Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER –I • EXAMINATION – WINTER 2012

Subject code: 110008 **Date: 21-01-2013**

Subject Name: Mathematics - I

Time: 10.30 am – 01.30 pm **Total Marks: 70**

Instructions:

1. **Attempt any 5 questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) 1 For what value of α — 3

$$f(x) = x^2 + x, \quad x \le 1$$
$$= 2\alpha x, \quad x > 1$$

Continuous at every x ?

2 Determine absolute extrema of — 3

$f(x) = x^2 + x,\ x \in [-5, 5]$.

(b)1 Write all possible functions whose derivative is $3x^2 + 5$. — 1

2 Show that $g(x) = 9x^2 + 2x - 22$ has at least one zero in the interval $\left(-2, -\frac{4}{3}\right)$. — 4

(c) Discuss Maclauarin's series for $f(x) = \sqrt{x}$ and $g(x) = x$. — 3

Q.2 (a)1 State Fundamental Theorem of Calculus and evaluate — 2

$$\frac{d}{dx}\int_0^{x^2} \cos t\, dt$$

2 State the rule which helpful to evaluate $\int_{1/x}^{x} \frac{1}{t}\, dt$ and then evaluate. — 3

(b)1 Determine volume of a sphere of radius a by the solids of revolution. — 3

2 The region bounded by the curve $y = \sqrt{6x - x^2}$, the x-axis and the line $x = 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid. — 3

(c) Let $f(x) = 1 \quad \text{if } x \text{ is rational number}$ — 3
$\quad\ = \ = \quad \text{if } x \text{ is irrational number}$.

Prove or disprove that it is reimann integrable.

Q.3 (a) 1 Determine the convergence of $\int_1^{\infty} \frac{dx}{x^p}$. — 4

2 Prove that $\int_1^{\infty} \frac{5}{e^x + 3}\, dx$ is convergent. — 3

(b) Expand $f(x) = x^3 - 6x^2 + 14x + 1$ in Taylor's series about $x = 2$. — 3

(c) 1 Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$. — 2

2 Evaluate $\lim_{x \to \infty} x \tan \frac{1}{x}$ — 2

Q.4 (a) 1 State the Integral Test and determine the convergence of $\sum_{n=1}^{\infty} \frac{2\tan^{-1} n}{1 + n^2}$. — 4

2 Determine the convergence of $\sum_{n=1}^{\infty} \frac{1}{5^n - 1}$. — 2

(b) 1 Investigate the convergence of $\sum_{n=1}^{\infty} \frac{2^n}{n!}$. 2

2 Show that 3

$$f(x,y)=\begin{cases}(xy)/(x^2+y^2), & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

Is continuous everywhere except at origin.

(c) Solve the system $u = 2x + y$, $v = x - 2y$ for x and y in terms of u and v. Also find $\frac{\partial(x,y)}{\partial(u,v)}$ 3

Q.5 (a) 1 Find $\frac{dw}{dt}$ at $t = 0$ if $w = xy^2 + z^2$ 3

where $x = \cos t,\ y = \sin t,\ z = t$.

2 Let $u = f(x,y)$ is a homogeneous function of degree n. 4

Then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

(b) Find the directions in which $f(x,y) = \left(x^2/2\right) + \left(y^2/2\right)$ 3

(a) increases most rapidly at the point (1,1)
(b) decreases most rapidly at (1,1)
(c) what are directions of zero change in f at $(1,1)$?

(c) Define Saddle Point. Find the local extreme values of 4
$f(x) = x^2 - y^2 - xy - x + y + 6$ if possible.

Q. 6 (a)1 Sketch the triangle R in the xy plane bounded by the x axis, the line $y = 2x$, and the line $x = 1$. 3

Evaluate $\iint_R \frac{\sin x}{x}\, dA$.

2 Sketch the region of integration and evaluate by reversing the order of integration. 3

$$\int_0^{3/2}\int_3^{6-2x} x\, dy\, dx$$

(b) Sketch the region of integration and change in to polar integral and then evaluate. 4

$$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\, dx$$

(c) Find the volume of tetrahedron whose vertices $(0,0,0),\ (1,0,0), (0,1,0), (0,0,1)$ by triple integral. 4

Q.7 (a) 4

1 Evaluate $\int_C (x^2 + y)\, ds$ where C is the straight line segment $x = 2t$, $y = 1 - t$, $z = 1$ for $0 \le t \le 1$.

3

2 Let $\overline{F} = 2x^2\hat{i} + xy\,\hat{j} + \hat{k}$ is the velocity field of a fluid in space. Find the flow along the curve $t\hat{i} + t\,\hat{j} + \hat{k}$, $0 \le t \le 1$.

(b) 4

Verify Green's theorem for $\overline{F} = x^2\hat{i} + xy\,\hat{j}$, C: The square bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1$.

(c) 3

Find a parametrization of the cylinder $x^2 + (y-2)^2 = 4,\ 0 \le z \le 4$

****