Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-II Examination June 2010

Subject code: 110009

Subject Name: Mathematics-II

Date: 23 /06 /2010 **Time: 02.30 pm – 05.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Attempt any two: **06**

i. Solve the following system for x, y and z:

$$-\frac{1}{x}+\frac{3}{y}+\frac{4}{z}=30,\ \frac{3}{x}+\frac{2}{y}-\frac{1}{z}=9,\ \frac{2}{x}-\frac{1}{y}+\frac{2}{z}=10.$$

ii. Find A^{-1} using row operations if $A=\begin{bmatrix}1 & 0 & 1\\ -1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$.

iii. Find the standard matrices for the reflection operator about the line $y = x$ on R^2 and the reflection operator about the yz – plane on R^3.

(b) Show that there is no line containing the points $(1,1)$, $(3,5)$, $(-1,6)$ and $(7,2)$. **03**

(c) i. Find all vectors in R^3 of Euclidean norm 1 that are orthogonal to the vectors $u_1 = (1,1,1)$ and $u_2 = (1,1,0)$. **02**

ii. Find the rank of the matrix $A=\begin{bmatrix}2 & -1 & 3\\ 4 & -2 & 6\\ -6 & 3 & -8\end{bmatrix}$ in terms of determinants. **02**

iii. Is $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ in row-echelon or reduced row-echelon form? **01**

Q.2

(a) i. What conditions must b_1, b_2 and b_3 satisfy in order for $x_1 + 2x_2 + 3x_3 = b_1,\ 2x_1 + 5x_2 + 3x_3 = b_2,\ x_1 + 8x_3 = b_3$ to be consistent? **04**

ii. Is $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + 3y, y, z + 2x)$. linear? Is it one-to-one, onto or both? Justify. **03**

(b) i. Show that the set $S = \{e^x, xe^x, x^2e^x\}$ in $C^2(-\infty,\infty)$ is linearly independent. **02**

ii. Check whether $V = R^2$ is a vector space with respect to the operations $(u_1,u_2)+(v_1,v_2)=(u_1+v_1-2, u_2+v_2-3)$ and $\alpha(u_1,u_2)=(\alpha u_1+2\alpha-2, \alpha u_2-3\alpha+3)$, $\alpha \in R$. **05**

OR

(b) i. State only one axiom that fails to hold for each of the following sets W to be subspaces of the respective real vector space V with the standard operations: **05**

$[A]\; W=\{(x,y)\mid x^2=y^2\}, \qquad V=R^2$

$[B]\; W=\{(x,y)\mid xy\geq 0\}, \qquad V=R^2$

$[C]\; W=\{(x,y,z)\mid x^2+y^2+z^2=1\}, \qquad V=R^3$

$[D]\; W=\{A_{n\times n}\mid Ax=0\Rightarrow x=0\}, \qquad V=M_{n\times n}$

$[E]\; W=\{f\mid f(x)\leq 0,\forall x\}, \qquad V=F(-\infty,\infty)$

ii. Check whether $S=\{\sin(x+1),\sin x,\cos x\}$ in $C(0,\infty)$ is linearly independent. **02**

Q.3

(a) i. Determine whether the following polynomials span P_2: **03**

$p_1=1-x+2x^2,\; p_2=5-x+4x^2,\; p_3=-2-2x+2x^2.$

ii. Show that $S=\{1-t-t^3,-2+3t+t^2+2t^3,1+t^2+5t^3\}$ is linearly independent in P_3. **03**

(b) i. Find a standard basis vector that can be added to the set $S=\{(-1,2,3),(1,-2,-2)\}$ to produce a basis of R^3. **03**

ii. Determine whether b is in the column space of A, and if so, express b as a linear combination of the column vectors of A if **03**

$$A=\begin{bmatrix}1 & -1 & 1\\ 1 & 1 & -1\\ -1 & -1 & 1\end{bmatrix} \text{ and } b=\begin{bmatrix}2\\0\\0\end{bmatrix}.$$

(c) i. If A is an $m\times n$ matrix, what is the largest possible value for its rank. **01**

ii. Find the number of parameters in the general solution of $Ax=0$ if A is a 5×7 matrix of rank 3. **01**

OR

Q.3

(a) i. Find basis and dimension of $W=\{(a_1,a_2,a_3,a_4)\in R^4\mid a_1+a_2=0,a_2+a_3=0,a_3+a_4=0\}$. **03**

ii. Find a basis for the subspace of P_2 spanned by the vectors $1+x,x^2,-2+2x^2,-3x.$ **03**

(b) i. Reduce $S=\{(1,0,0),(0,1,-1),(0,4,-3),(0,2,0)\}$ to obtain a basis of R^3. **03**

ii Find a basis for the row space of A and column space of A if **03**

$$A=\begin{bmatrix}1 & 4 & 5 & 2\\ 2 & 1 & 3 & 0\\ -1 & 3 & 2 & 0\end{bmatrix}.$$ Also verify the dimension theorem for matrices.

(c) Show that $S=\left\{\begin{bmatrix}1 & 2\\ 1 & -2\end{bmatrix},\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix},\begin{bmatrix}0 & 2\\ 3 & 1\end{bmatrix},\begin{bmatrix}0 & 0\\ -1 & 2\end{bmatrix}\right\}$ is a basis for M_{22}. **02**

Q.4

(a) i. Compute $d(f,g)$ for $f=\cos 2\pi x$ and $g=\sin 2\pi x$ in $V=C[0,1]$ with inner product $\langle f,g\rangle=\int_0^1 f(x)g(x)\,dx$. **02**

ii. Find a basis for the orthogonal complement of the subspace of R^3 spanned by the vectors $v_1=(1,-1,3)$, $v_2=(5,-4,-4)$ and $v_3=(7,-6,2)$. **03**

(b) i. Let $W=span\left\{\left(\frac{4}{5},0,\frac{-3}{5}\right),(0,1,0)\right\}$. Express $w=(1,2,3)$ in the form of $w=w_1+w_2$, where $w_1\in W$ and $w_2\in W^{\perp}$. **03**

ii. Define algebraic and geometric multiplicity. Show that $A=\begin{bmatrix}1 & 0 & 0\\ 1 & 2 & 0\\ -3 & 5 & 2\end{bmatrix}$ is not diagonalizable. **03**

(c) Show that P_3 and M_{22} are isomorphic. **03**

OR

Q. 4

(a) i. Let R^3 have the Euclidean inner product. Transform the basis $S=\{(1,0,0),(3,7,-2),(0,4,1)\}$ into an orthonormal basis using the Gram-Schmidt process. **03**

ii. For $U=\begin{bmatrix}u_1 & u_2\\ u_3 & u_4\end{bmatrix}$ and $V=\begin{bmatrix}v_1 & v_2\\ v_3 & v_4\end{bmatrix}$ in M_{22}, define $\langle U,V\rangle=u_1v_1+u_2v_2+u_3v_3+u_4v_4$. For the matrices A and B, verify Cauchy-Schwarz inequality and find the cosine of the angle between them, if $A=\begin{bmatrix}2 & 6\\ 1 & -3\end{bmatrix}$, $B=\begin{bmatrix}3 & 2\\ 1 & 0\end{bmatrix}$. **02**

(b) i. Find the least squares solution of the linear system $Ax=b$ and find the orthogonal projection of b onto the column space of A where $A=\begin{bmatrix}1 & 1\\ -1 & 1\\ -1 & 2\end{bmatrix}$ and $b=\begin{bmatrix}7\\ 0\\ -7\end{bmatrix}$. **03**

ii. Find the transition matrix from basis $B=\{(1,0),(0,1)\}$ of R^2 to basis $B'=\{(1,1),(2,1)\}$ of R^2. **03**

(c) For the matrix $A=\begin{bmatrix}\frac{1+i}{2} & \frac{1+i}{2}\\ \frac{1-i}{2} & \frac{-1+i}{2}\end{bmatrix}$, show that the row vectors form an orthonormal set in C^2. Also, find A^{-1}. **03**

Q.5

(a) i. For the basis $S=\{v_1,v_2,v_3\}$ of R^3, where $v_1=(1,1,1)$, $v_2=(1,1,0)$ and $v_3=(1,0,0)$, let $T:R^3\to R^3$ be a linear transformation such that $T(v_1)=(2,-1,4)$, $T(v_2)=(3,0,1)$, $T(v_3)=(-1,5,1)$. Find a formula for $T(x_1,x_2,x_3)$ and use it to find $T(2,4,-1)$. **04**

ii. Let $T_1:M_{22}\to R$ and $T_2:M_{22}\to M_{22}$ be the linear transformations given by $T_1(A)=tr(A)$ and $T_2(A)=A^T$. Find $(T_1\circ T_2)(A)$ where $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. **02**

(b) Find a matrix P that diagonalizes $A=\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, and hence find A^{10}. Also, find the eigenvalues of A^2. **04**

(c) Let $T:R^4\to R^3$ be the linear transformation given by $T(x_1,x_2,x_3,x_4)=(w_1,w_2,w_3)$ where $w_1=4x_1+x_2-2x_3-3x_4$, $w_2=2x_1+x_2+x_3-4x_4$, $w_3=6x_1-9x_3+9x_4$. Find bases for the range and kernel of T. **04**

OR

Q.5

(a) Let $T:R^2\to R^3$ be the linear transformation defined by

$$T(x_1,x_2)=(x_2,-5x_1+13x_2,-7x_1+16x_2).$$

Find the matrix for the transformation T with respect to the bases

$B=\{(3,1)^T,(5,2)^T\}$ for R^2 and

$B'=\{(1,0,-1)^T,(-1,2,2)^T,(0,1,2)^T\}$ for R^3. **04**

(b) i. Let $T:R^2\to R^2$ be defined by $T(x,y)=(x+y,x-y)$. Is T one-one? If so, find formula for $T^{-1}(x,y)$. **04**

ii. Find eigenvalues of $A=\begin{bmatrix} -420 & 1/2 & 576 \\ 0 & 0 & 0.6 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$. Is A invertible? **01**

(c) Translate and rotate the coordinate axes, if necessary, to put the conic $9x^2-4xy+6y^2-10x-20y=5$ in standard position. Find the equation of the conic in the final coordinate system. **05**

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