Seat No.: _____   *Enrolment No._____*

GUJARAT TECHNOLOGICAL UNIVERSITY
BE- Ist /IInd SEMESTER–EXAMINATION – MAY/JUNE - 2012

Subject code: 110008   **Date: 06/06/2012**
Subject Name: Maths-I
Time: 10:30 am – 01:30 pm   **Total Marks: 70**
Instructions:
   1. **Attempt any five questions.**
   2. **Make suitable assumptions wherever necessary.**
   3. **Figures to the right indicate full marks.**
   4. **Each question carry equal marks**

Q.1   (a)   Evaluate : (i) $\lim\limits_{x \to 0} \left(\dfrac{1}{x}\right)^{1-\cos x}$

                   (ii) $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x^2 \tan x}$.   **04**

         (b)   Find the directional derivative of $f = 4xz^3 - 3x^2 y^2 z$ at $(2,-1,2)$ **03**
               in the direction of $2\vec{i} + 3\vec{j} + 6\vec{k}$

         (c)   Find the Taylor series generated by $f(x) = \dfrac{1}{x}$ at a=2. Where ,if   **03**

               anywhere , does the series converge to $\dfrac{1}{x}$?

         (d)   Find Maclaurin series for the function $x^4 - 2x^3 - 5x + 4$.   **02**

         (e)   Evaluate $\int_0^1 \int_0^{1-y} \int_0^2 dx\,dy\,dz$.   **02**

Q.2   (a)   Discuss the convergence of the following series: **06**
               (i) $\sum\limits_{n=1}^{\infty} \dfrac{2n+1}{n^2 + n + 1}$ (ii) $\sum\limits_{n=1}^{\infty} \dfrac{n^2}{2^n}$ (iii) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n x^{n+1}}{2n - 1}$

         (b)   State Direct comparison test for improper integrals. Using it show   **04**
               that, $\int_1^{\infty} e^{-x^2} dx$ is convergent.

         (c)   Evaluate the following improper integrals: **04**
               (i) $\int_0^{\infty} \dfrac{dx}{x^2 + 1}$ (ii) $\int_0^{\infty} \dfrac{dv}{(1+v^2)(1+\tan^{-1} v)}$

Q.3   (a)   (i) Show that $f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2}, & (x, y) \neq 0 \\ 0 & (x, y) = 0 \end{cases}$ is continuous   **05**

               at every point except at the origin.

               (ii) Express $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$,

               $x = \dfrac{r}{s}, y = r^2 + \ln s, z = 2r$.

         (b)   State Euler's theorem on homogeneous function of two variable and   **05**

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apply it to evaluate $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ if $u = \tan^{-1}\left(\dfrac{x^2 + y^2}{x - y}\right)$.

(c) If $u = \log(x^3 + y^3 - x^2 y - xy^2)$ prove that   **04**

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = \frac{-4}{(x+y)^2}.$$

Q.4

(a) Find the extreme values of $\sin x + \sin y + \sin(x + y)$, $(0 \le x, y \le \dfrac{\pi}{2})$. **05**

(b) Find the maximum and minimum values of the function **05**
$f(x, y) = 3x + 4y$ on the circle $(x^2 + y^2) = 1$ using the method of Lagrange multipliers.

(c) Evaluate $\displaystyle\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x - y}{2}\,dx\,dy$ by applying the transformation **04**

$u = \dfrac{2x - y}{2}, v = \dfrac{y}{2}$ and integrating over an appropriate region tn the uv-plane.

Q.5

(a) Evaluate $\displaystyle\iint_R (y - 2x^2)\,dA$, Where R is the region bounded by the **05**
square $|x| + |y| = 1$.

(b) Sketch the region of integration. Change the order of integration and **05**
evaluate the integral $\displaystyle\int_0^1 \int_y^{\sqrt{y}} dx\,dy$.

(c) Find the area enclosed by the cardioid $r = a(1 + \cos\theta)$. **04**

Q. 6

(a) Find the volume of the region that lies under the paraboloid **05**
$z = x^2 + y^2$ and above the triangle enclosed by the lines
$y = x, x = 0$, and $x + y = 2$ in the xy-plane.

(b) Find the area of the region R enclosed by the parabola $y = x^2$ and **05**
the line $y = x + 2$.

(c) Determine whether the vector field $\vec{u} = y^2\vec{i} + 2xy\vec{j} - z^2\vec{k}$ is **04**
solenoidal or irrotational at a point $(1,2,1)$?

Q.7

(a) State Green's theorem and using it ,evaluate $\displaystyle\oint_c xy\,dy - y^2\,dx$, **05**

Where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

(b) Verify stoke's theorem for $\vec{F} = xy^2\vec{i} + y\vec{j} + z^2 x\vec{k}$ for the surface of a **05**
rectangular lamina bounded by $x = 0, y = 0, x = 1, y = 2, z = 0$

(c) Evaluate $\displaystyle\int_c \vec{F}.d\vec{r}$,where $\vec{F} = y^2\vec{i} + 2xy\vec{j}$ and (i) C is the straight line **04**
from (0,0) to (1,2). (ii) C is the parabola $y = 2x^2$ from (0,0) to (1,2)

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