Seat No.: ________ Enrolment No._______

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd • EXAMINATION – SUMMER • 2014

Subject Code: 110009 **Date: 16-06-2014**

Subject Name: Mathematics - II

Time: 02:30 pm - 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1

(a) Find the inverse of $A=\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ **07**

(b) What conditions must b_1, b_2, and b_3 satisfy in order for the system of equations $x + y + z = b_1,\ x + z = b_2,\ 2x + y + 3z = b_3$ to be consistent? **07**

Q.2 (a) Find the eigen values and one of the eigen vector of the matrix **07**

$$A=\begin{bmatrix} 8 & 5 & 0 \\ 0 & 3 & 0 \\ -9 & -1 & -1 \end{bmatrix}$$

(b) Find the basis for the null space of the matrix **07**

$$A=\begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$$

Q.3 (a) Use the inner product $\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx$ .To compute $\langle p, q \rangle$ for the vectors $p = p(x)$ and $q = q(x)$ in P_3 where **07**

1. $p = 1 - x + x^2 + 5x^3 \quad q = x$
2. $p = x - 5x^3 \qquad q = 2 + 8x^2$

(b) Find the orthonormal basis for the subspace spanned by { (1,1,1),(1,-2,1),(1,2,3)}. **07**

Q.4 (a) Consider the basis S={ v_1, v_2, v_3 } for R^3, where $v_1 = (1,1,1)$, $v_2 = (1,1,0)$ and $v_3 = (1,0,0)$. Let $T: R^3 \rightarrow R^2$ be the linear transformation such that $T(v_1) = (1,0)$ $T(v_2) = (2,-1)$ and $T(v_3) = (4,3)$.Find a formula for T(x, y, z) and compute T(2,1,3) **07**

(b) Let $T: R^2 \rightarrow R^3$ be the linear transformation defined by $T(x, y) = (y, -5x + 13y, -7x + 16y)$. Find the matrix for the linear transformation T with respect bases B={ $b_1 = (3,1)$ $b_2 = (5,2)$} for R^2 and $B^{/}$ ={ $v_1 = (1,0,-1)$, $v_2 = (-1,2,2)$, $v_3 = (0,1,2)$}. **07**

Q.5 (a) Solve the following system of the equations **07**

$x + 2y + z = 5,\ 3x - y + z = 6,\ x + y + 4z = 7.$

(b) Define: symmetric and skew-symmetric matrix. Show that every square matrix can be expressed as sum of symmetric and skew-symmetric matrix. **07**

Q.6 (a) Consider the vectors u = (2,-1,1) and v = (1,1,2) **04**

Find 1. u + v 2. u.v 3. $\| u - v \|$ and determine the angle between u and v.

(b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ **04**

(c) Let A = $\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$. Compute A^3, A^{-3} and $A^2 - 2A + I$. **06**

Q.7 **(a)** Show that the vectors $v_1 = (1, 2, 3)$, $v_2 = (4, 5, 6)$ and $v_3 = (2, 1, 1)$ in R^3 linearly independent. **04**

(b) Define: Basis for Vector Space. **04**
Show that the set of vectors S={ (1,2,1),(2,9,0),(3,3,4)} is a basis for R^3.

(c) Determine which of the following are subspace of R^3 **06**

1. $W = \{ (x,y,0) / x, y \in R \}$
2. $U = \{(x, 1, 1) / x \in R\}$.

****