GUJARAT TECHNOLOGICAL UNIVERSITY
BE- VIIth SEMESTER–EXAMINATION – MAY/JUNE- 2012

Subject code: 171901   **Date: 24/05/2012**
Subject Name: Operation Research
Time: 02:30 pm – 05:00 pm   **Total Marks: 70**
Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Explain significance of any two assumptions of Linear Programming Problem (LPP).   **07**

A small fabrication industry is faced with a problem of scheduling production and subcontracting for three products A, B and C. Each product requires casting, machining and assembly operations. Casting operation for product A and B can be subcontracted but product C requires special tooling hence it can not be subcontracted. Each unit of product A, B and C requires 6, 10 and 8 minutes of casting time in the foundry shop of a company. Machining times per unit of products A, B and C are 6, 3 and 8 minutes while assembly times are 3, 2 and 2 minutes respectively. The time available per week in foundry, machining and assembly shop are 8000, 12000 and 10000 minutes respectively. If product A, B and C are produced completely in the company, the overall profits per unit of product are Rs. 700, Rs. 1000 and Rs. 1100 respectively. When castings are obtained from subcontractors, the profit per unit of product A and B are Rs. 500 and 900 respectively. Formulate above problem as LPP so as to maximize the profit for company by scheduling its production and subcontracting.

(b) Solve the following LPP using Simplex method;   **07**
$Maximize\ (Z)=6x_1+4x_2$

$subject\ to$

$2x_1+3x_2 \leq 30$; $3x_1+2x_2 \leq 24$; $x_1+x_2 \geq 3$

$x_1, x_2 \geq 0$

Q.2 (a) A transport company has 5, 10, 7 and 3 trucks available at four different sites A, B, C and D. Its customers have requirement of 5, 8 and 10 trucks at three different destinations X, Y and Z respectively. The distance (in kms.) from an origin to destination is summarized in following table.   **07**

Sites	Customers		
	X	Y	Z
A	70	30	60
B	40	60	80
C	50	80	40
D	80	40	30

Formulate above problem as a transportation problem and determine strategy for a company using VAM. Test the optimality of VAM solution and determine optimum strategy for the transport company.

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(b) i. What do you mean by Infeasibility and Unboundness in LPP? How are the following issues identified from the simplex tableau? **03**

ii. Construct the dual of following Primal Problem; **04**

$Minimize\,(Z) = 5x_1 - 6x_2 + 4x_3$

subject to

$$3x_1 + 4x_2 + 6x_3 \geq 9\,;\quad x_1 + 3x_2 + 2x_3 \geq 5;\quad 7x_1 - 2x_2 - x_3 \leq 10$$

$$x_1 - 2x_2 + 4x_3 \geq 4\,;\quad 2x_1 + 5x_2 - 3x_3 = 3\,;\quad x_1, x_2, x_3 \geq 0$$

OR

(b) The following tableau for a maximization type LPP is produced after few iterations of simplex method; **07**

		8.5	10.5	0	0	0
Mix	Qty (b$_i$)	X$_1$	X$_2$	S$_1$	S$_2$	S$_3$
X$_2$	300	0	1	3/5	-2/5	0
X$_1$	300	1	0	-2/5	3/5	0
S$_3$	400	0	0	-1/5	-1/5	1

Answer the following questions with brief reasons from above table;
i. Does the tableau represent an optimal solution? If not, carry out necessary iterations and obtain an optimal solution.
ii. Is this solution degenerate?
iii. Are there more than single optimal solution to above problem?
iv. What are the shadow prices or dual values of resources?
v. What is optimum objective function value for the problem?
vi. If S$_1$ represent the slack for production capacity constraint, how much should company be willing to pay for each additional unit of production capacity?

Q.3 **(a)** What do you understand by 'zero-sum' in the context of game theory? Explain the meaning following terms used in game theory; **07**
i. Saddle Point
ii. Pure Strategy
iii. Mixed Strategy

(b) The captain of a cricket team has to allot five middle order batting positions to six batsmen available for selection. The average runs scored by each batsmen at these positions are summarized in a table below **07**

Batsman	Batting Position				
	III	IV	V	VI	VII
A	40	40	35	25	50
B	42	30	16	25	27
C	50	48	40	60	50
D	20	19	20	18	25
E	58	60	59	55	53
F	45	52	38	50	49

Using Assignment model, determine the assignment of batsmen to positions which would give maximum runs in favor of team. Which batsmen will not qualify for selection based on the solution obtained?

OR

Q.3 **(a)** What is 'dominance rule' in game theory? How can a 'two person-zero sum game' problem be converted into LP problem? Illustrate with example. **07**

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(b) A solicitors' firm employs typists on hourly piece-rate basis for daily work. There are five typists available with hourly charges and speed mentioned in table below. **07**

Typist	A	B	C	D	E
Rate per hour (Rs.)	5	6	3	4	4
No. pages typed/hour	12	14	8	10	11

There are five jobs available to the firm and it wishes to allocate one job to one typist only. The typist is paid for full hour even if he works for fraction of an hour. The details of job are given in table below.

Job	P	Q	R	S	T
No. of Pages	199	175	145	298	178

Find least cost allocation for the firm using Assignment model.

Q.4 (a) Following failure rates have been observed for certain type of light bulbs; **07**

Month	1	2	3	4	5
Percentage of items failing by end of month	10	25	50	80	100

There are total 1000 bulbs in use and it costs Rs. 10 to replace an individual bulb which has fused out. If all bulbs are replaced simultaneously, it would cost Rs. 4 per bulb. Two policies are being considered for replacement of bulbs; First, replace all bulbs simultaneously at fixed interval whether failed or not and do individual replacement in intermediate periods. Secondly, individual replacement of bulbs as and when it fails. Determine the optimum policy for replacement of bulbs based on above failure data and costs.

(b) What is simulation? What are different phases of simulation process? Differentiate between deterministic and stochastic simulation models. What are the advantages and limitations of simulation? **07**

OR

Q.4 (a) An electronic item contains 10000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is Rs. 1 only. If all resistors are replaced at the same time, the cost per resistor reduces to 35 paisa. The probability of failure of a resistor by the end of month is given in table below. **07**

Month	1	2	3	4	5	6
Prob. of items failing by end of month	0.03	0.07	0.2	0.4	0.15	0.15

Two policies are being considered for replacement of resistors; First, replace all items simultaneously at fixed interval whether failed or not and do individual replacement in intermediate periods. Secondly, individual replacement of items as and when it fails. Determine optimum policy for replacement of bulbs based on above failure data and costs.

Q.4 (b) i. Explain the meaning of following items in inventory management; **03**
   a. Re-order Level
   b. Buffer Stock

ii. A purchase manager has decided to place an order for a minimum quantity of 500 units of a particular item of inventory in order to get discount of 10%. Past records reveal that 8 orders (each of 200 units) were placed last year. Given ordering cost = Rs. 500 per year, Inventory carrying cost = 40% of inventory value and price of item = Rs. 400 per unit. What is the effect of this decision on company? **04**

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Q.5 **(a)** Arrival rate of telephone calls at a telephone booth follows Poisson distribution with an average time of 9 minutes between two consecutive calls. The length of telephone call is assumed to be exponentially distributed with mean of 3 minutes. **07**
  i. Determine the probability that a person arriving at the telephone booth have to wait.
  ii. Find the average queue length that is formed from time to time.
  iii. The telephone company will install a second booth when convinced that an arrival would expect to have to wait at least four minutes for the phone. Find increase in rate of arrival which will justify a second booth.

(b) A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning and ending node numbers. **07**

Activity		1-2	1-3	1-4	2-5	3-5	4-6	5-6
Time Estimates (weeks)	Optimistic	1	1	2	1	2	2	3
	Most Likely	1	4	2	1	5	5	6
	Pessimistic	7	7	8	1	14	8	15

  i. Draw the project network.
  ii. Find the expected duration and variance for each activity.
  iii. What is the expected project length and standard deviation?
  iv. What is the probability that the project will be completed 3 weeks later than the expected time?

OR

Q.5 **(a)** How is dynamic programming problem different from LPP? Explain the meaning of following terms used in dynamic programming; **07**
  i. Stages
  ii. States
  iii. Principle of optimality

(b) The time estimates and precedence relationships of different activities constituting a small construction project is given in following table; **07**

Activity	A	B	C	D	E	F	G	H	I
Predecessor	-	-	B	B	A	A	F	C, E, G	F
Duration (days)	3	8	6	5	13	4	2	6	2

  i. Draw the project network.
  ii. Determine project completion time.
  iii. What is critical path?

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