Seat No.: _____ Enrolment No.______

(EM-8)

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. all Sem-I Examination December 08/January 09

Maths-I (110008)

DATE: 22-12-2008, Monday **TIME: 12.00 to 3.00 p.m.** **MAX. MARKS: 70**

Instructions:

1. ***Attempt all questions.***
2. ***Make suitable assumptions wherever necessary.***
3. ***Figures to the right indicate full marks.***

Q.1(a) (i) Let the function f be defined by **02**

$$f(x) = \begin{cases} |x|/x & ; x > -2,\ x \neq 0 \\ x+1 & ; x < -2 \\ 3 & ; x = -2 \end{cases}$$

Evaluate each of the following limits if it exists.

$$\lim_{x\to -2} f(x),\ \lim_{x\to 0} f(x),\ \lim_{x\to 2/5} f(x).$$

(ii) Explain various types of discontinuities with an example. **02**

(b) (i) Uing L'Hospital rule, evaluate $\lim_{x\to 0} \frac{1}{x}(1 - x\cot x)$. **02**

(ii) State Sandwich theorem on sequences and using it show that, if $x \in$ R with $|x| < 1$, then $x^n \to 0$ as $n \to \infty$. **02**

(c) (i) Using mean value theorem, show that $3 + \frac{1}{28} < \sqrt[3]{28} < 3 + \frac{1}{27}$. **03**

(ii) Prove that $\tan^{-1}\left[\frac{x(3-4x^2)}{\sqrt{1-x^2}\,(1-4x^2)}\right] = 3\left(x + \frac{x^3}{6} + \frac{3x^5}{40} + \ldots\right)$. **03**

Q.2(a) Attempt **any two** of the following. **04**

(i) For $f(x) = x + 2$, $x \in [0,5]$, find $U(f,P)$ and $L(f,P)$ for $P = \{0,1,2,3,4,5\}$.

(ii) State Fundamental theorem of Integral calculus and give one example on which it is not applicable.

(iii) Evaluate the improper integral $\int_0^\infty \frac{1}{x^2}dx.$

(b) Evaluate $\int_3^7 \sqrt[4]{(x-3)(7-x)}\,dx$ using the concept of Gamma and Beta function. **04**

(c) (i) Test the convergence and divergence of the following series **(any two)** . **04**

[A] $\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{1}{n^2+n+1}\right)$ [B] $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots$ [C] $\sum_{n=1}^{\infty} \frac{2n^2+3n}{5+n^5}$.

(ii) Evaluate $\int_0^1\int_0^x (x^2+y^2)\,dA$, where dA indicates small area in xy – plane. **02**

Q.3(a) Suppose that $u = f(x, y, z)$ and $x = g_1(t)$, $y = g_2(t)$, $z = g_3(t)$.
Then write the chain rule for derivative of u w.r.t. t.
Express $\partial w/\partial r$ and $\partial w/\partial s$ in terms of r and s if $w = x + 2y + z^2$,
$x = r/s$, $y = r^2 + \log(s)$, $z = 2r$. **05**

(b) (i) Evaluate $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2}$. **02**

(ii) Find the absolute maximum and minimum values of
$f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$. **03**

(c) Find the length of the arc of the curve $x = a\cos\theta$ and $y = a\sin\theta$; $a > 0$. **04**

OR

Q.3(a) State Euler's theorem on homogeneous function. Using it show that **05**

$$\text{(i) } x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u \quad \text{(ii) } x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u\cos 3u$$

if $u = \tan^{-1}(x^2 + 2y^2)$.

Explain whether we can apply Euler's theorem for the function

$$u = f(x,y) = \frac{x^2+y^2+1}{x+y}.$$

(b) (i) If $u = f(x, y)$ then what geometrically $\partial u/\partial x$ indicates? **01**

(ii) If $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{uv}$ and $u = r\sin\theta\cos\phi$,
$v = r\sin\theta\sin\phi$, $w = r\cos\theta$. Calculate $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. **02**

(iii) The pressure P at any point (x, y, z) in space is $P = 400\,x y z^2$.
Find the highest pressure on the surface of the unit sphere
$x^2 + y^2 + z^2 = 1$. **02**

(c) Find the volume generated by revolving the parabola $y^2 = 4ax$; $a > 0$ about the latus rectum. **04**

Q.4(a) Evaluate $\iint_R (x+y)\,dydx$; R is the region bounded by $x = 0$, $x = 2$,
$y = x$, $y = x + 2$. **05**

(b) Find the volume of the solid that lies under the plane $3x + 2y + z = 12$ and above the rectangle $R = \{(x, y) \mid 0 \le x \le 1, -2 \le y \le 3\}$. **05**

(c) For the vector field $\vec{A} = k\,\vec{i}$ and $\vec{A} = k\,\vec{r}$ .Find $\nabla\bullet\vec{A}$ and $\nabla\times\vec{A}$.
Draw the sketch in each case. **04**

OR

Q.4 (a) Change the order of integration and evaluate $\displaystyle\int_0^{a/\sqrt{2}}\int_x^{\sqrt{a^2-x^2}} y^2\,dA$. **05**

(b) Find the area common to $r = a$ and $r = 2a\cos\theta$; $a > 0$. **05**

(c) Find the scalar potential function f for $\vec{A} = y^2\vec{i} + 2xy\vec{j} - z^2\vec{k}$. **04**

Q.5(a) Evaluate $\oint_C -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$, where $C = C_1 \cup C_2$ with

$C_1 : x^2 + y^2 = 1$ and $C_2 : x = \pm 2, y = \pm 2$. **05**

(b) Evaluate $\iint_S 6xy\,dS$, where S is the portion of the plane $x + y + z = 1$

that lies in front of the yz – plane. **05**

(c) Trace the curve $y = x + (1/x)$. **04**

OR

Q.5 (a) Find the surface area of a sphere of radius a. **05**

(b) Evaluate $\int_C \vec{F} \bullet d\vec{r}$, where $\vec{F}(x,y,z) = -y^2\vec{i} + x\vec{j} + z^2\vec{k}$ and C is the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$. **05**

(c) Trace the curve $9ay^2 = x(x-3a)^2$, $a > 0$. **04**