B.Tech.

Fourth Semester Examination

Fluid Mechanics (ME-208-F)

Note : Attempt any *five* questions.

1. (a) What do you understand by the hydrostatic equation ? With the help of this equation, derive the expression for the buoyant force acting on submerged bodies. 10

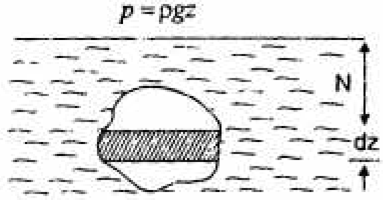
Ans. Hydrostatic law states that the rate of increase of pressure in vertically downward direction must be equal to specific weight of fluid at that point.

i.e.,
$$\frac{\partial p}{\partial z} = \rho \cdot g$$

When a body is immersed in fluid, an upward force is exerted by fluid on body which is equal to weight of fluid displaced by body and is called buoyant force.

Upon integrating $\frac{\partial p}{\partial z} = \rho \cdot g$, we get

$$p = \rho g z$$



Consider a surface submerged in liquid. Pressure force on small strip taken

$$dF = p \times \text{area}$$
$$= \rho g z \cdot dA$$

Buoyant force

$$\Rightarrow \quad F = \int dF = \rho g \int z \times (b dz)$$
$$= \rho g A \times \bar{z}$$

Q. 1. (b) A rectangular tank 4m long and 1.5 m wide contains water upto the height of 2 m. Calculate the force due to water pressure on the base of the tank. Find also the depth of centre of pressure from the free surface. 10

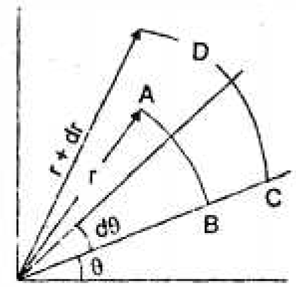
Ans. Force on the base of tank

$$= \rho g A \bar{z}$$
$$= 1000 \times 9.81 \times (1.5 \times 4) \times 2$$
$$= 1,17,720 \text{N}$$

The resultant passes through the centroid of base and is directed vertically downwards.

Q. 2. (a) Derive continuity equation in cylindrical coordinates. Also list the assumption made.

Ans. Consider fluid element *ABCD* between r and $(r + dr)$



Side, $AB = rd\theta$, $BC = dr$, $DC = (r + dr)\,d\theta$, $AD = dr$

Mass of fluid entering the face AB per unit time $= \rho \times$ velocity in r-direction $\times$ Area

$$= \rho \times u_r \times (AB \times 1)$$

$$= \rho u_r\,(rd\theta)$$

Mass of fluid leaving face CD per second

$$= \rho\left(u_r + \frac{\partial u_r}{\partial r}\cdot dr\right) \times CD$$

$$= \rho\left[u_r + \frac{\partial u_r}{\partial r}\cdot dr\right] \times (r + dr)\,d\theta$$

$$= \rho\left[u_r r + u_r dr + r\frac{\partial u_r}{\partial r}\cdot dr\right] d\theta$$

(small terms neglected)

$\therefore$ Gain of mass in r direction

$$= \rho u_r r d\theta - \rho\left[u_r r + u_r dr + r\frac{\partial u_r}{\partial r}\,dr\right] d\theta$$

$$= -\rho\left[u_r dr + r\frac{\partial u_r}{\partial r}\cdot dr\right]\cdot d\theta$$

$$= -\rho\left[\frac{u_r}{r} + \frac{\partial u_r}{\partial r}\right] r\cdot dr\cdot d\theta$$

Similarly gain in mass in θ direction

$$= \rho u_\theta dr \times 1 - \rho\left(u_\theta + \frac{\partial u_\theta}{\partial \theta}\,d\theta\right) \times dr \times 1$$

$$= -\rho\left(\frac{\partial u_\theta}{\partial \theta}\cdot d\theta\right) dr \times 1$$

$$= -\rho\,\frac{\partial u_\theta}{\partial \theta}\cdot\frac{rd\theta dr}{r}$$

Total gain of mass per second

$$= -\rho\left[\frac{u_r}{r} + \frac{\partial u_r}{\partial r}\right] r\,dr\,d\theta - \rho\,\frac{\partial u_\theta}{\partial \theta}\cdot\frac{rd\theta dr}{r}$$

But rate at increase of fluid mass per second

$$= \frac{\partial}{\partial t}[\rho r d\theta\,(dr \times 1)] = \frac{\partial \rho}{\partial t} r d\partial dr$$

$\therefore$

$$-\rho\left(\frac{u_r}{r} + \frac{\partial u_r}{\partial r}\right) rdr\,d\theta - \rho\,\frac{\partial u_\theta}{\partial \theta}\,\frac{rd\theta \cdot dr}{r}$$

$$= \frac{\partial \rho}{\partial t} rd\theta\,dr$$

$$\Rightarrow \quad \frac{\partial \rho}{\partial t}\left[\frac{u_r}{r} + \frac{\partial u_r}{\partial r}\right] + \rho\,\frac{\partial u_\theta}{\partial \theta}\cdot\frac{1}{r} = 0$$

Q. 2. (b) A fluid is given by :

$$V = xy^2 i - 2yz^2 j - (zy^2 - 2z^3/3)\,k$$

Prove that it is a case of steady incompressible fluid flow. Calculate velocity and acceleration at a point [1, 2, 3]. **10**

Ans. For steady three dimensional incompressible fluid flow, the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Now, $u = xy^2$

$\therefore$ $\dfrac{\partial u}{\partial x} = y^2$

$$v = -2yz^2 \text{ so } \frac{\partial v}{\partial y} = -2z^2$$

$$w = -\left(zy^2 - \frac{2z^3}{3}\right) \text{so } \frac{\partial w}{\partial z} = -y^2 + 2z^2$$

So, $y^2 + (-2z^2) - y^2 + 2z^2 = 0$

Hence, flow is steady incompressible.

Now substituting $x = 1,\ y = 2,\ z = 3$

$$v = 1 \times 2^2 i - 2 \times 2 \times 3^2 j - \left(3 \times 2^2 - \frac{2 \times 3^3}{3}\right) k$$

$$= 4i - 36j + 6k$$

$\therefore$ Resultant velocity $= \sqrt{4^2 + 36^2 + 6^2}$

$= 36.71$ m/s **Ans.**

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\frac{\partial u}{\partial x} = y^2, \frac{\partial u}{\partial y} = 2xy, \frac{\partial u}{\partial z} = 0$$

$\therefore$ $$a_x = xy^2 \cdot y^2 + (-2yz^2)\cdot(2xy) = 16 - 144 = -128$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$\frac{\partial v}{\partial x} = 0, \frac{\partial v}{\partial y} = -2z^2, \frac{\partial v}{\partial z} = -4yz$$

$$a_y = xy^2 \cdot 0 + (-2yz^2)(-2z^2) + \left[-\left(zy^2 - \frac{2z^3}{3}\right)\right](-4yz)$$

$\Rightarrow$ $$a_y = 648 + (12-18)(24) = 504$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$\frac{\partial w}{\partial x} = 0, \frac{\partial w}{\partial y} = -(2zy), \frac{\partial w}{\partial z} = -y^2 + 2z^2$$

$$a_z = xy^2 \cdot 0 + (-2yz^2)(-2zy) + \left[-\left(zy^2 - \frac{2z^3}{3}\right)\right](-y^2 + 2z^2)$$

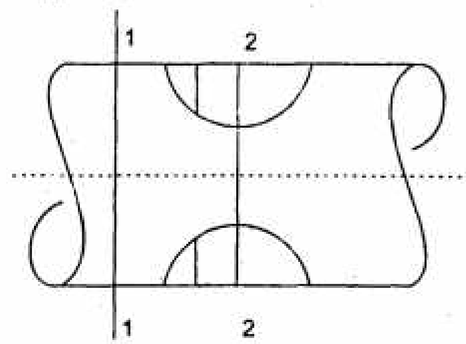
$$= 432 + [-(12-18)(-4+18)] = 516$$

Acceleration $$= \sqrt{(128)^2 + (504)^2 + (516)^2}$$

$$= 732.56 \text{ m/s}^2$$ **Ans.**

Q. 3. (a) Explain the principle of orifice-meter. Derive an expression for flow through it. What are its merits and demerits over the venturimeter. **10**

Ans. Orifice meter works on principle of Bernoulli's theorem.



Apply theorem at (1) and (2)

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v^2}{zg} + z_2$$

/

$$\Rightarrow \quad \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\Rightarrow \quad h = \frac{v_2^2 - v_1^2}{2g}$$

$$\Rightarrow \quad v_2^2 - v_1^2 = 2gh$$

$$\Rightarrow \quad 8\ v_2 = \sqrt{v_1^2 + 2gh}$$

Now, coefficient at contraction

$$C_c = \frac{a_2}{a_0}$$

Where a_2 and a_0 are area at vena-contracta and orifice, respectively,

Also,

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow \quad v_1 = a_2 v_2 / a_1 = \frac{a_0 C_c v_2}{a_1}$$

$$\Rightarrow \quad v_2 = \sqrt{\left(\frac{a_0 C_c v_2}{a_1}\right)^2 + 2 \times gh}$$

$$\Rightarrow \quad v_2^2 - \frac{a_0^2 C_c^2 v_2^2}{a_1^2} = 2gh$$

$$\Rightarrow \quad v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0 C_c}{a_1}\right)^2}}$$

Discharge

$$Q = v_2 a_2 = v_2 \times a_0 C_c$$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0 C_c}{a_1}\right)^2}}$$

Taking C_d = coefficient of discharge

$$= C_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$\therefore \quad Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

Orifice meter is cheaper device than venturimeter but losses are more in orifice meter.

Q. 3. (b) An orifice-meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure gauge fitted upstream and downstream of the orifice meter gives reading of 14.715 N/cm² and 9.81 N/cm² respectively. Find the rate of flow of water through the pipe in liters/sec. Take $C_d = 0.6$.

Ans. Area of orifice $a_0 = \frac{\pi}{4} \times (15)^2$

$= 176.7 \text{ cm}^2$

Area of pipe, $a_1 = \frac{\pi}{4} \times (30)^2 = 706.85 \text{ cm}^2$

$$\frac{p_1}{\rho g} = \frac{14.715 \times 10^4}{1000 \times 9.81} = 15 \text{ m of water}$$

$$\frac{p_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10 \text{ m of water}$$

$$h = 15 - 10 \text{ m} = 500 \text{ cm of water}$$

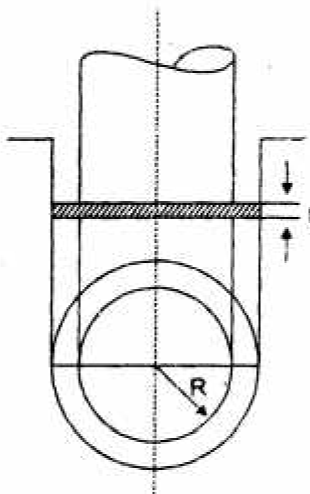
$$Q = C_d \frac{a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

$$= 0.6 \frac{176.7 \times 706.85 \times \sqrt{2 \times 981 \times 500}}{\sqrt{(706.85)^2 - (176.7)^2}}$$

$= 108451 \text{ cm}^3/\text{s} = 108.4 \text{ litres/s}$ **Ans.**

Q. 4. (a) Derive the relation for power absorbed in foot step bearing and colour bearing. **10**

Ans. Let N = Speed of shaft



t = Thickness of oil film

Area of elementary ring $= 2\pi r dr$

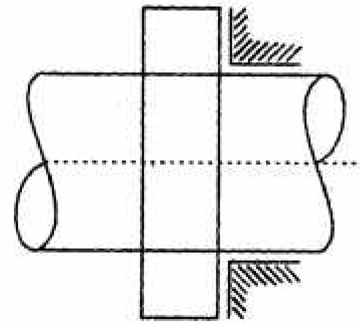
$$\tau = \mu \frac{du}{dy} = \mu \frac{v}{t}$$

v = Tangential velocity at radius $r = \frac{2\pi N}{60} \times r$

Shear force, $dF = \tau \times \text{area}$

$$= \mu \frac{2\pi N}{60} \times \frac{r}{t} \times 2\pi r dr$$

$$= \frac{\mu}{15} \frac{\pi^2 N r^2}{t} dr$$



Torque, $dT = dF \times r = \frac{\mu}{15t} \pi^2 N r^3 dr$

$$T = \int_0^R \frac{\mu}{15t} \pi^2 N r^3 dr$$

$$= \frac{\mu}{15t} \pi^2 N \left[\frac{r^4}{4}\right]_0^R = \frac{\mu}{60t} \pi^2 N R^4$$

Power absorbed, $P = \frac{2\pi NT}{60}$

$$= \frac{2\pi N}{60} \times \frac{\mu}{60t} \pi^2 N R^4$$

$$= \frac{\mu \pi^2 N^2 R^4}{60 \times 30t}$$

For colour bearing

$$dT = \frac{\mu}{15t} \pi^2 N r^3 dr$$

$$T = \int_{R_1}^{R_2} \frac{\mu}{15t} \pi^2 N r^3 dr$$

$$= \frac{\mu}{15t} \pi^2 N \left[\frac{r^4}{4}\right]_{R_1}^{R_2}$$

$$= \frac{\mu}{60t} \pi^2 N [R_2^4 - R_1^4]$$

$\therefore$ $$P = \frac{\mu \pi^3 N^2}{60 \times 30t} (R_2^4 - R_1^4)$$

Q. 4. (b) Determine : **10**

(i) The pressure gradient,

(ii) Shear stress at the two horizontal plates,

(iii) The discharge per meter width for laminar flow of oil with a maximum velocity of 2 m/s between two plates which are 150 mm apart. Take viscosity of the oil as 2.5 Ns/m^2.

Ans. Maximum velocity, $U_{max} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2$

$$\Rightarrow \quad 2 = -\frac{1}{8 \times 2.5}\frac{\partial p}{\partial x}(0.15)^2$$

$$\Rightarrow \quad \frac{\partial p}{\partial x} = -1778 \text{ N/m}^2 \text{ per m} \quad \textbf{Ans.}$$

Shear stress,
$$\tau_0 = -\frac{1}{2}\frac{\partial p}{\partial x} \times t$$

$$= -\frac{1}{2}(-1778) \times (0.55) = 133.35 \text{ N/m}^2$$

Discharge per meter width,

$$Q = \text{Mean velocity} \times \text{Area}$$

$$= \frac{2}{3}U_{max} \times (t \times 1)$$

$$= \frac{2}{3} \times 2 \times 0.15 = 0.2 \text{ m}^3/\text{s} \quad \textbf{Ans.}$$

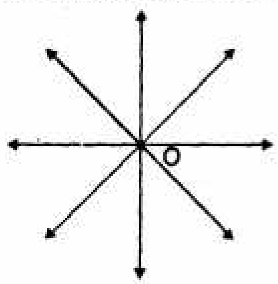
Q. 5. (a) Explain doublet and define the strength of the doublet. **5**

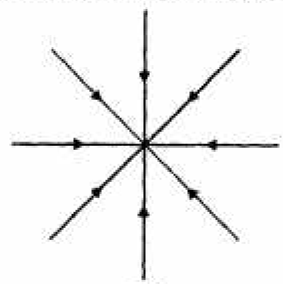
Ans. Doublet is case of source and sink pair both of equal strength when two approach each other in such a way the distance $2a$ between them approaches zero and product $2a \cdot q$ remains constant. This product is called doublet strength and is denoted by μ.

$\therefore \quad \mu = 2a \cdot q$

Q. 5. (b) Distinguish between source and sink. **5**

Ans. Source flow is coming from a point and move out radially in all directions of plane at uniform rate.





In sink flow fluid moves radially inwards towards a point where it disappears at constant rate.

Q. 5. (c) A uniform flow of velocity 6 m/s is flowing along x-axis over a source and a sink which are situated along x-axis. The strength of the source and the sink is 15 m^2/s and they are at a distance of 1.5 m apart. Determine : **10**

(i) Location of stagnation point

(ii) Length and width of Rankine oval

(iii) Equation of profile of Rankine body

Ans. Distance between source and sink= $2a$

$$\Rightarrow \quad a = \frac{1.5}{2} = 0.75\text{m}$$

$$x_s = a\sqrt{1+\frac{q}{\pi a U}} = 0.75\sqrt{1+\frac{15}{\pi \times 0.75 \times 6}} = 1.076\text{m}$$

This is distance of stagnation points from origin. Then will be two stagnation points. The distance of stagnation points from source and sink $= x_s - a = 1.076 - 0.75 = 0.326\,\text{m}$.

Length of Rankine oval, $L = 2x_s = 2.152\,\text{m}$

Width, $B = 2xy_{max}$

$$y_{max} = a\cot\left(\frac{\pi U y_{max}}{q}\right) = 0.75\cot\left(0.4\pi y_{max} \times \frac{180^\circ}{\pi}\right)$$

$$= 0.75\cot\,(72y_{max})^\circ$$

Using hit and trial $y_{max} = 0.67\,\text{m}$

$\therefore$ $B = 1.34\,\text{m}$ **Ans.**

Equation of profile of Rankine body

$$r = \frac{q}{2\pi}\frac{\theta_2-\theta_1}{U\sin\theta} = \frac{15}{2\pi}\frac{\theta_2-\theta_1}{6\sin\theta} = \frac{0.398\,(\theta_2-\theta_1)}{\sin\theta}\ \textbf{Ans.}$$

Q. 6. (a) Explain the different types of losses in a pipe line. Derive the relation for measuring any one type of loss. **10**

Ans. There are two kind of losses in pipe flow :

(i) Major losses : This is due to friction present in pipe.

(ii) Minor losses : This is due to :

(a) Sudden expansion of pipe.

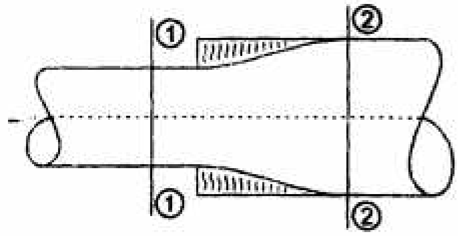
(b) Sudden contraction of pipe.

(c) Bend in pipe.

(d) Pipe fittings.

(e) An obstruction in pipe.

Loss due to sudden expansion can be derived by considering two sections (1) and (2) and applying Bernoulli's equation.



$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2+\text{loss}$$

$\because$ Pipe is horizontal, $z_1=z_2$ and

Let loss $= h_e$

$$\therefore \qquad h_e=\frac{p_1-p_2}{\rho g}+\frac{v_1^2-v_2^2}{2g}$$

Let p' is pressure intensity of liquid eddies. Then force acting on liquid

$$F_x=p_1A_1+p'(A_2-A_1)-p_2A_2$$

$$p'=p_1 \text{ (assuming)}$$

$$F_x=(p_1-p_2)A_2$$

Change of momentum of liquid/sec between (1) and (2)

$$=\rho A_2v_2^2-\rho A_1v_1^2$$

From continuity equation, $\quad A_1=\dfrac{A_2v_2}{v_1}$

Change of momentum $\quad =\rho A_2v_2^2-\rho\dfrac{A_2v_2}{v_1}\cdot v_1^2$

$$=\rho A_2[v_2^2-v_1v_2]$$

$$\Rightarrow \qquad (p_1-p_2)A_2=\rho A_2(v_2^2-v_1v_2)$$

$$\Rightarrow \qquad \frac{p_1-p_2}{\rho}=v_2^2-v_1v_2$$

$$\Rightarrow \qquad \frac{p_1-p_2}{\rho g}=\frac{v_2^2-v_1v_2}{g}$$

$$\Rightarrow \qquad h_e-\frac{v_1^2-v_2^2}{2g}=\frac{v_2^2-v_1v_2}{g}$$

$$\Rightarrow \qquad h_e=\frac{(v_1-v_2)^2}{2g}$$

Q. 6. (b) A nozzle is fitted at the end of a pipe of length 400 m and 150 mm diameter. For the maximum transmission of the power. **10**

Ans. Question is incomplete, it seems diameter of nozzle is required. Taking $f = 0.009$

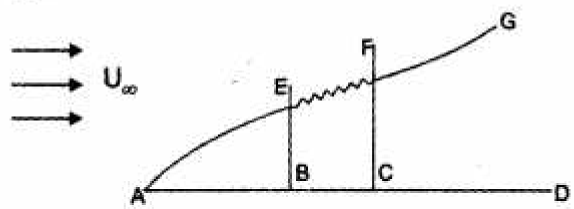
$$d = \left(\frac{D^5}{8fL}\right)^{1/4}$$

$$= \left[\frac{(0.15)^5}{8 \times 0.009 \times 400}\right]^{1/4} = 0.04\,\text{m}$$

$$= 40\text{mm}\ \textbf{Ans.}$$

Q. 7. (a) What is meant by boundary layer ? Why does it increase with distance from the upstream edge. **10**

Ans. When real fluid flows past a solid body, the fluid particles adhere to boundary and velocity of fluid is same close to boundary. If boundary is stationary, velocity of fluid will be zero. Further away from boundary velocity gradient is set up in the fluid which develops shear resistance, which retards the fluid. Thus, the fluid with uniform free stream velocity is retarded in the vicinity of solid surface of plate and boundary layer begins at sharp leading edge.



At subsequent point downstream the leading edge, the boundary layer region increases because the retarded fluid is further retarded. Near the leading edge the flow is laminar though main flow is turbulent.

Q. 7. (b) Find the displacement thickness, the momentum thickness and energy thickness for velocity distribution in the boundary layer given by $u/U = 2(y/\delta) - (y/\delta)^2$ where δ = boundary layer thickness. **10**

Ans. Displacement thickness, $$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$\Rightarrow$ $$\delta^* = \int_0^\delta \left\{1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right\} dy$$

$$= \left[y - \frac{2y}{2\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta = \frac{\delta}{3}\ \textbf{Ans.}$$

Momentum thickness, $$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$\Rightarrow$ $$\theta = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) \cdot \left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4} \right] dy$$

$$= \left[\frac{2y^2}{2\delta} - \frac{5y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4} \right]_0^\delta$$

$$= \frac{2\delta}{15} \textbf{ Ans.}$$

Energy thickness,
$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

$$\Rightarrow \quad \delta^{**} = \int_0^\delta \left[\frac{2y}{\delta} - \left(\frac{y}{\delta}\right)^2 \right] \left[1 - \left[\frac{2y}{\delta} - \left(\frac{y}{\delta}\right)^2 \right]^2 \right] dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} \right) \left[1 - \left(\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - \frac{4y^3}{\delta^3} \right) \right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{8y^2}{\delta^3} - \frac{2y^5}{\delta^5} + \frac{8y^4}{\delta^4} - \frac{y^2}{\delta^2} + \frac{4y^4}{\delta^4} + \frac{y^6}{\delta^6} - \frac{4y^5}{\delta^5} \right] dy$$

$$= \left[\frac{2y^2}{2\delta} - \frac{y^3}{3\delta^2} - \frac{8y^4}{4\delta^3} + \frac{12y^5}{5\delta^4} - \frac{6y^6}{6\delta^5} + \frac{y^7}{7\delta^6} \right]_0^\delta$$

$$= \frac{22\delta}{105} \textbf{ Ans.}$$

Q. 8. (a) What is a velocity defect ? Derive an expression for velocity defect in pipes. 10

Ans. The difference between maximum velocity 'u_{max}' and local velocity at any point 'u' in case of turbulent flow in pipes is called velocity defect.

According to Prandtl

$$\bar{\tau} = \rho l^2 \left(\frac{d\bar{u}}{dy} \right)^2$$

Let
$$l = ky$$

$$\bar{\tau} = \rho k^2 y^2 \left(\frac{du}{dy} \right)^2$$

$$\Rightarrow \quad \frac{du}{dy} = \sqrt{\frac{\tau}{\rho k^2 y^2}}$$

Putting
$$\sqrt{\frac{\tau}{\rho}} = u^* \text{ (shear velocity)}$$

$$\frac{du}{dy} = \frac{1}{ky} u^*$$

Upon integration $$u = \frac{u^*}{k} \ln y + C$$

At $y = R$ radius of pipe $$u = u_{max}$$

$\therefore$ $$u_{max} = \frac{u^*}{k} \ln R + C$$

$\Rightarrow$ $$C = u_{max} - \frac{u^*}{k} \ln R$$

$\therefore$ $$u = \frac{u^*}{k} \ln y + u_{max} - \frac{u^*}{k} \ln R$$

$$= u_{max} + \frac{u^*}{k} (\ln y - \ln R)$$

$$= u_{max} + \frac{u^*}{0.4} \ln\left(\frac{y}{R}\right) (\because K = 0.4 \text{ Karman constant})$$

$$= u_{max} + 2.5u^* \ln\left(\frac{y}{R}\right)$$

$\Rightarrow$ $$u_{max} - u = -2.5u^* \ln\left(\frac{y}{R}\right) = 2.5u^* \ln\left(\frac{R}{y}\right)$$

Q. 8. (b) What do you mean by Prandtl Mixing Length Theory ? Find an expression for shear stress due to Prandtl. **10**

Ans. According to Prandtl, mixing length l is that distance between two layers in transverse direction such that lumps of fluid particles from one layer could reach other layer and particles are mixed in other layer in such a way momentum in x-direction is same. Prandtl assumes that

$$\overline{u'} = l\frac{du}{dy} \text{ and } \overline{v'} = l\frac{du}{dy}$$

$$\overline{u'v'} = l^2\left(\frac{du}{dy}\right)^2$$

Substituting in Reynolds expression

$$\tau = \rho u'v' = \rho l^2\left(\frac{du}{dy}\right)^2$$

Total shear stress $$\tau_{total} = \mu\frac{du}{dy} + \rho l^2\left(\frac{du}{dy}\right)^2$$