Seat No.: _____   Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2013

Subject Code: 110008   **Date: 10-06-2013**
Subject Name: Maths-I
Time: 02:30 pm – 05:30 pm   **Total Marks: 70**

Instructions:
1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) If $3x \le f(x) \le x^3 + 2$ for $0 \le x \le 2$, Evaluate $\lim_{x \to 1} f(x)$. **02**

(b) Find the constant c that makes f continuous on $(-\infty, \infty)$, **03**
$$f(x) = \begin{cases} x^2 - c^2 & if \ x < 4 \\ cx - 20 & if \ x \ge 4 \end{cases}$$

(c) Express the polynomial $x^3 + 7x^2 + x - 6$ in power of $(x - 1)$. **03**

(d) Evaluate: **06**
  1. $\lim\limits_{x \to 0} \dfrac{xe^x - \log(1+x)}{x^2}$.
  2. $\lim\limits_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.

Q.2 (a) Find the absolute maximum and absolute minimum values of **04**
$f(x) = 3x^2 - 12x + 5$ on the interval $[0,3]$.

(b) Expand $e^{\sin x}$ by Maclaurins series up to the terms containing x^4. **05**

(c) Find two positive numbers whose product is 100 and whose sum is minimum. **05**

Q.3 (a) If $f(x) = x^3 + x^2 y^3 - 2y^2$, find $f_x(2,1)$ and $f_y(2,1)$. **04**

(b) If $u = sin^{-1} \dfrac{x+2y}{x^8 + y^6}$ find the value of $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y}$. **05**

(c) Find the local extreme values of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$. **05**

Q.4 (a) For what values of a, m and b does the function **04**
$$f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$$
Satisfies the hypotheses of the Mean Value theorem on the interval $[0,2]$?

(b) Find $\dfrac{d}{dx} \int_1^{x^2} \cos t \, dt$. **04**

(c) Determine whether series converges or diverges. **06**
  1. $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^2 + n + 1}$.   2. $\sum\limits_{n=1}^{\infty} \left(\dfrac{2n+3}{3n+12}\right)^n$.

Q.5 (a) Evaluate: $\int_1^2 \int_y^2 xy \, dx \, dy$. **04**

(b) Evaluate: $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2) \, dz \, dx \, dy$. **05**

(c) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\dfrac{16}{3} a^2$. **05**

Q.6 (a) Find the jacobian of the transformation $x = u + 4v$ and $y = 3u - 2v$. **04**

(b) Evaluate the integral $\int_c xy \, dy - y^2 \, dx$, where c is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$. **05**

(c) If $\vec{f} = 3xyi - y^2 j$, evaluate $\int \vec{f} \cdot dr$, where c is the curve in the xy- plane $y = 2x^2$ from (0,0) to (1,2). **05**

Q.7 (a) Find the area enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.   04

(b) Evaluate: 04
1. $div \left[3x^2 i + 5xy^2 j + xyz^3 k \right]$.
2. $curl[e^{xyz}(i + j + k)]$.

(c) Verify Stoke's theorem for $\vec{f} = (x^2 + y^2)\, i - 2xy\, j$ taken around the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$.   06

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