B.E.

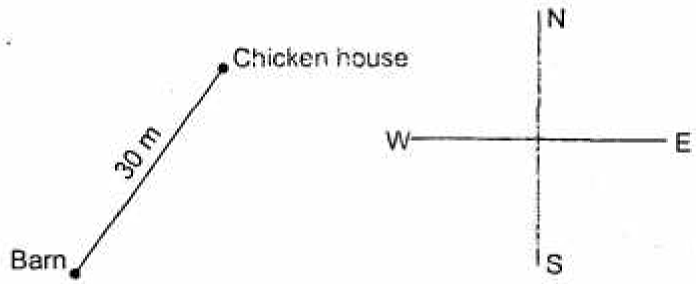
Third Semester Examination, May-2009

Engineering Mechanics (ME-205-E)

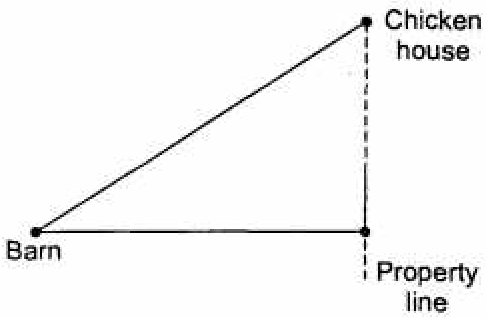
Note : Attempt any five questions. All questions carry equal marks.

Q. 1. A farmer needs to build a fence from the corner of his barn to the corner of his chicken house 30m away in the NE direction. However he wants to enclose as much of the barnyard as possible. Thus he runs the fence east, from the corner of his barn to the property line and then NNE to the corner of his chicken house. How long is the fence?

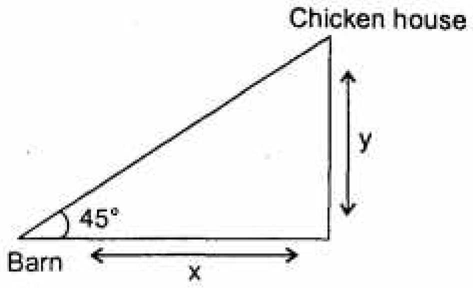
Ans. Length of the fence from the corner of his barn to the corner of his chicken house = 30 m



He runs the fence east, from the corner of his barn to the property line. Let it be x.



And then NNE to the corner to his chicken house



$$\sin 45^\circ = \frac{y}{30}$$

$$y = 30 \times \frac{1}{\sqrt{2}} = 15 \times \sqrt{2}$$

Now x

$$x = 30\cos 45$$
$$= 30 \times \frac{1}{\sqrt{2}}$$
$$= 30 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{30}{2} \times \sqrt{2}$$
$$= 15\sqrt{2}$$

Total fence required

$$= 30 + 15\sqrt{2} + 15\sqrt{2}$$
$$= 30 + 30\sqrt{2}$$
$$= 30(1+\sqrt{2})$$

Hence the length of total fence is $30(1+\sqrt{2})$ **Ans.**

Q. 2. Given the Couple Moments

$$C_1 = 100i + 30j + 82k \text{ lb-ft}$$
$$C_2 = -16i + 42j \text{ lb-ft}$$
$$C_3 = 15kt \text{ lb-ft}$$

What couple will restrain the twisting action of this system about an axis going through

$$r_1 = 6i + 3j + 2k \text{ ft to}$$
$$r_2 = 10i - 2j + 3k \text{ ft}$$

While giving a moment of 100 lb-ft about the x axis and 50 lb-ft about the y axis?

Ans.

$$C_1 = 100i + 30j + 82k$$
$$C_2 = -16i + 42j$$
$$C_3 = 15k$$

Axis is passing through

$$r_1 = 6i + 3j + 2k \text{ ft}$$
$$r_2 = 10i - 2j + 3k \text{ ft}$$

Sum of moment is about the axis

$$100i + 30j + 82k - 16i + 42j + 15k$$
$$= 84i + 72j + 97k$$

Force F component along x is F_x; component along y is F_y

Moment of force along the axis going through r_1 and r_2

$$(F_x i + F_y j + F_z k).(r_1 - r_2) \quad \text{... (i)}$$

$$r_1 - r_2 = -4i + 5j - k$$

Equation (i) reduces to $+4F_x - 5F_y + F_z$ lb-ft

Moment along the x axis $= 100$ lb ft

$$4F_x + 84 = 100$$

$$4F_x = 100 - 84$$

$$4F_x = 16$$

$$F_x = 4 \text{ lb}$$

Moment along y axis $= 50$ lb-ft

$$-5F_y + 72 = 50$$

$$F_y = \frac{50-72}{-5}$$

$$= \frac{+22}{+5} = 4.5$$

$$F_y = 4.5 \text{ lb}$$

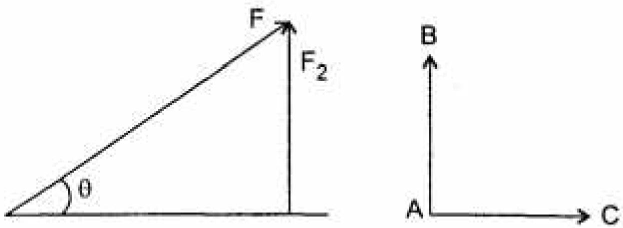
Force will be $= (4i + 4.5j)$ **Ans.**

Q. 3. Explain why equilibrium of a concurrent force system is guaranteed by having $\Sigma(F_y)_i = 0$, $\Sigma(M_d)_i = 0$, and $\Sigma(M_e)_i = 0$. Axes d and e are not parallel to the xz plane. Moreover the axes are oriented so that the line of action of the resultant force cannot intersect both the axes.

Ans. Forces which have zero, linear resultant and zero turning effect well not cause any change in the motion of the object to which they are applied such forces are said to be in equilibrium.

For understanding the equilibrium of an object under two or more concurrent or coplanar forces let us first discuss the resolution of forces and moment of force some point.

Resolution of Forces :



When a force is replaced by an equivalent set of components, it is said to be resolved. One of the most useful ways in which to resolve a force may be resolved in three or more components which are at right angle also. The magnitude of these components can be very easily found using trignometry.

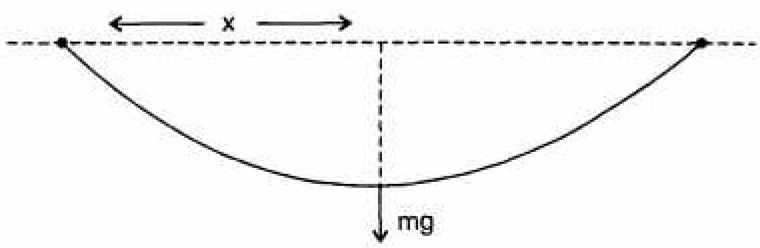
In figure $F_1 = F\cos\theta =$ Component of $\vec{F}$ along AC

$$F_2 = F \sin\theta = \text{Component of } \vec{F} \text{ perpendicular to AC or along AB}$$

Finding such components is referred to as resolving a force in a pair of perpendicular directions. Note that the component of a force in a direction perpendicular to itself is zero. For example, if a force of 10N is applied on the object in horizontal direction then its component along vertical is zero. Similarly the component of a force in a direction parallel to the force is equal to magnitude of the force.

Q. 4. Derive the equation for deflection of a cable when the loading is the weight of the cable itself.

Ans.



Let the mass of cable be m

Then the weight of cable $= mg$

Now $EI\dfrac{d^2y}{dx^2} = \text{Total moment}$

$$EI\frac{d^2y}{dx^2} = -mg \times \frac{x}{2}$$

Where l is the length of cable $EI\dfrac{d^2y}{dx^2} = -mg\dfrac{x}{2}$

$$EI\frac{d^2y}{dx^2} = -mg\frac{x^2}{2} + C_1$$

$$EIy = -mg\frac{x^3}{6} + C_1x + C_2$$

Apply boundary condition $x = 0, \quad y = 0$

$$x = l, \quad y = 0$$

$$0 = -mg.\frac{l^3}{6} + 0 + C_2$$

$$C_2 = \frac{+mgl^3}{6}$$

$$EIy = \frac{-mgx^3}{6} + \frac{mgl^3}{6}$$

Now weight of cable act on the middle of the cable

Therefore $$EIy = -\frac{mg.l^3}{8\times 6} + \frac{mgl^3}{6}$$

$$EIy = \frac{mgl^3}{6}\left[-\frac{1}{8}+1\right] = \frac{mgl^3}{6\times 8}\times 7$$

$\Rightarrow$ $$EIy = \frac{7mgl^3}{56}$$ **Ans.**

Q. 5. Find the I_{yy} for the area between the curves

$$y = 2\sin x \text{ ft}$$

$$y = \sin 2x \text{ ft}$$

from $x = 0$ to $x = \Pi$ ft

Ans. $$y = 2\sin x \text{ ft}$$

$$y = \sin 2x \text{ ft}$$

Limits $x = 0$ to $x = \pi$ ft

$$A = \int_0^\pi [2\sin x - \sin 2x]dx$$

$$= \int_0^\pi 2\sin x dx - \int_0^\pi 2\sin x \cos x\, dx$$

$$= -2[\cos x]_0^\pi - 2\int_0^\pi t\, dt$$

Let $\cos x = t$, $\sin x \cos x = dt$

$$= -2[\cos x]_0^\pi - 2\int_0^\pi t dt$$

$$= -2[\cos x]_0^\pi - 2\frac{t^2}{2}$$

$$= 2[\cos x]_0^\pi - \left[\cos^2 x\right]_0^\pi$$

$$= -2[\cos\pi - \cos 0] - [\cos^2\pi - \cos^2 0]$$

$$= -2[-2]$$

$$= 4 \text{ ft}^2$$ **Ans.**

Q. 6. A particle at a position (3, 4, 6) ft at time $t_0 = 1$ sec is given a constant acceleration having the value $6i + 3j$ ft/sec^2. If the velocity at the time t_0 is $16i + 20j + 5k$ ft/sec what is the velocity of the particle 20 sec later? Also give the position of particle.

Ans. Particle position $(3, 4, 6) = 3\hat{i} + 4\hat{j} + 6\hat{k}$

At time $t_0 = 1$ sec

$$v = 6\hat{i} + 3\hat{j} \text{ ft/sec}^2$$

Velocity at time $t_0 = 16\hat{i} + 20\hat{j} + 5\hat{k}$

Velocity after 20 sec $= v + at$

$$= 16\hat{i} + 20\hat{j} + 5k + 20(6\hat{i} + 3\hat{j})$$

$$= 16\hat{i} + 20\hat{j} + 5\hat{k} + 120\hat{i} + 60\hat{j}$$

$$= 136\hat{i} + 80\hat{j} + 5k$$

Velocity after 20sec $= 136\hat{i} + 80\hat{j} + 5\hat{k}$

For position $h = ut + \frac{1}{2}\text{ft}^2$

$$h = (16\hat{i} + 20\hat{j} + 5\hat{k})20 + \frac{1}{2}(6\hat{i} + 3\hat{j})20^2$$

$$= 320\hat{i} + 400\hat{j} + 100\hat{k} + 200(6\hat{i} + 3\hat{j})$$

$$= 320\hat{i} + 450\hat{j} + 100\hat{k} + 1200\hat{i} + 600\hat{j}$$

$$h = 1520\hat{i} + 1000\hat{j} + 100\hat{k}$$

Position of the particle after 20 sec $= (1520,\ 1000,\ 100)$ **Ans.**

Q. 7. Write short notes on the following :

(a) Hamilton's Principle

(b) Lagrange's equations

Ans. (a) Hamilton's Principle : Hamilton's principle is William Rowan Hamilton's formulation of the principle of stationery action. It states that the dynamics of a physical system is determined by a variational problem for a functional based on a single function, the langarian which contains all the physical information concerning the system and the forces acting on it. The variational problem is equivalent to and allows for the derivation of the differential equations of motion of the physical system. Although formulated originally for classical mechanics, Hamilton's principle also applies to classical fields such as the electromagnetic and gravitational fields, and has even been extended to quantum mechanics, quantum field theory and criticality theories.

Mathematical Formulation : Hamilton's principle states that the true evolution $q(t)$ of a system described by N generalised co-ordinates.

$$q = (q_1, q_2 \ldots\ldots\ldots\ldots, q_n)$$

Between two specified states $q_1 \overset{\text{dif}}{=} q(t_1)$ and

$q_2 \overset{\text{dif}}{=} q(t_2)$ at two specified times t_1 and t_2 is a extremum i.e., a stationery point, a minimum, maximum or saddle point of the action functional

$$S|q| \stackrel{\text{dif}}{=} \int_{t_1}^{t_2} L(q(t),\, q(t),\, t)dt$$

Where $L(q,\, q,\, t)$ is the langarian function for the system. In other words, any first order perturbation of the true evolution results in (at most) second order changes in S. It should be noted that the action S is a functional, i.e., someting that takes as it input a function and returns a single number, a scalar. In terms of functional analysis. Hamilton's principle states that the true evolution of a physical system is the solution of the function equation

$$\frac{\delta S}{\delta q(t)} = 0$$

(b) Lagrange's Equations : Lagrange's equation is a differential equation whose solutions are functions for which a given functional is stationery. It was developed by Swiss mathematician Leonhard Euler and Italo-French mathematician Joseph Louis Langrange in the 1750's.

Because a differentiable function is stationery at its local maxima and minima, the Euler-Lagrange's equation is useful for solving optimization problems in which, given some functional, one seeks the function minimizing (or maximizing) it. This is analogous to Fermat's theorem in calculus stating that where a differential function attains its local extrema, its derivative is zero.

The Euler-Lagrange's equation satisfied by a function q of a real argument t which is a stationery point of the function

$$S(q) = \int_a^b L(t,\, q(t),\, q'(t))dt$$

Where, q is the function to be found

$$q : [a,\, b] \subset R \to X$$

$$t \to x = q(t)$$

Such that q is differentiable, $q(a) = X$

& $q(b) = X_b$;

q' is the derivative of q : $q' : [a,\, b] \to T_{q(t)}X$

$$t \to v = q(t)$$

TX being the tangent bundle of X (the space of possible values of derivatives of functions with values in X).

L is a real valued function with continues first partial derivatives :

$$L : [a,\, b] \times X \times TX \to R$$

$$(t,\, x,\, v) \to L(t,\, x,\, v)$$

The Euler-Lagrange equation then is the ordinary differential equation.

$$L_x(t,\, q(t),\, q'(t)) - \frac{d}{dt} L_v(t,\, q(t),\, q'(t)) = 0$$

Where L_x and L_v denotes the partial derivatives of L with respect to the second and third arguments respectively.

If the dimension of the space x is greater than 1, this is a system of differential equations, one for each component.

$$\frac{\partial L(t, q(t), q'(t))}{\partial x_i} - \frac{d}{dt}\frac{\partial L(t, q(t), q'(t))}{\partial v_i} = 0$$

For $i = 1, 2, \ldots\ldots, n$

Q. 8. A high speed land racer is moving at a speed of 100m/sec. The resistance to motion is primarily due to the aerodynamic drag, which for this speed can be approximated as $0.2V^2$N with V in m/sec. If the vehicle has a mass of 4000 kg what distance will the vehicle coast before it speed is reduced to 70 m/sec?

Ans. Speed of land racer $= 100$ m/s

Resistance applied $= 0.2\,v^2$ N

Thus, the total resistance applied $= 0.2 \times (100)^2$

$= 2000$ N

Now $F = m \times$ acceleration

$$\Rightarrow \quad 2000 = 4000 \times \frac{(v_1 - v_2)}{t}$$

$$= 4000 \times \frac{(70-100)}{t}$$

$$\Rightarrow \quad t = 60 \text{ sec}$$

Now $S = ut + \frac{1}{2}at^2$

$$= 100 \times 60 - \frac{1}{2} \times \frac{30}{60} \times (60)^2$$

$$S = 5100 \text{ m}$$