Seat No.: ________ Enrolment No.____________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEM- I / II Winter Examination-Dec.-2011

Subject code: 110008 **Date: 23/12/2011**

Subject Name: Mathematics-I

Time: 10.30 am -1.30 pm **Total marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Verify Rolle's theorem for $f(x) = \dfrac{\sin x}{e^x}$ in $(0, \pi)$. **03**

(ii) Find the maximum and minimum values of $f(x) = 8x^5 - 15x^4 + 10x^2$. **04**

(b) (i) Evaluate $\lim\limits_{x \to 0}\left(\dfrac{1}{\sin x} - \dfrac{1}{x}\right)$. **02**

(ii) Evaluate $\lim\limits_{x \to \frac{\pi}{2}}(\sin x)^{\tan x}$. **02**

(iii) Expand e^x in the powers of $(x - 1)$ up to four terms. **03**

Q.2 **(a)** Discuss the convergence of the following:

(i) $\sum\limits_{n=1}^{\infty}\left[\sqrt{n^2+1} - n\right]$ **02**

(ii) $\dfrac{1}{2\sqrt{1}} + \dfrac{x^2}{3\sqrt{2}} + \dfrac{x^4}{4\sqrt{3}} + \dfrac{x^6}{5\sqrt{4}} + \cdots \text{to } \infty$ **02**

(iii) $\int\limits_{-\infty}^{\infty}\dfrac{1}{1+x^2}dx$ **03**

(b) (i) Find the area of the loop of the curve $ay^2 = x^2(a - x)$. **04**

(ii) Find the entire length of the Cardiod $r = a(1 + \cos\theta)$. **03**

Q.3 **(a)** (i) If $f(x, y) = \dfrac{y - x}{y + x}$ and $f(0,0) = 0$, discuss the continuity of $f(x, y)$ at $(0, 0)$. **03**

(ii) State Euler's theorem on homogeneous functions. **04**

Verify Euler's theorem when $f(x, y) = \dfrac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}}$

(b) Show that $\dfrac{\partial^2 z}{\partial x \partial y} = -[x\log(ex)]^{-1}$ at the point $x = y = z$ for the surface $x^x y^y z^z = c$. **07**

Q.4 **(a)** If $u = f(r, s, t)$ and $r = \dfrac{x}{y}$, $s = \dfrac{y}{z}$, $t = \dfrac{z}{x}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **07**

	(b)	Determine the points where the function $f(x, y) = x^3 + y^3 - 3axy$ has a maxima or minima.	**07**
Q.5	**(a)**	Change to polar coordinates and then evaluate $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dxdy$.	**07**
	(b)	Change the order of integration and then evaluate $\int_0^\infty \int_0^x xe^{\frac{-x^2}{y}} dydx$.	**07**
Q.6	**(a)**	Show by double integration that area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $\frac{16}{3}a^2$.	**07**
	(b)	Find the volume under the plane $x + y + z = 6$ and above the triangle in the xy-plane bounded by $2x = 3y$, $y=0$, $x=3$.	**07**
Q.7	**(a)**	(i) Find the unit vector normal to the surface $x^2y + 2xz = 4$ at the point (2, −2, 4).	**03**
		(ii) Find the directional derivative of $f(x, y, z) = x^2 - y^2 + 2z^2$ at the point (1, 2, 3) in the direction $4\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$. In what direction will it be maximum? Also find the maximum value.	**04**
	(b)	State Green theorem in the plane. Use it to evaluate the integral $\int_c \left[\left(2x^2 + y^2\right)dx + \left(x^2 + y^2\right)dy\right]$ where c boundary of the surface xy-plane enclosed by the x-axis and the semi-circle $y = \sqrt{(1 - x^2)}$.	**07**

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