*Seat No.:* ________ *Enrolment No.*___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**B. E. - SEMESTER – I • EXAMINATION – WINTER • 2014**

**Subject code: 110008** **Date: 29-12-2014**

**Subject Name: Mathematics - I**

**Time: 10:30 am - 01:30 pm** **Total Marks: 70**

**Instructions:**

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** (i) If $3-x^3 \le g(x) \le 3\sec x$, for all x, find $\lim\limits_{x\to 0} g(x)$. **02**

(ii) Find the value of c so that function becomes continuous **02**

$$f(x) = cx+1 \quad ; \; x \le 3$$
$$\quad = cx^2-1 \quad ; \; x > 3$$

(iii) Verify the Lagrange's mean value theorem for the function **03**
$f(x) = 2x^2 + 3x + 4$ in $[a, b]$ .

**(b)** (i) Find the Maclaurin series of function tanx upto terms containing $x^5$ . **03**

(ii) Evaluate using L' Hospital rule $\lim\limits_{x\to 1}\left[\frac{1}{\log x} - \frac{x}{x-1}\right]$ **02**

(iii) Evaluate using L' Hospital rule $\lim\limits_{x\to 0} (a^x + x)^{1/x}$ **02**

**Q.2** **(a)** (i) Find the local maximum and local minimum value of the function **04**
$f(x) = x^3 - 9x^2 + 15x + 11$

(ii) Evaluate $\int_0^\infty \frac{dx}{a^2+x^2}$ ; $a > 0$ **03**

**(b)** (i) Trace the curve $x^3 + y^3 = 3axy$ **04**

(ii) Discuss the convergence of the integral $\int_0^\infty \frac{1}{x^2}\,dx$ **03**

**Q.3** **(a)** (i) Does the sequence whose $n^{th}$ term is $a_n = [(n+1)/(n-1)]^n$ converge? If so, find $\lim\limits_{n\to\infty} a_n$ **04**

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ **03**

**(b)** (i) Test the convergence of the series $\sum_{n=1}^{\infty}\left[\left(\left(\frac{3}{4}\right)^{n-1} - \frac{4}{n(n+1)(n+2)}\right)\right]$ **04**

(ii) Show that the sequence $[3/(n+3)]$ is a decreasing sequence. **03**

**Q.4** **(a)** (i) If $u = 2(ax+by)^2 - (x^2+y^2)$ and $a^2 + b^2 = 1$ then prove that **03**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

(ii) Find the equation of the tangent plane and normal line to the surface **04**
$2xz^2 - 3xy - 4x = 7$ at $(1,-1,2)$

**(b)** (i) Discuss the continuity of the function **03**

$$f(x,y) = (x^2 - y^2)/(x^2 + y^2) \quad ; \; (x,y) \neq (0,0)$$
$$= 0 \quad ; \; (x,y) = (0,0)$$

(ii) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then find the value of **04**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

**Q.5** **(a)** (i) If $u = x^2 + y^2$, $x = a\cos t$, $y = b\sin t$ then find $\frac{du}{dt}$ **03**

(ii) Find $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$ where $x = r\cos\theta$, $y = r\sin\theta$ and $z = z$. **04**

**(b)** (i) Change the order of integration in the integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dydx$ and evaluate it. **03**

(ii) Find the volume of the region bounded by the surface $x = 0$, $y = 0$, $x + y + z = 1$ and $z = 0$. **04**

**Q.6** **(a)** (i) Find the directional derivatives of $f(x,y,z) = x^2z + 2xy^2 + yz^2$ at the point P (1,2,-1) in the direction of the vector $a = 2i+3j-4k$. **03**

(ii) Using Green's Theorem evaluate $\int_c (x^2y\,dx + x^2dy)$; where c is the boundary of the triangle whose vertices (0,0),(1,0), (1,1). **04**

**(b)** (i) Show that $F(x, y, z) = y^2z^3 i + 2xyz^3 j + 3xy^2z^2 k$ is a conservative vector Field. **03**

(ii) If $F = 3xy i - y^2 j$ then evaluate $\int_c F.dr$, where c is the arc of the parabola $y = 2x^2$ from (0, 0) to (1,2). **04**

**Q.7** **(a)** (i) Test the convergence of the series **03**

$$\sum_{n=1}^{\infty}\frac{n}{e^{n}}$$

(ii) If $u = f(x-y, y-z, z-x)$ then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ **04**

**(b)** (i) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dxdy$. **03**

(ii) Find the Taylor's series expansion of $f(x) = \sin x$ in the power of $(x - \pi/2)$. Hence obtain $\sin 91^0$. **04**

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