

Roll No.

Total No. of Questions : 09]

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Paper ID [A0459]

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B.Tech. (Sem. - 3rd/4th)

MATHEMATICS - III (CS - 203/204)

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

- 1) Section - A is **Compulsory**.
- 2) Attempt any **Four** questions from Section - B.
- 3) Attempt any **Two** questions from Section - C.

Section - A

Q1)

(10 × 2 = 20)

a) Discuss the relationships between limit, continuity and differentiation of a function.

b) Define fundamental theorem of Integral Calculus.

c) Distinguish between Trapezoidal and Simpson's $\frac{1}{3}$ rd rule of numerical integration.

d) Define analytic function and Cauchy-Riemann equations.

e) Poles and residues of analytic functions.

f) Define residue theorem of functions of complex variables.

g) Explain the conditions for the existence of Laplace transform of a function.

h) Define conformal mapping.

i) Discuss the categorization of Laplace, wave and heat equations.

j) State Rolle's theorem.

Section - B

(4 × 5 = 20)

Q2) Determine the analytic function $f(z) = u + iv$ if

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$$

and $f\left(\frac{\pi}{2}\right) = 0$.

Q3) Evaluate

$$\int_C \frac{z-3}{z^2+2z+5} dz$$

where C is the circle $|z+i+1|=2$.

Q4) An impulse I (kg - sec) is applied to a mass m attached to a spring having a spring constant k . The system is damped with damping constant μ . The differential equation governing the phenomena is

$$m \frac{d^2x}{dt^2} + kx + \mu \frac{dx}{dt} = I \delta(x)$$

where $\delta(x)$ is unit impulse function. Derive expression for displacement $x(t)$ of the mass, assuming initial conditions $x(0) = x'(0) = 0$.

Q5) Find the area included between the curves $y^2(2a-x) = x^3$ and its asymptote.

Q6) Employ Taylor's method to obtain approximate solution of y at $x = 0.2$ for

the differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$.

Section - C

(2 × 10 = 20)

Q7) Using Runge-Kutta method of order 4, compute $y(0.2)$ and $y(0.4)$ from

$$10 \frac{dy}{dx} = x^2 + y^2, \quad y(0) = 1.$$

taking $h = 0.1$.

Q8) A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement $y(x, t)$, using separation of variable technique.

Q9) Show that the transformation

$$w = z + \frac{a^2}{z}$$

transforms circles with origin at the centre in the z -plane into coaxial concentric, confocal ellipses in the w -plane.

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