B.Tech.

Fourth Semester Examination

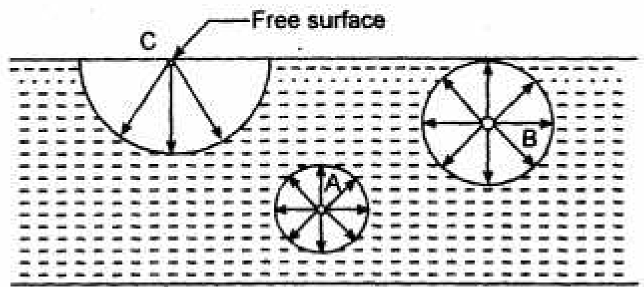
Fluid Mechanics (ME-208-F)

Note : Attempt any **five** questions.

Q. 1. (a) Discuss the phenomenon of surface tension and capillarity. Obtain the expression for both.

Ans. Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

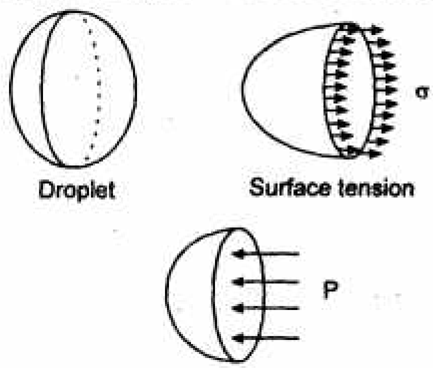
Consider three molecules *A,B,C* of a liquid in a mass of liquid. The molecule *A* is attracted in all directions equally by the surrounding molecules of the liquid. Thus, the resultant force acting on the molecule *A* is zero. But the moledule *B*, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced. Thus, a net resultant force on molecule *B* is acting in the downward direction. The molecule *C*, situated on the free surface of liquid, does experience a resultant downward force. All the molecules on the free surface experience a downward force. Thus, the free surface of liquid acts like a very thin film under tension of the surface of liquid act as though it is an elastic membrane under tension.



Surface tension

Surface Tension on Liquid Droplet :

Tensite force due to surface tension acting around the circumference of the cut portion is equal to



$$= \sigma \times \text{Circumference}$$
$$= \sigma \times \pi d$$

Pressure force on the area $\frac{\pi}{4} d^2$

$$= p \frac{\pi}{4} d^2$$

These two forces will be equal and opposite under equilibrium conditions,

i.e., $$P \times \frac{\pi}{4} d^2 = \sigma \pi d$$

$$P = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{4\sigma}{d}$$

Surface Tension on Hollow Bubble : A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and one outside. The two surfaces are subjected to surface tension. In such case, we have

$$P \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$$P = \frac{2\sigma \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d}$$

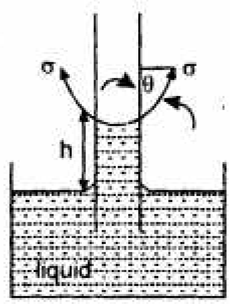
Capillarity : Capillarity is defined as the phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of liquid.

Expression for Capillary Rise

h = height of liquid in the tube

σ = Surface tension of liquid

θ = Angle of contact between liquid and glass tube



The weight of liquid of height h in the tube

$$= (\text{Area of tube} \times h) \times \rho \times g$$
$$= \frac{\pi}{4} d^2 h \rho g$$

Vertical component of the surface tensile force

$$= (\sigma \times \text{circumference}) \times \cos\theta$$
$$= \sigma \times \pi d \times \cos\theta$$

For equilibrium, we get

$$\frac{\pi}{4} d^2 \times h\rho g = \sigma \times \pi d \times \cos\theta$$

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma \cos\theta}{\rho g d}$$

The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos\theta$ is equal to unity.

$$\therefore \qquad h = \frac{4\sigma}{\rho g d}$$

Q. 1. (b) A rectangular pontoon 2.5 m deep, 10 m long, 7 m broad and weighs 686.7 kN. It carries on its upper deck an empty boiler of 5m diameter weighing 588.6 kN. The centre of gravity of the boiler and pontoon are at their respective centers along a vertical line. Find the metacentric height. Weight density of sea water is 10.104 kN/m³.

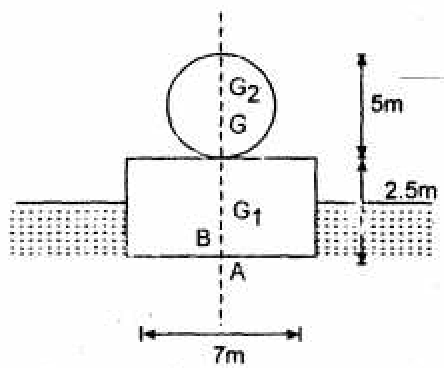
Ans. Dimension of pontoon = $10 \times 7 \times 2 \cdot 5$

Weight of Pontoon, $w_1 = 686 \cdot 7$ KN

Dia. of boiler $D = 5 \cdot 0$m

Weight of boiler $w_2 = 588 \cdot 6$ KN

w for sea water $= 10 \cdot 104$ KN/m^3



Let G_1 and G_2 are the centre of gravities of pontoon and boiler respectively. Then

$$AG_1 = \frac{2 \cdot 5}{2} = 1 \cdot 25\text{m}$$

$$AG_2 = 2 \cdot 5 + \frac{5 \cdot 0}{2} = 2 \cdot 5 + 2 \cdot 5 = 5\text{m}$$

The distance of common centre of graivty G from A is gives as

$$AG = \frac{w_1 \times AG_1 + w_2 \times AG_2}{2}$$

$$=\frac{686\cdot7\times1\cdot25+588\cdot6\times5\cdot0}{(686\cdot7+588\cdot6)}=2\cdot98\text{m}$$

Let h be the depth of immersion.

Total wt. of pontoon and boiler

$$= wt \text{ of sea water displaced } (686.7+588.6)$$
$$= w\times \text{volume of the pontoon in water}$$
$$= 10\cdot104\times L\times G\times \text{Depth of immersion}$$
$$1275\cdot3 = 10.104\times10\times7\times h$$
$$h=\frac{1275\cdot3}{10\times7\times10\cdot104}=1\cdot803\text{ m}$$

∴ Distance of common centre of buoyancy B from A is

$$AB=\frac{h}{2}=\frac{1\cdot803}{2}=.9015\text{ m}$$
$$BG = AG - AB = 2\cdot98 - \cdot9015 = 2\cdot0785\text{m}$$
$$\approx 2\cdot078\text{ m}$$

Meta-centric height is given by

$$GM=\frac{I}{\forall}-BG$$

Where I = M.O.I of the plan of the body at the water level along $y = y$

$$=\frac{1}{12}\times10\cdot0\times7^3=\frac{10\times49\times7}{12}\text{m}^4$$

$\forall$ = Volume of the body in water = $LGh = 10\times7\times1\cdot857$

$$\therefore\ \frac{I}{\forall}=\frac{10\times49\times7}{12\times10\times7\times1\cdot857}$$
$$=2\cdot198\text{ m}$$
$$GM=\frac{I}{\forall}-BG=2\cdot198-2\cdot078$$
$$=\cdot12\text{m}$$

∴ Metacentric height of both the pontoon and boiler = 0·12m **Ans.**

Q. 2. (a) What do you mean by equipotential lines and streamlines? Show them on a neat diagram.

Ans. Steam Function : It is defined as the scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction.

For steady flow it is defined as $\psi = f(x, y)$ such that

$$\left.\begin{aligned}\frac{\partial\psi}{\partial x}&=v\\ \frac{\partial\psi}{\partial y}&=-u\end{aligned}\right\}$$

The velocity components in cylindrical polar co-ordinates in terms of stream function are given as

$$\mu_r=\frac{1}{r}\frac{\partial\psi}{\partial\theta}\qquad\text{and}\qquad\mu_\theta=-\frac{\partial\psi}{\partial r}$$

Where μ_r = radial velocity and μ_θ = tangential velocity. The continuity equation for two-dimensional flow is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Substituting the values μ and V from equation

$$\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right)=0$$

$$\frac{-\partial^2 \psi}{\partial x \partial y}+\frac{\partial^2 \psi}{\partial x \partial y}=0$$

Hence existence of ψ means a possible case of fluid flow. The flow may be rotational or irrotational. The rotational component ω_2 is given by

$$\omega_2=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

Substituting the value of μ and v

$$\omega_2=\frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right)-\frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right]$$

$$=\frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}\right]$$

For irrotational flow $\omega_2=0$

$$\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}=0$$

which is Laplace equation for ψ.

Equipotential Line : A line along which the velocity potental ϕ is constant, is called equipotential line

For equipocntial line $\qquad \phi =$ Constant

$\therefore \qquad \partial\psi=0$

But $\phi=f(x, y)$ for steady flow

$$\therefore \qquad \partial\phi=\frac{\partial \phi}{\partial x}dx+\frac{\partial \phi}{\partial y}dy$$

$$=-udx-vdy$$

$$=-(udx+vdy) \qquad \left\{\because \frac{\partial \phi}{\partial x}=-u, \frac{\partial \phi}{\partial y}=-v\right\}$$

For equipotential line $d\phi=0$

or $\qquad -(udx+vdy)=0$

$$\frac{dy}{dx}=\frac{-u}{v}$$

But $\frac{dy}{dx}$ = Slope of equipotential line

Q. 2. (b) For a velocity component given as u = ay sin xy and v = ax sin xy. Obtain an expression for velocity potential function and stream function.

Ans.

$$u = ay \sin xy$$

$$v = ax \sin xy$$

The velocity components in term of velocity potential function

$$\frac{\partial \phi}{\partial x} = -u = -ay \sin xy \quad \ldots(i)$$

$$\frac{\partial \phi}{\partial y} = -v = -ax \sin xy \quad \ldots(ii)$$

Integrating equation (i) w.r.t x

$$\phi = a \cos xy + C \quad \ldots(iii)$$

Where C is constant of integration which is independent of x but can be a function of y

Differentiating equation (iii) w.r. to y

$$\frac{\partial \phi}{\partial y} = -ax \sin xy + \frac{\partial C}{\partial y}$$

But
$$\frac{\partial \phi}{\partial y} = -ax \sin xy$$

On comparing the values

$$\frac{\partial C}{\partial y} = 0$$

$$\partial C = 0$$

$$C = K$$

$$\therefore \quad \phi = a \cos xy + K \qquad \textbf{Ans.}$$

Q. 3. (a) Derive impulse momentum relationship and discuss its applications.

Ans. Impulse Momentum Relationship : It is based on the law of conservation of momentum, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

$$F = ma$$

But
$$a = +\frac{dv}{dt}$$

$$\therefore \quad F = m\frac{dv}{dt}$$

$$= \frac{d(mv)}{dt} \text{ (} m \text{ is constant)}$$

$$F = \frac{d(mv)}{dt}$$

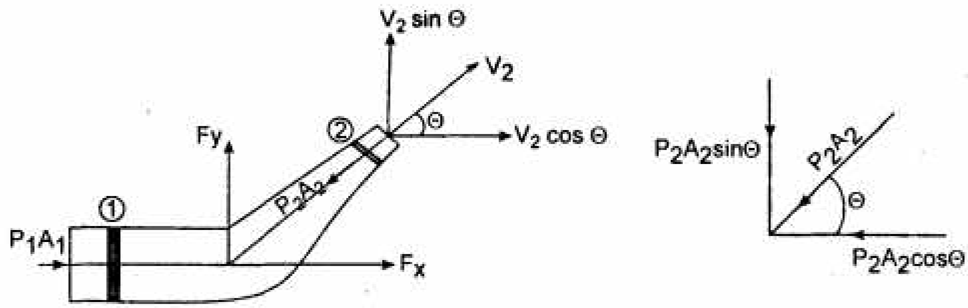
Equation is known as momentum principle.

Equation can be written as

$$F \bullet dt = d(mv)$$

Which is known as impulse momentum equation and states that the impulse of a force F acting on a fluid of mass m in a short interval of tune dt is equal to the change of momentum $d(mv)$ in the direction of force.

Application : The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.



Net force acting on fluid in the direction x

$$= \text{rate of change of momentum in } x \text{ direction}$$

$$\therefore \quad P_1A_2 - P_2 A_2 \cos\theta - F_x = (\text{Mass per sec})\,(\text{Change in velocity})$$

$$= \rho Q(V_2 \cos\theta - V_1)$$

$$\therefore \quad F(x) = \rho Q(V_1 - V_2 \cos\theta) + P_1 A_1 - P_2 A_2 \cos\theta$$

Similarly the momentum equation in y direction gives

$$O - P_2 A_2 \sin\theta - F_y = \rho Q(V_2 \sin\theta - O)$$

$$\therefore \quad F_y = \rho Q(-V_2 \sin\theta) - P_2 A_2 \sin\theta$$

Now resultant force acting on bend

$$F_R = \sqrt{F_x^2 + F_y^2}$$

& angle mode by resultant force

$$\tan\theta = \frac{F_y}{F_x}$$

Q. 3. (b) A 45° reducing bend is connected in a pipeline, the diameter at the inlet and the outlet of the bend being 40 cm and 20 cm respectively. Find the force exerted by the water on the bend if the intensity of pressure at inlet of bend is 21.58 N/cm^2. The rate of flow of water is 500 liters/sec.

Ans. $\theta = 45°$

Dia. at inlet = 40 cm = ·4 m

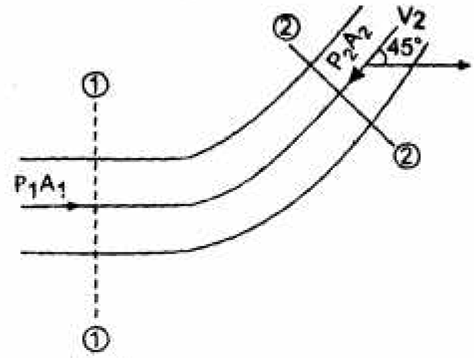
$$\text{Area} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(\cdot 4)^2$$

$$= .125 \text{ m}^2$$

Dia. at outlet = 20 cm = ·2 m

$$\text{Area} = \frac{\pi}{4}(\cdot 2)^2$$

$$= \cdot 031 \text{ m}^2$$



Pressure at inlet $P_1 = 21 \cdot 58 \text{ N/cm}^2$

$$= 21 \cdot 58 \times 10^4 \text{ N/m}^2$$

$$Q = 500 \text{ lit/sec}$$

$$= \cdot 5 \text{ m}^3/\text{s}$$

$$V_1 = \frac{Q}{A_1} = \frac{0 \cdot 5}{\cdot 125} = 4 \text{m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0 \cdot 5}{\cdot 031} = 16 \cdot 1 \text{m/s}$$

Applying Bernoulli's equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

But $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{21 \cdot 58 \times 10^4}{1000 \times 9 \cdot 81} + \frac{4^2}{2 \times 9 \cdot 81} = \frac{P_2}{\rho g} + \frac{(16 \cdot 1)^2}{2 \times 9 \cdot 81}$$

$$21 \cdot 997 + \cdot 8154 = \frac{P_2}{\rho g} + \frac{259 \cdot 21}{19 \cdot 62}$$

$$22 \cdot 812 - 13 \cdot 211 = \frac{P_2}{\rho g}$$

$$\frac{P_2}{\rho g} = 9 \cdot 601$$

$$P_2 = 9 \cdot 601 \times 1000 \times 9 \cdot 81$$

$$= 9 \cdot 418 \times 10^4 \text{ N/m}^2$$

Forces on bend in x and y directions are given by

$$F_x = \rho Q \{V_1 - V_2 \cos\theta\} + P_1 A_1 - P_2 A_2 \cos\theta$$

$$= 1000 \times \cdot 5 [4 - 16 \cdot 1 \cos 45°] + 21 \cdot 58 \times 10^4 \times \cdot 125 - 9 \cdot 418 \times 10^4 \times \cdot 031 \times \cos 45°$$

$$= -3692 \cdot 20 + 26975 - 2064 \cdot 45$$

$$= 21218 \cdot 345 \, N$$

& $F_y = \rho Q[-V_2 \sin\theta] - P_2 V_2 \sin\theta$
$= 1000 \times \cdot 5[-16\cdot 1 \sin 45°] - 9\cdot 418 \times \cdot 031 \times 10^4 \times \sin 45°$
$F_y = -5692\cdot 209 - 2064\cdot 454$
$= -7756\cdot 663$

∴ Resultant force

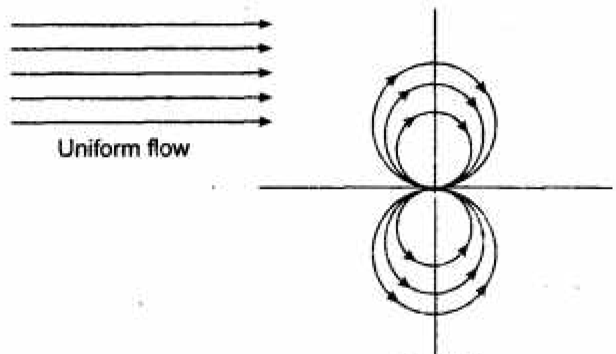
$$F_R = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{(21218\cdot 345)^2 + (-7756\cdot 663)^2}$$
$$= 22591\cdot 679\ N \text{ Ans.}$$

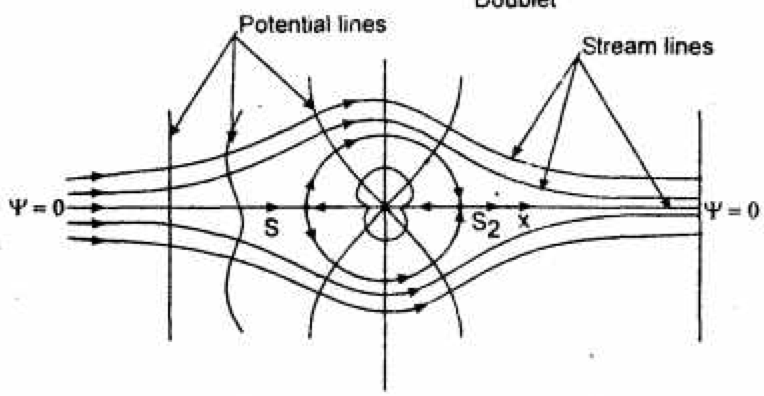
Angle made by resultant force

$$\tan\theta = \frac{F_y}{F_x} = \frac{7756\cdot 663}{21218\cdot 345}$$
$$= 36\cdot 55$$
$$= 20\cdot 08° \text{ Ans.}$$

Q. 4. (a) Discuss and sketch the flow pattern of an ideal fluid flow past a cylinder with circulation.

Ans. Figure shows a uniform flow of velocity u in the positive x-direction and a doublet at the origin. Doublet is a special case of a source and a shk combination in which both of equal strength approach each other such that distance between them tends to zero. When the uniform flow is flowing over the doublet, a resultant flow will be obtained. This resultant flow is known as flow past a Rankine oval of equal axes or flow past a circular cylinder.





The stream function (ψ) and velocity potential function (ϕ) for the resultant flow is obtained as

ψ = stream function due to uniform flow + stream function due to doublet

$$= u \times y + \left(\frac{-\mu}{2\pi r}\sin\theta\right)$$

$$= u \times r \times \sin\theta - \frac{\mu}{2\pi r}\sin\theta$$

$$= \left(u \times r - \frac{\mu}{2\pi r}\right)\sin\theta$$

& ϕ = potential function due to uniform flow + potential function due to doublet

$$= u \times x + \frac{\mu}{2\pi} \times \frac{\cos\theta}{r}$$

$$= u \times r\cos\theta + \frac{\mu}{2\pi} \times \frac{\cos\theta}{r}$$

$$= \left(u \times r \times \frac{\mu}{2\pi r}\right)\cos\theta$$

Q. 4. (b) A uniform flow of 10m/s is flowing over a doublet of strength 15m^2/s. The doublet is in the line of the uniform flow. The polar coordinates of the point P in the flow field are 0.9 m and 30°. Find;

(i) Stream function **(ii) The resultant velocity at the point.**

Ans. $u = 10\text{ m/s}$; $\mu = 15\text{ m}^2/\text{s}$; $r = 0.9\text{m}$; $\theta = 30°$

$$R = \sqrt{\frac{\mu}{2\pi u}} = \sqrt{\frac{15}{2\pi \times 10}} = .488\text{m}$$

Polar coordinates of point P are 0.9m and 30°

$\therefore$ $r = 0.9$ and $\theta = 30°$

(i) Value of stream line function at point P

$$\psi = u\left(r - \frac{R^2}{r}\right)\sin\theta$$

$$= 10\left(0.9 - \frac{0.488^2}{0.9}\right)\sin 30°$$

$$= 3.177\text{ m}^2/\text{s} \textbf{ Ans.}$$

(ii) Resultant velocity at point P

$$\mu_r = u\left(1 - \frac{R^2}{r^2}\right)\cos\theta$$

$$= 10\left(1 - \frac{0.488^2}{0.9^2}\right)\cos 30°$$

$$= 611\text{ m/s}$$

Positive sign shows radial velocity is outward.

&

$$\mu_\theta = -u\left(1+\frac{R^2}{r^2}\right)\sin\theta$$

$$= -10\left(1+\frac{0\cdot488^2}{0\cdot9^2}\right)\sin 30^\circ$$

$$= -6\cdot47 \text{ m/s}$$

Negative sign shows the clockwise direction of tangential velocity

∴ Resultant velocity

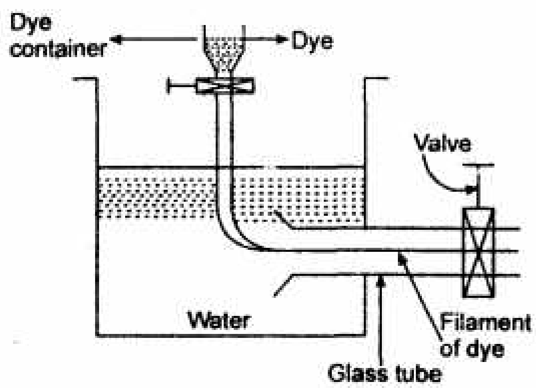
$$V = \sqrt{\mu_r^2 + \mu_\theta^2}$$

$$= \sqrt{6\cdot11^2 + (-6\cdot47)^2}$$

$$= 8\cdot89 \text{ m/s } \textbf{Ans.}$$

Q. 5. (a) Describe Reynold's experiment to demonstrate two type of flow.

Ans. The type of flow is determined from the Reynold number, i.e. $\frac{\rho V \times d}{\mu}$. This was demonstrated by O. Reynold in 1883.



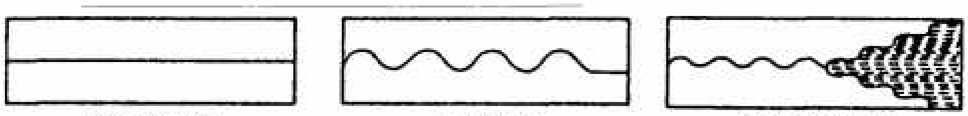
Reynold apparatus

The water from the tank was allowed to flow through the glass-tube. The velocity of the flow was varied by the regulating value. A liquid dye of some specific wt. as water was introduced into the glass tube.

The following observations were made by Reynold :

(i) When the velocity of flow was low, the dye filament in the glass tube was in the form of straight line. This straight line of dye filament was parallel to the glass tube, which was a case of laminar flow.

(ii) With the increase of velocity of flow, the dye filament as no longer a straight line but it become a wavy one. This shows that flow is no longer laminar.



Laninar Flow　　Transition　　Turbulent flow

(iii) With the further increase of velocity of flow, the wavy dye filament broke up and finally diffused in water. This means that the fluid particles of the dye at this higher velocity are moving in random fashion, which shows the case of turbulence flow.

Reynold observed that loss of head is approximately proportional to the square of velocity. More exactly the loss of head $h_f \propto V^n$, where n varies from $1\cdot75$ to $2\cdot0$.

Q. 5. (b) There is a horizontal crack 50 mm wide and 3 mm deep in a wall of thickness 150 mm. Water leaks through the crack. Find the rate of leakage of water through the crack if the difference of pressure between two ends of the crack is 245.25 N/m². Take viscosity of water as 0.01 poise.

Ans. Given :

$$l = 150 \text{ mm} = \cdot 15 \text{ m}$$

$$b = 50 \text{ m} = \cdot 05 \text{ m}$$

$$t = 3\text{mm} = \cdot 003\text{m}$$

$$\text{Pressure} = 245\cdot 25 \text{ N/m}^2$$

$$\text{Viscosity} = \cdot 01 \text{ poise}$$

$$= \frac{\cdot 01}{10} \frac{\text{kg}}{\text{sm}}$$

$$= \cdot 001 \text{ kg/sm}$$

$$\text{Volume} = l \times b \times t$$

$$= \cdot 15 \times \cdot 05 \times \cdot 003$$

$$= 2\cdot 25 \times 10^{-5} \text{ m}^3$$

Q. 6. (a) What do you mean by "equivalent pipe" and "flow through parallel pipes"?

Ans. Equivalent Pipe : This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters.

Let L_1 = length of pipe 1 and d_1 = dia

L_2 = length of pipe 2 and d_2 = dia

L_3 = length of pipe 3 and d_3 = dia

H = total head loss

L = length of equivalent pipe

d = diameter of equivalent pipe

Then $L = L_1 + L_2 + L_3$

Total head loss in the compound pipe, neglecting minor losses

$$H = \frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{4 f_3 L_3 V_3^2}{d_3 \times 2g}$$

Assuming $f_1 = f_2 = f_3 = f$

Discharge, $Q = A_1 V_1 = A_2 V_2 = A_3 V_3 = \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2 = \frac{\pi}{4} d_3^2 V_3$

$$\therefore \quad V_1 = \frac{4Q}{\pi d_1^2}, V_2 = \frac{4Q}{\pi d_2^2}, V_3 = \frac{4Q}{\pi d_3^2}$$

Substituting the values, we have

$$H = \frac{4fL_1\left(\frac{4Q}{\pi d_1^2}\right)^2}{d_1 \times 2g} + \frac{4fL_2\left(\frac{4Q}{\pi d_2^2}\right)^2}{d_2 + 2g} + \frac{4fL_3\left(\frac{4Q}{\pi d_3^2}\right)^2}{d_3 \times 2g}$$

$$= \frac{4 \times 16 f Q^2}{\pi^2 (2g)}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]$$

Head loss in the equivalent pipe, $H = \frac{4fLV^2}{d2g}$

When $V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$

$$\therefore \quad H = \frac{4fL\left(\frac{4Q}{\pi d^2}\right)^2}{d \times 2g} = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right]$$

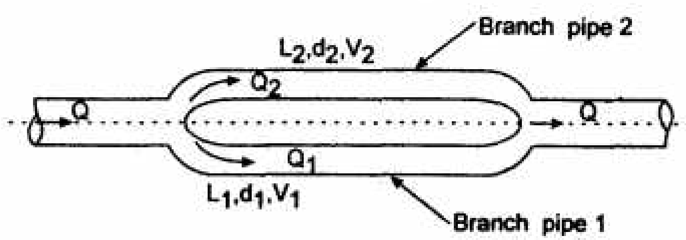
Head loss in compound pipe and equivalent pipe is same

$$\therefore \quad \frac{4 \times 16 f Q^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right]$$

$$\therefore \quad \frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

Flow through Parallel Pipes : The rate of flow in main pipe is equal to the sum of rate of flow through branch pipes.

$$Q = Q_1 + Q_2$$



Loss of head for each branch is same.

$\therefore$ Loss of head for branch pipe 1 = loss of head for branch pipe 2.

or
$$\frac{4f_1 L_1 V_1^2}{d_1 2g}=\frac{4f_2 L_2 V_2^2}{d_2 2g}$$

If $f_1 = f_2$, $\quad \dfrac{L_1 V_1^2}{d_1 \times 2g}=\dfrac{l_2 V_2^2}{d_2 \times 2g}$

Q. 6. (b) Determine the difference in the elevations between the water surfaces in the two tanks which are connected hy horizontal pipe of diameter 400 mm and length 500 m. The rate of flow of water through the pipe is 200 liters/sec. Consider all losses and take f = 0.009.

Ans. $d = 400\text{ mm} = 4\text{m}$

Length $= 500\text{m} = \cdot 5\text{m}$

Flow of water $= 200$ litres/sec

$$Q = \cdot 2\text{ m}^3/\text{s}$$

$$f = 0 \cdot 009$$

$$H = \frac{4 \times f L \left(\dfrac{4Q}{\pi d^2}\right)^2}{d \times 2g}$$

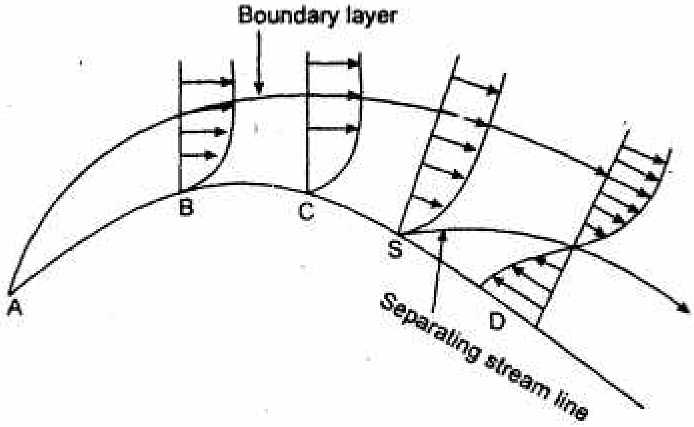
$$= \frac{4 \times \cdot 009 \times \cdot 5 \times \left(\dfrac{4 \times \cdot 2}{\pi \times 16}\right)^2}{4 \times 2 \times 9 \cdot 81}$$

$$= 5 \cdot 8 \times 10^{-7} \textbf{ Ans.}$$

Q. 7. (a) What do you mean by boundary layer separation? What is the effect of pressure gradient on boundary layer separation?

Ans. Boundary Layer Separation : When a solid body is immersed in a flowing fluid, a thin layer of fluid called the boundary layer is formed adjacent to the solid body. In this thin layer of fluid, the velocity varies from zero to free stream velocity in the direction normal to the solid body. Along the length of the solid body, the thickness of the boundary layer increases. The fluid layer adjacent to the solid surface has to do work against surface friction at the expense of its kinetic energy. The loss of kinetic energy is recovered from the immediate fluid layer in contact with the layer adjacent to solid surface through momentum exchange process. Thus, the velocity of layer goes on decreasing. Along the length of the solid body, at a certain point a stage may come when the boundary layer may not be able to keep sticking to the solid body if it cannot provide kinetic energy to overcome the resistance offered by the solid body. In other words, boundary layer will be separated from the surface. This phenomenon is called the boundary layer separation.

Effect of Pressure Gradient on Boundary Layer Separation : The effect of pressure gradient $\left(\dfrac{dP}{dx}\right)$ on boundary layer separation can be explained by considering the flow over a curved surface *AB CSD*. In the region *ABC* of the curved surface, the area of flow decreases and hence velocity increases. This means that flow gets accelerated in this region. Due to the increase of velocity, the pressure decreases in the direction of flow and hence pressure gradient $\frac{dP}{dx}$ is negative in the region. As long as $\frac{dP}{dx} < 0$, the entire boundary layer moves forward.



Q. 7. (b) Oil with a free stream velocity of 1.5 m/s flow over a thin plate 1.4m wide and 2.2 m long. Calculate the boundary layer thickness and shear stress at the trailing end and determine the total surface resistance of the plate. Take specific gravity of the oil as 0.80 and kinematic viscosity as 0.1 stoke.

Ans.

$$u = 1.5 \text{ m/s}$$

$$b = 1 \cdot 4 \text{ m}$$

$$l = 2 \cdot 2 \text{ m}$$

$$\text{Area of plate} = b \times l = 1 \cdot 4 \times 2 \cdot 2 = 3 \cdot 08 \text{ m}^2$$

$$\text{Specific gravity} = 0 \cdot 80$$

$$\therefore \quad \text{Density of oil} = 0 \cdot 80 \times 100 = 800 \text{ kg/m}^3$$

$$\text{Kinematic viscosity} = \cdot 1 \text{ stoke} = \cdot 1 \times 10^{-5} \text{ m}^2/\text{s}$$

$$R_{eL} = \frac{uL}{\nu} = \frac{1 \cdot 5 \times 2 \cdot 2}{\cdot 1 \times 10^{-4}} = 3 \cdot 3 \times 10^5$$

$$R_{eL} < 5 \times 10^5$$

$$\therefore \quad \delta = \frac{4 \cdot 91 \times L}{\sqrt{R_{eL}}} = \frac{4 \cdot 91 \times 2 \cdot 2}{\sqrt{3 \cdot 3 \times 10^5}} = \cdot 0188 \text{ m} = 18 \cdot 8 \text{ mm} \quad \textbf{Ans.}$$

Shear stress at the end of the plate is

$$\tau_0 = \cdot 332 \times 18 \cdot 8 = 6 \cdot 24 \text{ N/m}^2 \quad \textbf{Ans.}$$

Q. 8. (a) Obtain an expression for velocity distribution in turbulent flow for

(i) smooth pipe

(ii) rough pipe

Ans. Velocity Distribution for Turbulent Flow in Smooth Pipes :

The velocity distribution for turbulent flow is given by

$$u = \frac{\mu_*}{K} \log_e y + C$$

At $y = 0$ velocity $\quad u = -\infty$

At some finite distance from wall the velocity will be equal to zero. Let this distance from pipe wall is y'.

at $y = y'$, $\quad u = 0$

$$O = \frac{u_*}{K} \log_e y' + C$$

or

$$C = \frac{-u_*}{K} \log_e y'$$

Substituting the value of C, we get

$$u = \frac{u_*}{K} \log_y e - \frac{u_*}{K} \log_e y' = \frac{u_*}{K} \log_e \left(\frac{y}{y'}\right)$$

Substituting the value of $K = 0.4$

$$u = \frac{u_*}{0.4} \log_e \left(\frac{y}{y'}\right)$$

$$= 2.5\, u_* \log_e \left(\frac{y}{y'}\right)$$

$$\frac{u}{u_*} = 2 \cdot 5 \times 2 \cdot 3 \log_{10} \left(\frac{y}{y'}\right)$$

or

$$\frac{u}{u_*} = 5 \cdot 75 \log_{10} \left(\frac{y}{y'}\right)$$

Velocity Distribution for Turbulent Flow in Rough Pipes : In case of rough boundaries, the thickness of laminar sub layer is very small. The surface irregularities are above the laminar sub-layer and hence the laminar sub-layer is completely destroyed. Thus, y' can be considered proportional to the height of protrusions K, Nikruadse's experiment shows the value of y' for pipes coated with uniform sand as $y' = \frac{K}{30}$

$$\frac{u}{u_*} = 5 \cdot 75 \log_{10} \left(\frac{y}{y'}\right)$$

$$= 5 \cdot 75 \log_{10} \left(\frac{y}{K/30}\right)$$

$$= 5 \cdot 75 [\log_{10} (y/K) \times 30]$$

$$= 5 \cdot 75 \log_{10} \frac{4}{K} + 5 \cdot 75 \log_{10} 30 \cdot 0$$

$$= 5 \cdot 75 \log_{10} (y/K) + 8 \cdot 5 \ldots\ldots$$