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BE.

Third Semester Examination, May-2009

MATHEMATICS-III

Note : Attempt any five questions, selecting at least one question from each part.

Part-A

Q. 1. (a) Find the Fourier series for the function,

$$f(x) = \begin{cases} x & -1 < x \le 0 \\ x+2 & 0 < x < 1 \end{cases}$$

where $f(x) = f(x+2)$

Ans.

$$f(x) = \begin{cases} x & -1 < x \le 0 \\ x+2 & 0 < x < 1 \end{cases}$$

Here interval is $(-1, 1)$. Here $c = 1$

$$a_0 = \frac{1}{C}\int_{-C}^{C} f(x)dx = \frac{1}{1}\int_{-1}^{1} f(x)dx$$

$$= \int_{1}^{0} x\,dx + \int_{0}^{1}(x+2)\,dx$$

$$= \left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} + 2x\right]_{0}^{1}$$

$$= -\frac{1}{2} + \frac{1}{2} + 2 = 2$$

$$a_n = \frac{1}{C}\int_{-C}^{C} f(x)\cos\frac{n\pi x}{C}\,dx$$

$$= \frac{1}{1}\int_{-1}^{1} f(x)\cos n\pi x\,dx$$

$$= 1\int_{-1}^{0} x\cos nx\,dx + \int_{0}^{1}(x+2)\cos n\pi x\,dx$$

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$$= \left[x\left(\frac{\sin n\pi x}{n\pi}\right) - (1)\left(-\frac{\cos n\pi x}{n^2\pi^2}\right)\right]_{-1}^{0} + \left[(x+2)\frac{\sin n\pi x}{n\pi} + \frac{\cos n\pi x}{n^2\pi^2}\right]_0^1$$

$$= \frac{1}{n^2\pi^2} - \frac{(1)^n}{n^2\pi^2} + \frac{(-1)^n}{n^2\pi^2} - \frac{1}{n^2\pi^2} = 0$$

$$b_n = \frac{1}{+1}\int_{-1}^{1} f(x)\frac{\sin n\pi x}{1}dx$$

$$= \int_1^0 x \sin n\pi x \, dxt + \int_0^1 (x+2)\sin n\pi x \, dx$$

$$\Rightarrow \quad \left[x\left(\frac{-\cos n\pi x}{n\pi}\right) - (1)\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)\right]_{-1}^{0} + \left((x+2)\left(\frac{-\cos n\pi x}{n\pi}\right) + (1)\frac{\sin n\pi x}{n^2\pi^2}\right)_0^1$$

$$-\left(\frac{(-1)(-1)(-1)^n}{n\pi}\right) + \left(\frac{-3(-1)^n}{n\pi} + \frac{2}{n\pi}\right) \Rightarrow \frac{-4(-1)^n}{n\pi} + \frac{1}{n\pi}$$

$$f(x) = \frac{a_0}{2} + \Sigma a_n \cos\frac{n\pi x}{C} + \Sigma b_n \frac{\sin n\pi x}{C}$$

$$f(x) = 1 + \sum_{n=1}^{\infty}\left(\frac{-4(-1)^n}{n\pi} + \frac{2}{2\pi}\right)\frac{\sin n\pi x}{1}$$

$$f(x) = 1 + \frac{6}{\pi}\sin n\pi - \frac{2}{2\pi}\sin 2\pi x + \ldots\ldots\infty$$

Q. 1. (b) Find the Fourier cosine and sine series of

$f(x) = 1$, $0 \le x \le 2$.

Ans. $f(x) = 1 \qquad 0 < x < 2$

(i) For the Half Range Since Series :

$$1 = \Sigma b_n \sin\frac{n\pi x}{2} \qquad (\text{since } l = 2)$$

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$$b_n = \frac{2}{2}\int_1^2 1 \sin\frac{n\pi x}{2}dx$$

$$= \left[\frac{-\cos\frac{n\pi x}{2}}{\frac{n\pi}{2}}\right]_0^2 = -\frac{2}{n\pi}\left((-1)^n - 1\right)$$

$$= \frac{4}{n\pi}, \text{ if n is odd}$$

0, if n is even

$$1 = \frac{4}{\pi}\sin\frac{\pi x}{2} + \frac{4}{3\pi}\sin\frac{3\pi x}{2} + \frac{4}{5\pi}\sin\frac{5\pi 4}{2.} + \ldots$$

(ii) For Half Range Cosine Series :

$$1 = \frac{a_0}{2} + \Sigma a_n \cos\frac{n\pi x}{2}$$

$$a_0 = \frac{2}{2}\int_0^2 1dx = [x]_0^2 = 2$$

$$a_n = \frac{2}{2}\int_1^2 1\cos\frac{n\pi x}{2}dx = \left(\frac{\sin\frac{n\pi x}{2}}{\frac{n\pi}{2}}\right)_0^2 = 0$$

$$\boxed{f(x) = 1 + 0}.$$

Q. 2. (a) Find the Fourier transform of

$$f(x) = xe^{-ax^2}, a > 0.$$

Ans. $F[f(x)] = F\left[e^{-ax^2}\right]$

First to find $F\left[e^{-x^2}\right]$

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Let $$f(x) = e^{-x^2}$$

Then

$$F[f(x)] = \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \int_{-\infty}^{\infty} e^{-x^2} e^{isx} dx$$

$$= \int_{-\infty}^{\infty} e^{-\left(x^2 - isx\right)} dx = \int_{-\infty}^{\infty} e^{-\left\{\left(x - \frac{is}{2}\right)^2 + \frac{s^2}{4}\right\}} dx$$

$$= e^{-s^2/4} \int_{-\infty}^{\infty} e^{-\left(x - \frac{is}{2}\right)^2} dx$$

$$= e^{-s^2/4} \int_{-\infty}^{\infty} e^{-z^2} dz \qquad \text{where } Z = x - \frac{is}{2}$$

$$= 2e^{-s^2/4} \int_0^{\infty} e^{-z^2} dz = 2e^{-s^2/4} \frac{\sqrt{\pi}}{2}$$

$$= \sqrt{\pi}\, e^{-s^2/4} = F(s)$$

Now,

$$e^{-ax^2} = e^{-\left(\sqrt{ax}\right)^2} = f\left(\sqrt{a}x\right)$$

By change of scale property, we have

$$F\left[f\sqrt{ax}\right] = \frac{1}{\sqrt{a}} F\left(\frac{s}{\sqrt{a}}\right)$$

$$F\left[e^{-ax^2}\right] = \frac{1}{\sqrt{a}} \sqrt{\pi}\, e^{-\frac{1}{4}\left(\frac{s}{\sqrt{a}}\right)^2}$$

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$$= \sqrt{\frac{\pi}{a}} e^{-s^2/49}$$

Thus,

$$\boxed{F\left[e^{-ax^2}\right] = \sqrt{\frac{\pi}{a}} e^{-s^2/49}}$$

Q. 2. (b) State and prove time-shifting and frequency-shifting properties for Fourier transformation.

Ans. Shifting an time axis : If f(s) is the complex Fourier transform of f(t) and to in any real number, then

$$F[f(t-t_0)] = e^{ist_0} F(s)$$

$$\therefore \quad F(f(t-t_0)) = \int_{-\infty}^{\infty} f(t-t_0) e^{ist} dt \quad \ldots(1)$$

Putting $t - t_0 = T$, i.e. $t = t_0 + T$, we have $dt = dT$, when $t \to -\infty$, $T \to -\infty$ and when $t \to \infty$, $T \to \infty$. From equation (1), we have

$$F[f(t-t_0)] = \int_{-\infty}^{\infty} f(T) e^{is(t_0+T)} dT$$

$$= \int_{-\infty}^{\infty} f(T) e^{ist_0} e^{isT} dT$$

$$= e^{ist_0} \int_{-\infty}^{\infty} f(T) e^{isT} dT$$

$$= e^{ist_0} F(s)$$

Shifting on frequency axis :

If F(s) is the complex Fourier transform of f(t) and so is any real number than,

$$F\left[e^{ist_0} f(t)\right] = F(s+s_0)$$

Proof : By definition we have

$$F(s) = F(f(t)) = \int_{-\infty}^{\infty} f(t) e^{ist} dt$$

$$\therefore \quad F\left(e^{ist_0} f(t)\right) = \int_{-\infty}^{\infty} e^{is_0t} f(t) e^{ist} dt$$

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$$= \int_{-\infty}^{\infty} f(t)\, e^{i(s+s_0)t}\, dt = F(s+s_0).$$

Part-B

Q. 3. (a) If f(z) is analytic function with constant modulus, then prove that f(z) is constant.

Ans. Let $f(z) = u + iv$ be an analytic function.

Since $|f(z)| = \text{constant} = c \text{ (say) } (c \neq 0)$

$\therefore$ $$|f(z)|^2 = u^2 + v^2 = c^2 \quad \text{...(1)}$$

Diff., (1) partially w.r.t x and y respectively, we have

$$2u\frac{\partial u}{\partial x} + 2v\frac{\partial v}{\partial x} = 0$$

or $$\frac{u\partial u}{\partial x} + \frac{v\partial v}{\partial x} = 0 \quad \text{...(2)}$$

and $$2u\frac{\partial u}{\partial y} + 2v\frac{\partial v}{\partial y} = 0$$

or $$\frac{u\partial u}{\partial y} + v\frac{\partial v}{\partial y} = 0 \quad \text{...(3)}$$

Using C-R equations,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

and $$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Equation (3) becomes,

$$-u\frac{\partial v}{\partial x} u\frac{\partial u}{\partial x} = 0 \quad \text{...(4)}$$

Squaring and adding (2) and (4), we have

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$$\left(u^2+v^2\right)\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right]=0$$

$$C^2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2\right]=0 \quad \because C \neq 0$$

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial u}\right)^2=0$$

$$|f'(z)|^2=0 \text{ since } f'(z)=\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}$$

$$\therefore \quad f'(z)=0 \Rightarrow f(z)=\text{constant.}$$

Q. 3. (b) Show that the function

$$v(x, y)=\ln\left(x^2+y^2\right)+x-2y$$

is harmonic. Find its conjugate harmonic function u (x, y) and the corresponding analytic function f(z).

Ans.

$$v(x, y)=\ln\left(x^2+y^2\right)+x-2y$$

$$v_x=\frac{1}{x^2+y^2}2x+1$$

$$v_{xx}=\frac{\left(x^2+y^2\right)(2)-(2x)(2x)}{\left(x^2+y^2\right)^2}$$

$$=\frac{2y^2-2x^2}{\left(x^2+y^2\right)^2}=\frac{2\left(y^2-x^2\right)}{\left(x^2+y^2\right)^2}$$

$$v_y=\frac{2y}{x^2+y^2}-2$$

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$$v_{yy} = \frac{(x^2+y^2)(2)-(2y)(2y)}{(x^2+y^2)^2} = \frac{2x^2-2y^2}{(x^2+y^2)^2}$$

Since $v_{xx} + v_{yy} = 0 \Rightarrow$ v is harmonic

Let u be harmonic conjugate of v the

$$du = u_x dx + uy\,dy$$

$$= v_y dx - v_x dy$$

$$= \left(\frac{2y}{x^2+y^2} - 2\right)dx - \left(\frac{2x}{x^2+y^2} + 1\right)dy$$

$$u = \log\left|\frac{x+y}{x-y}\right| - 2x - \frac{2x}{2x}\log\frac{y+x}{y-y} - y$$

$$\log\frac{x+y}{x-y} - 2x - \log\frac{x+y}{x-y} - y$$

$$= -2x - y + c$$

$$f(z) = \left(\ln(x^2+y^2)(x-2y)\right).\hat{i} - (y+2x)$$

$$= -(y+2x) + \left[\ln(x^2+y^2) + (x-2y)\right]\hat{i}$$

$$\boxed{Z + iZ + 2\ln Z}.$$

Q. 4. (a) Solve $\tan z = e^{i\alpha}$, where α is real.

Ans. $\tan z = e^{i\alpha} = \cos\alpha + i\sin\alpha$

As $\tan(x+iy) = \cos\alpha + i\sin\alpha$

$$\tan(x-iy) = \cos\alpha - i\sin\alpha$$

$$\tan 2\alpha - \tan(x + xy + x - iy)$$

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$$= \frac{\tan(x+iy)+\tan(x-iy)}{1-\tan(x+iy)\tan(x-iy)}$$

$$= \frac{(\cos\alpha + i\sin\alpha)+(\cos\alpha - i\sin\alpha)}{1-(\cos\alpha + i\sin\alpha)(\cos\alpha - i\sin\alpha)}$$

$$= \frac{2\cos\alpha}{1-\left(\cos^2\alpha+\sin^2\alpha\right)} = \frac{2\cos\alpha}{0}$$

$$\tan 2x = \infty$$

Or $$2x = \tan^{-1}\infty = 2n\pi + \frac{\pi}{2}$$

$$x = n\pi + \frac{\pi}{4} \qquad \text{...(i)}$$

Again $$2iy = (x+iy)-(x-iy)$$

$$\tan(2iy) = \tan[(x+iy)-(x-iy)]$$

$$= \frac{\tan(x+iy)-\tan(x-iy)}{1+\tan(x+iy)\tan(x-iy)}$$

$$= \frac{(\cos\alpha + i\sin\alpha)-(\cos\alpha - i\sin\alpha)}{1+(\cos\alpha + i\sin\alpha)(\cos\alpha - i\sin\alpha)}$$

$$= \frac{2i\sin\alpha}{2} = i\sin\alpha$$

Or $$i\tanh 2y = i\sin\alpha$$

$$\tanh 2y = \sin\alpha$$

$$2y = \tanh^{-1}\sin\alpha$$

Using $$\tanh^{-1}x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$$

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$$=\frac{1}{2}\log\left[\frac{1+\sin\alpha}{1-\sin\alpha}\right]$$

$$y=\frac{1}{4}\log\left[\frac{1-\cos\left(\frac{\pi}{2}+\alpha\right)}{1+\cos\left(\frac{\pi}{2}+\alpha\right)}\right]$$

$$=\frac{1}{4}\log\left[\frac{2\sin^2\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)}{2\cos^2\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)}\right]$$

$$y=\frac{1}{2}\log\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right) \qquad \text{...(ii)}$$

From equation (i) and (ii)

$$Z=x+iy=n\pi+\frac{\pi}{4}+\frac{1}{2}i\log\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right).$$

Q. 4. (b) Evaluate $\oint_C \frac{dz}{z^2+9}$, where C is

(i) $|z+3i|=2$

(ii) $|z|=5$

taken in counter-clockwise sense.

Ans. $\oint_C \frac{dz}{z^2+9}$

Here $f(z)=\frac{1}{z^2+9}$ has simple poles at $z=\pm 3i$

(i) $|z+3i|=2$

Only the pole $z=3i$ lies inside the circle

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Residue at $z=3i$

$$\underset{z\to 3i}{\mathrm{Lt}}(z-3i)\frac{1}{(z^2+3i)(z-3i)}=\frac{1}{6i}=-\frac{i}{6}$$

$\therefore$ By residue the,

$$\oint_C \frac{dz}{z^2+9}=2\pi i\left(-\frac{i}{6}\right)=\frac{\pi}{3}$$

(ii) $|z|=5$ is circle of centre (0, 0) and radius 5

Both the poles $z=3i$ and $z=-3i$ lies inside the circle.

Residue at $z=3i$ is already calculate it comes $-\frac{i}{6}$.

Residue at $z=-3i$

$$\underset{z\to -3i}{\mathrm{Lt}}(z+3i)\frac{1}{(z+3i)(z-3i)}=\frac{1}{-6i}=\frac{i}{6}$$

By Residue theorem,

$$\oint_C \frac{dz}{z^2+9}=2\pi i\ \text{[Sum of residue at both poles]}$$

$$=2\pi i\left[-\frac{i}{6}+\frac{i}{6}\right]=0.$$

Q. 5. (a) Explain $f(z)=\frac{7z-2}{z(z+1)(z-2)}$ as a Laurent series in the region $1<|z+1|<3$.

Ans. $$\frac{7z-2}{z(z+1)(z-2)}\quad 1<|z+1|<3$$

$$\frac{7z-2}{z(z+1)(z-2)}=\frac{A}{z}+\frac{B}{z+1}+\frac{C}{z-2} \qquad ...(1)$$

$$7z-2=A(z+1)(z-2)+Bz(z-2)+Cz(z+1)$$

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$$A = 1 \qquad B = -2 \qquad C = 2$$

Equation (1) reduces to,

$$\frac{1}{z} - \frac{3}{z+1} + \frac{2}{z-2} \qquad \text{Since } 1 < |z+1| < 3$$

$$\frac{1}{z} - \frac{3}{z+1} + \frac{2}{-2\left(1 - \frac{z}{2}\right)} \qquad 0 < z < 2$$

$$\frac{1}{z} - \frac{3}{z+1} + \frac{2}{-2}\left(1 - \frac{z}{2}\right)^{-1} \qquad \left|\frac{z}{2}\right| < 1$$

$$\Rightarrow \qquad \frac{1}{z} - \frac{3}{z+1} - \left(1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + \left(\frac{z}{2}\right)^3 + \ldots\ldots\infty\right)$$

$$= \frac{1}{z} - 3(1+z)^{-1} - \left(1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + \left(\frac{z}{2}\right)^3 + \ldots\ldots\infty\right)$$

$$= \frac{1}{z} - 3(1+z)^{-1} + \left(1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + \left(\frac{z}{2}\right)^3 + \ldots\ldots\infty\right)$$

$$\frac{1}{z} - 3\left(1 - z + z^2 - z^3 + z^4 + \ldots\ldots\right) - \left(1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + \left(\frac{z}{2}\right)^3 + \ldots\ldots\infty\right).$$

Q. 5. (b) Evaluate the integral $\int_{-\infty}^{\infty} \frac{\cos ax}{(x^2+\alpha^2)(x^2-\beta^2)}\,dx$.

Ans. $$\int_{-\infty}^{\infty} \frac{\cos ax}{(x^2+\alpha^2)(x^2-\beta^2)}\,dx$$

Consider $\int_C \frac{e^{iax}}{z^2+\alpha^2}$; where C is closed curve consisting of upper half of the circle C_R whose equation

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$|z| = R$ and a part of real axis from $-R$ to R.

$$\oint_C \frac{e^{inz}}{z^2+a^2} = \int_{C_R} \frac{e^{iaz}}{z^2+\alpha^2} + \int_{-R}^{R} \frac{e^{iaz}}{x^2+\alpha^2}$$

Using Cauchy's residue theorem,

$$2\pi i[S_R] = \oint_C \frac{e^{inz}}{z^2+a^2} = \int_{C_R} \frac{e^{iaz}}{z^2+\alpha^2} + \int_{-R}^{R} \frac{e^{iaz}}{x^2+\alpha^2}$$

$$f(z) = \frac{e^{iaz}}{z^2+\alpha^2} \qquad \text{...(1)}$$

$z = i\alpha$ is a simple pole inside 'C' $z = -i\alpha$ is a simple pole outside. Residue of f(z) at $z = \alpha i$ is a simple pole.

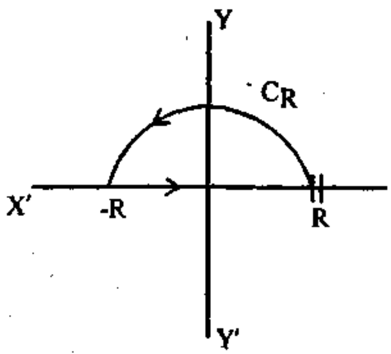
$$= \lim_{z\to\alpha i}(z-\alpha i)f(z)$$

$$\text{Resi} = \lim_{z\to\alpha i}(z-\alpha i)\frac{e^{iax}}{z^2+\alpha^2}$$

$$\text{Resi} = \frac{e^{-m\theta}}{2i\alpha} \qquad \text{...(2)}$$

C_R is semicircle $\quad |z| = R,\ z = Re^{i\theta}$

$$(dz) = Rd\theta$$



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$\theta : \theta \to 0$

$$\left|\int_{C_R} \frac{e^{iaz}}{z^2+\alpha^2}\right| < \int_{C_R} \frac{\left|e^{iaz}\right| dz}{|z|^2-\alpha^2} \le \int_0^{\pi} \frac{e^{-aR\sin\theta}}{R^2-\alpha^2} . R d\theta$$

Using Jordan in equality $0 \le \theta \le \frac{\pi}{2}$

$$\frac{2}{\pi} \le \frac{\sin\theta}{\theta} \le 1$$

Hence $\sin\theta \ge \frac{2\theta}{\pi}$

$\therefore$

$$e^{-aR\sin\theta} > e^{-aR\left(\frac{2\theta}{R}\right)}$$

$$\left|\int_{C_R} \frac{e^{iaz}}{z^2+\alpha^2}\right| \le \frac{R}{R^2-\alpha^2} \int_0^{\pi} e^{-aR\sin\theta}$$

i.e.,

$$\le \frac{R}{R^2-\alpha^2} 2\int_0^{\pi} e^{-9R\frac{2\theta}{\pi}} d\theta \le \frac{2R}{R_1^2-\alpha^0}\left[\frac{e^{-9R\frac{2\theta}{\pi}}}{\frac{2aR}{\pi}}\right]_0^{\pi/2}$$

$$\le \frac{\pi}{a(R^2-\alpha^2)}\left[1-e^{-9R}\right] \to 0 \quad \text{as } R \to \infty$$

and

$$\int_{-R}^{R} \frac{e^{iax}}{x^2+\alpha^2} \to \int_{-\infty}^{\infty} \frac{e^{iax}}{x^2+\alpha^2} \quad \text{an } R \to \infty \qquad \text{-(3)}$$

Using equation (3) and (2) in (1)

$$2\pi i \frac{e^{-a\alpha}}{2\alpha i} = 0 \int_{-\infty}^{\infty} \frac{e^{iax}}{x^2+\alpha^2} dx$$

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Hence,
$$\int_{-\infty}^{\infty} \frac{e^{iax}}{x^2+\alpha^2}\,dx = \frac{\pi e^{-a\alpha}}{a}$$

Equation real part,

$$\int_{-\infty}^{\infty} \frac{\cos ax}{x^2+\alpha^2}\,dx = \frac{\pi e^{-a^2}}{\alpha}.$$

Part-C

Q. 6. (a) Six cards are dealt from a well-shuffled deck of playing cards. Given that all six cards are black, find the probability that they are all of the same suit.

Ans. Probability that the six cards are drawn are of black colour,

$$\frac{{}^{26}C_6}{{}^{52}C_6}$$

Probability that six blade cards drawn and of same suit.

$$\frac{4\left({}^{13}C_6\right)}{{}^{52}C_6} \times \frac{{}^{26}C_6}{{}^{52}C_6}.$$

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Q. 7. (b) A cubical die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the die cannot be regarded as an unbiased one and find the extreme limits between which the probability of a throw of 3 or 4 lies.

Ans. Here, $n = 9000$

P = Probability of success (i.e., getting 5 or 6 induce)

$$P = \frac{2}{6} = \frac{1}{3},\ 8 = 1 - \frac{1}{3} = \frac{2}{3}$$

$$p = \frac{X}{m} = \frac{3240}{9000} = 0.36$$

H_0 : is unbiased, i.e., $P = \frac{1}{3}$.

H_1 : $P \neq \frac{1}{3}$ (two tailed test)

The test statistic $Z = \frac{p-d}{\sqrt{\frac{PQ}{n}}}$

$$= \frac{0.36 - 0.33}{\sqrt{\frac{1}{3} \times \frac{2}{3} \times \frac{1}{9000}}} = 0.03496$$

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$$|z| = 0.03494 < 1.96$$

Conclusion : Accept the hypothesis,

As $|z| < z_\alpha$, z_α is the tabulated value of z at 5% level of significance.

$\because$ H_0 is accepted, we conclude that the die is unbiased.

To find 95% confidence, limits of the proportion it is given by $p \pm z_\alpha \sqrt{PQ/n}$

$$= 0.36 \pm 1.96 \sqrt{\left(\frac{2}{3} \times \frac{2}{3}\right)/900}$$

$$= 0.36 \pm 0.01377$$

$$= (0.37377, .24623)$$

Q. 8. (a) Using Simplex method, maximize

$\mathbf{Z = 10x_1 + x_2 + 2x_3}$ **subject to** $\mathbf{x_1 + x_2 - 2X_3 \le 10,\ 4x_1 + x_2 + x_3 \le 20,\ x_1, x_2, x_3 \ge 0}$**.**

Ans. $Z = 10x_1 + x_2 + 2x_3$

Introducing slack variable

$$x_1 + x_2 - 2x_3 + s_1 = 10$$

$$4x_1 + x_2 + x_3 + s_2 = 20$$

		C_j	10	1	2	0	0	Ratio
C_B	Basic	Sol $b(X_R)$	x_1	x_2	x_3	s_1	s_2	x_B / x
0	s_1	10	1	1	-2	1	0	$10/1 = 10$
$\leftarrow 0$	s_2	20	4	1	1	0	1	$20/4 = 5$
$z = 0$		z_5	$10 \uparrow$	1	2	0	0	
		$c_j = c_j - z_j$						

c_j		10	1	2	0	0	$2 - \frac{10}{4}$

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	z_5	s_1	5	0	3/4	−9/4	1	−1/4
10	x_1	5	1	1/4	1/4	0	1/4	
	z_5	50	0	$-\frac{2}{4}$	$-\frac{2}{4}$	0	0	

All z_j are zero or (–ve).

So, we obtain optimal solution.

$$x_1 = 5,\ x_2 = 0,\ x_3 = 0,\ \text{Max } z = 50$$

Which is required solution.

Q. 8. (b) Using Graphical method, solve the following LPP

Max $Z = 5x_1 + 3x_2$ subject to $3x_1 + 5x_2 \le 15$, $5x_1 + 2x_2 \le 10$, $x_1, x_2 \ge 0$.

Ans. Max. $Z = 5x_1 + 3x_2$

Subject to $3x_1 + 5x_2 \le 15$

$$5x_1 + 2x_2 \le 10$$

Corresponding equality is,

$$3x_1 + 5x_2 = 15$$

x_1	0	5
x_2	3	0

$$5x_1 + 2x_2 = 10$$

x_1	0	2
x_2	5	0

Corner points are A (2, 0), B (5, 0), $C\left(\frac{20}{19}, \frac{45}{19}\right)$

Max $z = 5x_1 + 3x_2$

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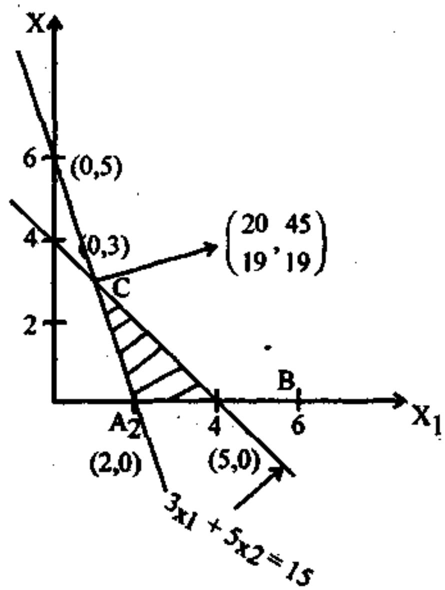

At $\qquad A(2, 0) = 10$

$B(5,0) = 15$

$C\left(\frac{25}{19}, \frac{45}{19}\right) = \frac{235}{19}$

Max value,

$$\text{Max } z = \frac{235}{19}$$

At $\qquad x_1 = \frac{20}{19},\ x_2 = \frac{45}{2}.$