Roll No. [ ] Total No. of Pages : 02

Total No. of Questions : 09

B.Tech. (CE/ECE/ETE/EE/EEE) (Sem.–3rd)

ENGINEERING MATHEMATICS-III

Subject Code : BTAM-301 (2011 Batch)

Paper ID : [A1128]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students has to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students has to attempt any TWO questions.

SECTION-A

1. Write briefly :

a) Define a periodic function and period of a periodic function.

b) Define unit step function and write its Laplace Transform.

c) Let $f(t) = e^t \sin t$. Find $L[f(t)]$.

d) Prove the recurrence formula: $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$.

e) Form a partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$

f) Solve the linear partial differential equation :
$(2D^2 + 5DD' + 2D'^2)z = 0$.

g) Illustrate the method of separation of variables to solve one dimensional heat flow equation : $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

h) Prove that $f(z) = \bar{z}$ is not analytic at any point in the complex plane.

i) Find the value of $\int_C \frac{e^z}{(z-3)^2} dz$ where C is the circle $|z| = 2$.

j) Find all invariant/fixed points of the transformation $w = \frac{1+z}{1-z}$

SECTION-B

2. Let $f(x) = \begin{cases} \pi x, & 0 \le x \le 1, \\ \pi(2-x), & 1 \le x \le 2. \end{cases}$ Show that in the interval (0,2),

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \ldots\right]$$

Also deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$ (4+1)

3. a) Find the Laplace Transform of $f(t) = t^2 \cos 2t$.

b) Find the inverse Laplace Transform of $f(s) = \frac{1}{s(s+2)^3}$ (2,3)

4. Obtain the solution of following differential equation in terms of Bessel functions :

$$y'' + \frac{y'}{x} + \left(1 - \frac{1}{9x^2}\right)y = 0$$ (5)

5. Solve the following partial differential equation :
$(D^3 - 3D^2D' + 4D'^3)z = e^{x+2y}$. (5)

6. State Cauchy's integral formula and use it to evaluate $\int_C \frac{2z+1}{z^2+z} dz$, where C is $|z| = \frac{1}{2}$ (5)

SECTION-C

7. A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y = y_0 \sin^3(\pi x/l)$. If it is released from rest from this position, find the displacement $y(x,t)$. (10)

8. a) Use the method of residues to evaluate the integral $\int_C \frac{e^z dz}{z^2+1}$, C : $|z|=2$.

b) Find the bilinear transformation which maps 1,i,–1 to 2,i,–2, respectively. (5,5)

9. Use the method of Laplace Transforms to solve the following differential equation :
$y''(t) + 2y(t) + 5y(t) = e^t \sin t$; where $y(0) = 0$, $y'(0) = 1$. (10)