

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER- 1st / 2nd • EXAMINATION – SUMMER 2013

Subject Code: Maths-II**Date: 05-06-2013****Subject Name: 110009****Time: 02:30 pm – 05:30 pm****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Let $u = (4, 1, 2, 3)$ and $v = (0, 3, 8, -2)$. Evaluate following as directed. **04**
1. Find the norm of $u + v$.
 2. Find the Euclidean inner product of u and v .
 3. Find the Euclidean distance between u and v .
 4. Find $2u - 3v$.
- (b) Find the inverse of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. **05**
- (c) If $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$, Prove that $A^2 - 13A + 12 = 0$. **05**
- Q.2** (a) Define symmetric and skew symmetric matrix. Express $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ as the **07**
sum of the symmetric and the skew symmetric matrix.
- (b) Determine whether $W = \{ (a, b, c) / a^2 + b^2 < 1 \}$ is a subspace of \mathbb{R}^3 . **03**
- (c) Show that the set of vectors $\{(2, 1, 1), (1, 2, 2), (1, 1, 1)\}$ is linearly independent **04**
in \mathbb{R}^3 .
- Q.3** (a) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$. **07**
- (b) Find the least square solution of the linear system $Ax = b$ given by **07**
 $x_1 - x_2 = 4, 3x_1 + 2x_2 = 1, -2x_1 + 4x_2 = 3$ and find the orthogonal
projection of b on the column space of A .
- Q.4** (a) Define linear transformation. Find which of the following function are linear **07**
transformations.
1. $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, Defined by $T(x, y, z) = (x^2, y, x + y)$.
 2. $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, Defined by $T(x, y) = (x, -y)$.
- (b) Let \mathbb{R}^3 have the Euclidean inner product. Use the Gram-Schmidt process to **07**
transform the basis $S = \{ (1, 0, -1), (2, 1, 1), (0, -1, 3) \}$ into an
orthonormal basis.
- Q.5** (a) Let V be the set of all positive real number with the operations $x + y = xy$ and **07**
 $kx = x^k, k \in \mathbb{R}$. Show that the set V is a vector space.
- (b) Let $v_1 = (1, 2, 1), v_2 = (2, 9, 0)$ and $v_3 = (3, 3, 4)$ **07**
1. Show that the set $S = \{ v_1, v_2, v_3 \}$ is a basis for \mathbb{R}^3 .
 2. Find the coordinate vector of $v = (5, -1, 9)$ with respect to S .
- Q.6** (a) Solve $x + y + 2z = 8, -x - 2y + 3z = 1, 3x - 7y + 4z = 10$ by Gauss-Jordan **07**
elimination.
- (b) Find a basis for the null space, the row space and the column space **07**
of $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$.

- Q.7** (a) Consider the basis $S = \{v_1, v_2\}$ for \mathbb{R}^2 , where $v_1 = (-2, 1)$ and $v_2 = (1, 2)$ and **07**
Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation such that $T(v_1) = (-1, 2, 0)$ and
 $T(v_2) = (0, -3, 5)$. Find a formula for $T(x_1, x_2)$ and use that formula to find
 $T(x_1, x_2)$.
- (b) Define: Real inner product space. Let the vector space P_2 have the inner product **07**
space $\langle p, q \rangle = \int_{-1}^1 p(x)q(x) dx$ then
1. Find $\|p\|$ for $p = 1, q = x$.
 2. Find $d(p, q)$ if $p = x, q = x^3$.

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