automatic called improper subgroup of the group other are proper subgroups.

Q. 6. (b) Define the following terms by giving two examples in each case :

(i) Group Homomorphism

(ii) Integral domain

(iii) Field

(iv) Cyclic group.

Ans. (i) Group Homomorphism : The homomorphism illustrates how closely the two structures remarks each other for this reason, the term homomorphic is rarely used and the function, the homomorphism are structured.

Homomorphism Let [G; *] and [G'; #] be groups. $Q: G \to G'$ is a homomorphism if $Q(x * y) = Q(x) \# Q(y)$ for all $x, y \in G$.

(ii) Integral Domain : A commutative ring with unity containing no discussion of zero is called integral domains. The are universally denoted by D.

(iii) Field : A field is a commutative ring with unity such that each non-zero element has a multiplicative universe.

A field is frequently designated generically by latter f. For eg : $[Q;+,\bullet]$

$$[R;+,\bullet]$$

(iv) Cyclic Group : Group G is cyclic if those exists $a \in G$ such that the cyclic subgroup generated by a, (a), equals all of G. That is $G = \{na / n \in z\}$ in which a is called a generator of G. The reader should note that additive notation is used for G.

Q. 7. (a) Evaluate the following post fix forms :

(i) $432+-5\times 42\times 5\times 3++$

(ii) $x2-3+23y+-w3-x+$, where x is 7, y is 2, and w is 1.

Ans.

Q. 7. (b) Define the following :

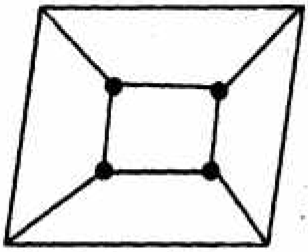
(i) Eulerian path **(ii) Hamiltonian path**

(iii) Planar graph **(iv) Isomorphic graphs** **(v) Binary tree.**

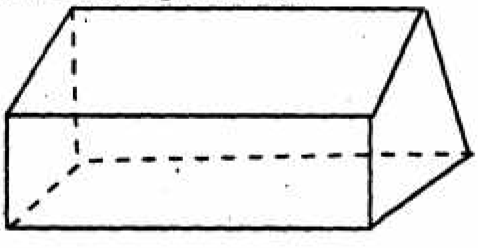
Ans. (i) Eulerian Path : If some closed walk in a graph containing all the edges of the graph then the work is called euler line.

(ii) Hamiltonian Path : A circuit is a connected graph is defined as a closed mark that transverses every vertex of G exactly once except of course the structing vertex. The graph containing the Hamiltonian circuit is called.

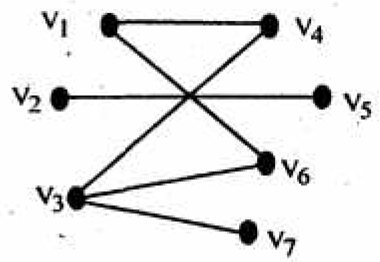
Hamiltopnian graph :



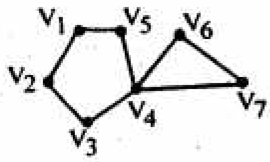
(iii) Planar Graph : A graph G is said to be planar if there exists same geometric relation of G which can be drawn on a plane such that no two of its edges intersects.

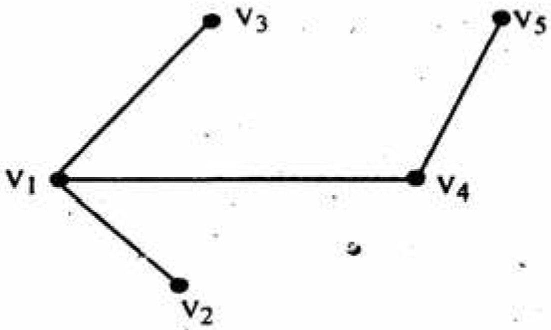


(iv) Isomorphic Graphs : A graph is called isomorphic if its vertex set V can be decomposed into two disjoint subsets V_1 & V_2 such that every edge in G join a vertex in V_1 with a vertex in V_2 .



(v) Binary Tree : A tree is a connected graph without clicks & there is only one path between every pain of vertices in a true T. A true with n vertices has $(n-1)$ edges.





Q. 8. (a) State Euler formula for connected planar graph. Verify this by giving two examples.

Ans. State Euler Formula : Since a planar graph may have different plane relation so Euler formula gives the no. of regions in any planer graph.

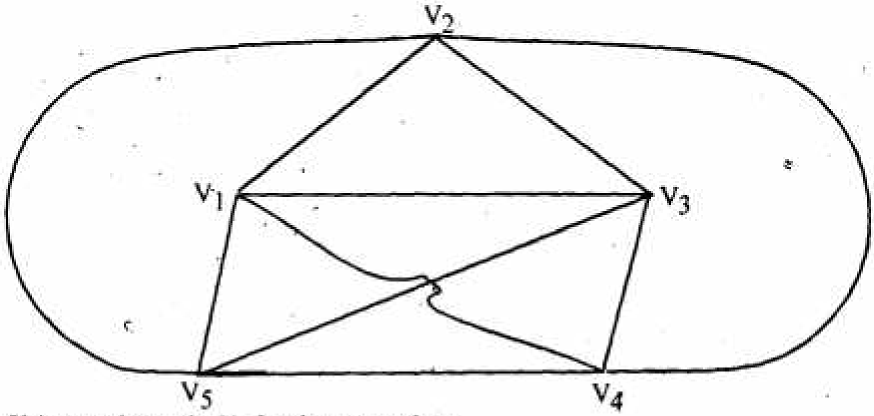
Fog e.g.: A connected planar graph with n vertices & e edges has $e-n+2$ regions. (Theorem).

Corollary : In any simple, connected graph (planar graph) with of regions, n vertices & e edges $(e>2)$ the following in equalities must hold :

$$\boxed{\begin{array}{l} e \geq 3/2f \\ e \geq 3n-6 \end{array}}$$

In case of k5, the complete graph of the 5 vertices are as :

$n = 5$, $e = 10$, $3n-6 = 3\times 5-6 = 9 < e$, Here k_5, is non planar.



Q. 8. (b) Using mathematical induction prove that :

$$1+2+2^2+2^3+\ldots\ldots+2^n = n^{n+1}-1$$

Ans. Using mathematical induction prove that :

$$\boxed{1+2+2^2+2^3+\ldots\ldots+2^n = 2^{n+1}-1}$$

Assume that,

$n \geq 1$ and

$2^{n+1}-1$ is a $q_0 - q_1, \ldots\ldots q_{n-1} \to q_n q_0$

Are the propositions and its justification is premises.

$$1+2+2^2+2^3+\ldots\ldots2^n$$

$$= 2^{n+1}-1$$

$$= (1+2+\ldots+n)+(n+1)$$

$$= n(n+1)/2+(n+1)(p(n)\text{ used here})$$

$$= n(n+1)/2+2(n+1)/2$$

$$= (n+1)(n+2)/2$$

$$= ((n+1)(n+1)+1)/2$$

Since there is a chain reaction.

Since $p(1) \Rightarrow p(2)$ is true then $(p(2)) \Rightarrow p(3)$ is also true and so on.