Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER –I • EXAMINATION – WINTER 2012

Subject code: 110009 **Date: 11-01-2013**

Subject Name: Mathematics - II

Time: 10.30 am – 01.30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Define dot product of vectors as matrix multiplication. State and prove Cauchy- Schwarz Inequality. Verify it for vectors $u = (1,2,3)$ & $v = (-1,0,3)$ **5**

(b) Define Vector Space over the field K. Check whether the structure $(R^2, +, \cdot)$ is Vector Space over the field R. Where vector addition $\bar{x} + \bar{y} = (x_1 + y_1\ ,\ x_2 + y_2)$ & Scalar Multiplication $\alpha \cdot \bar{x} = (\ \alpha^2 x_1, \alpha^2 x_2)$ for vectors $\bar{x} = (x_1, x_2)$ & $\bar{y} = (y_1, y_2)$ **5**

(c) Define Distance between two vectors. Find $d\ (u \times v\ ,\ \hat{v})$ for vectors $= (1, 2, -1)$ & $v = (-2, 1, 2)$, where $\hat{v}$ is unit vector in the direction of vector v. **4**

Q.2 (a) Define Linear combination of vectors, Linearly Dependent vectors and Linearly Independent vectors. Check whether vectors $1, sin^2 x$ & $\cos 2x$ of $F\ (-\infty, \infty)$are Linearly Dependent or Linearly Independent vectors. **5**

(b) Define Basis of Vector Space. Find the standard basis vector(s) that can be added to the following set of vectors to produce a basis for R^3. Set of vectors $\{v_1 = (-1, 2, 3), v_2 = (1, -2, -2)\}$. **5**

(c) Define Sub Space of a vector space. State the necessary and sufficient condition for a subset of a vector space to be subspace. Check whether sub set $W = \left\{A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : where\ a, b\ c\ d\ \in Z\ with\ |A| = 0\right\}$ of a vector space M_{22} is sub space. **4**

Q.3 (a) Define the rank of a matrix. Find the rank of the following by reducing to row echelon form. $A = \begin{pmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{pmatrix}$ **5**

(b) Find the basis for row and column spaces of matrix **5**

$$A = \begin{bmatrix} 1 & 4 & 5 & 4 \\ 2 & 9 & 8 & 2 \\ 2 & 9 & 9 & 7 \\ -1 & -4 & -5 & -4 \end{bmatrix}$$

(c) What is trivial solution of homogeneous system of equations? Solve the homogeneous system of linear equations : **4**

$2x_1 + 2x_2 - x_3 + x_5 = 0, -x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0,$
$x_1 + x_2 - 2x_3 - x_5 = 0$ & $x_3 + x_4 + x_5 = 0$

Q.4 (a) Define Linear Transformation.Consider the basis $S = \{v_1, v_2, v_3\}$for R^3. Where $v_1 = (1,1,1), v_2 = (1,1,0)$ & $v_3 = (1,0,0)$. A Linear Transformation $T: R^3 \rightarrow R^2$ such that $T(v_1) = (1,0)$, $T(v_2) = (2,-1)$ & $T(v_3) = (4,3)$, then find the formula for **5**

Linear Transformation T.

(b) A Linear Transformation $T: R^2 \rightarrow R^3$ defined by $T(x_1, x_2) = (x_2, -5x_1 + 13x_2, -7x_1 + 16x_2)$. Find the matrix of Transformation T with respect to the bases $B = \{u_1 = (3,1), u_2 = (5,2)\}$ for R^2 & $B' = \{v_1 = (1,0,-1), v_2 = (-1,2,2), v_3 = (0,1,2)\}$ for R^3. **5**

(c) Find the standard matrix for the Linear operator on R^3 that (1) its reflection through the 'xz – plane' (2) rotate each vector 90° counterclockwise about z - axis (along the positive z - axis toward the origin). Check your answer geometrically by sketching the vector (1,1,1) and T (1,1,1). **4**

Q.5 (a) Define inner product space. Find the matrix generated the inner product $\langle u, v\rangle = 3u_1v_1 + 5u_2v_2$ **5**

(b) Use Gram- Schmidt Process to transform the basis $\{u_1, u_2, u_3\}$ of R^3 (with usual inner product space) into orthonormal bases. Where $u_1 = (1,1,1), u_2 = (0,1,1), \& \ u_3 = (0,0,1)$. **5**

(c) Find the least square solution of the linear system $AX = B$ given by $x_1 - x_2 = 4, 3x_1 + 2x_2 = 1 \ \& -2x_1 + 4x_2 = 3$. **4**

Q. 6 (a) Define Hermitian Matrix and Unitary Matrix. Check whether the given matrix is hermitian matrix or unitary matrix : $A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$. **5**

(b) Find Algebraic Multiplicity & Geometric Multiplicity for the Matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ -8 & -10 & -8 \\ 4 & 4 & 2 \end{bmatrix}$. **5**

(c) Verify Dimension (Rank - Nullity) theorem for a linear Transformation T multiplication by a matrix A given by **4**

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 5 & 13 \\ -2 & -1 & -4 \end{bmatrix}$$

Q.7 (a) Find an orthogonal matrix P which diagonalizes the matrix **5**

$A = \begin{bmatrix} 1 & 4 \\ 4 & 1 \end{bmatrix}$.

(b) Reduced the quadratic form into canonical form for $2x_1^2 + 5x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_1x_3 - 8x_3x_2$. **5**

(c) What is Consistent solution of non -homogeneous system of equations? Solve the system of linear equations by Gaussian Elimination : $x + y + 2z = 9$, $2x + 4y - 3z = 1$ & $3x + 6y - 5z = 0$. **4**

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