Seat No.: _____                                          Enrolment No._____

# GUJARAT TECHNOLOGICAL UNIVERSITY
### BE- I$^{st}$ /II$^{nd}$ SEMESTER–EXAMINATION – MAY/JUNE - 2012

**Subject code: 110009**                                          **Date: 26/05/2012**
**Subject Name: Mathematics - II**
**Time: 10:30 am – 01:30 pm**                               **Total Marks: 70**
**Instructions:**
1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Each question carry equal marks**

| Q.1 | (a) | Solve by Gauss Jordan Method<br>$x + y + z = 3, x + 2y - z = 4, x + 3y + 2z = 4$ | 3 |
|---|---|---|---|
| | (b) | Express the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ as a product of Elementary Matrix. | 7 |
| | (c) | Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ | 4 |
| Q.2 | (a) | Prove that $\left\| \bar{u} + \bar{v} \right\|^2 + \left\| \bar{u} - \bar{v} \right\|^2 = 2\left\| \bar{u} \right\|^2 + 2\left\| \bar{v} \right\|^2$ | 3 |
| | (b)1 | Let $\bar{u} = (2,4,6)$ and $\bar{a} = (1,-1,-1)$. Find the vector component of $\bar{u}$ along $\bar{a}$ and orthogonal to $\bar{a}$. | 4 |
| | 2 | Give a $3 \times 3$ singular matrix such that all the entries in the matrix are non zero. | 2 |
| | (c)1 | Consider $\bar{u} = (6,2,3), \bar{v} = (2,2,-1)$ and $\bar{w} = (1,0,1)$ with same initial point. Determine whether they are in the same plane? | 2 |
| | 2 | State Cauchy Schwarz inequality in $R^n$ and verify for the vectors $\bar{u} = (1,2,-3)$ and $\bar{v} = (1,0,2)$. | 3 |
| Q.3 | (a)1 | Write the standard matrix for linear transformation<br>$T(x, y, z, w) = (3x + z + w,\ x - y - w)$ | 2 |
| | 2 | Use matrix multiplication to find the reflection of $(2,-1,4)$ about $xy$ - plane. | 2 |
| | 3 | Use matrix multiplication to find the image of the vector $(1,-1,3)$ if it is rotated $-30^0$ about the x – axis. | 2 |
| | (b) | Show that $T : R^2 \rightarrow R^2$ defined by the equation<br>$w_1 = x_1 + 2x_2,\quad w_2 = 2x_1 + 3x_2$<br>Is one – to – one and find $T^{-1}(w_1, w_2)$. | 5 |
| | (c) | Is $T(x, y) = (x + 3y, y - 2)$ linear operator? | 3 |
| Q.4 | (a) | Consider real vector space $P_2$ with standard operations.<br>Is $S_1 = \{a_0 + a_1 x + a_2 x^2 / a_0 = 0\}$ and $S_2 = \{a_0 + a_1 x + a_2 x^2 / a_0 \neq 0\}$ are subspaces? Give reasons. | 4 |

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| | (b) | Find an equation of the plane spanned by the vectors $\bar{u} = (1,1,2), \ \bar{v} = (3,2,0)$ | **4** |
|---|---|---|---|
| | | | |
| | (c)1 | Is the set $S = \{3 + x + x^2, \ 2 - x + 5x^2, 4 - 3x^2\}$ linearly independent in $P_2$ with standard operations? | **3** |
| | 2 | Write any three bases of $P_2$. | **3** |
| **Q.5** | (a) | Determine the basis of plane $x + 4y - 3z = 0$. | **4** |
| | (b) | Let $S = \{\bar{v}_1, \bar{v}_2, \bar{v}_3, \bar{v}_4, \bar{v}_5\}$. Find a subset of $S$ that forms a basis for the space spanned by $S$. Also express other vectors as linear combination of basis vectors. Here, $\bar{v}_1 = (1, -1, 2), \bar{v}_2 = (0, 1, 1), \bar{v}_3 = (1, 2, 8), \bar{v}_4 = (0, 3, 6), \bar{v}_5 = (-1, 3, 3)$ | **4** |
| | (c)1 | Find a basis for the null space of $A = \begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 0 \\ -1 & 1 & 3 \end{bmatrix}$. | **3** |
| | 2 | Find the rank and nullity of the matrix $A = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -1 & 1 & 1 \\ 2 & 0 & 3 & 1 \end{bmatrix}$ | **3** |
| **Q. 6** | (a) | Define an inner product on real vector space $V$. Let $\bar{u} = (u_1, u_2), \ \ \bar{v} = (v_1, v_2)$ are any vectors in $R^2$. Show that $<\bar{u}, \bar{v}> = 9u_1 v_1 + 4u_2 v_2$ is inner product on $R^2$. | **4** |
| | (b) | Find a basis for the orthogonal complement of $W = \{\bar{v}_1, \bar{v}_2, \bar{v}_3, \bar{v}_4\}$, where $\bar{v}_1 = (1, -1, 1, 0, 2), \ \bar{v}_2 = (0, 3, 1, 0, 0),$ $\bar{v}_3 = (-1, 0, 0, 1, -1), \ \bar{v}_4 = (0, -1, 1, 1, 1).$ | **4** |
| | (c)1 | Apply Gram – Schmidt process to transform the basis vectors $\bar{u}_1 = (1,1,1), \ \bar{u}_2 = (0,1,1), \ \bar{u}_3 = (0,0,1)$ in to an orthogonal basis. | **4** |
| | 2 | Write any three orthogonal matrices of the type $2 \times 2$. | **2** |
| **Q.7** | (a) | Find eigen basis of the matric $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$ | **5** |
| | (b) | Find a matrix $P$ that diagonalizes $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$. | **4** |
| | (c)1 | Is the function $T : P_2 \to P_2$, $\quad T(a_0 + a_1 x + a_2 x^2) = a_0 + a_1(x + 2) + (a_2 + 2)x^2$ linear transformation? | **3** |
| | 2 | Let $T : R^2 \to R^2$, $T(x, y) = (x + y, x - y)$. Determine $T^{-1}$ if possible. | **2** |

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