B.E.

Third Semester Examination, December-2008

Engineering Mechanics (ME-205-E)

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. The crew of a submarine patrol plane with three dimensional radar sights a surfaced submarine 10000 yards north and 5000 yards east while flying at an elevation of 3000 ft above sea level. Where should the pilot instruct a second plane flying at an elevation of 4000 ft at a position 40000 yards east of the first plane to confirm the sighting ?

Ans. Given : Diameter of grindstone $= D = 90$ cm.

$$R = \frac{90}{2} = 45\,\text{cm.}$$

Thickness of grindstone $= t = 10\,\text{cm.}$

Mass per unit volume $= m = 0.0026\,\text{kg/cm}^3$

$$M = m \times \text{volume of grindstone}$$

$$= m \times \pi R^2 \times t$$

$$= 0.0026 \times \pi \times 45^2 \times 10$$

$$= 165.4\ \text{kg.}$$

Moment of inertia (I_{zz}) of grindstone, about the axis of rotation is given by equation as :

$$I_{zz} = \frac{MR^2}{2}$$

$$= \frac{165.4 \times 45^2}{2}$$

$$= 167467.5\,\text{kg/cm}^2$$

Radius of gyration (k) is given by : $I_{zz} = \frac{MR^2}{2} = \frac{165.4 \times 45^2}{2}$

$$= 167467.5\,\text{kg/cm}^2$$

$$K = \frac{R}{\sqrt{2}} = \frac{45}{\sqrt{2}} = 31.819\text{cm}$$

Q. 2. Given the Couple Moments

$C_1 = 100i + 30j + 82k$ **lb-ft**

$C_2 = -16i + 42j$ **lb-ft**

$C_3 = 15k\ lb - ft$

What couple will restrain the twisting action of this system about an axis going through $r_1 = 6i + 3j + 2k$ **ft to**

$r_2 = 10i - 2j + 3k$ ft

while giving a moment of 100-lb-ft about the x-axis and 50 lb-ft about the y-axis ?

Ans. (i) For a Circular Lamina :

$$D = 60\,\text{cm}$$

$$R = \frac{D}{2}$$

$$= \frac{60}{2} = 30\,\text{cm}$$

$$m = 0.001\,\text{kg/cm}^2$$

$$M = m \times \text{area of circular lamina}$$

$$= 0.001 \times \pi R^2 = 0.001 \times \pi \times 30^2$$

$$= 2.827\,\text{kg}$$

$$I_{zz} = \frac{MR^2}{2} = \frac{2.827}{2} \times 30^2 = 1272.15\,\text{kg/cm}^2$$

Radius of gyration of circular section

$$K = \frac{R}{\sqrt{2}} = \frac{30}{\sqrt{2}} = 21.21\,\text{cm.}$$

(ii) For a Circular Cylinder :

$$D = 80\,\text{cm}$$

$$R = \frac{D}{2} = \frac{80}{2} = 40\text{ cm}$$

$$h = 15\,\text{cm}$$

$$m = 0.002\,\text{kg/cm}^2$$

$$M = m \times \pi R^2 \times h$$

$$= 0.002 \times \pi \times 40^2 \times 15$$

$$= 150.796\,\text{kg}$$

$$I_{zz} = \frac{MR^2}{2}$$

$$= \frac{150.796 \times 40^2}{2} = 120636.8\ \text{kg/cm}^2$$

$$K = \frac{R}{\sqrt{2}} = \frac{40}{\sqrt{2}}$$

$$K = 28.28\,\text{cm.}$$

(iii) For a Solid Sphere : $D = 40\,\text{cm}$

$$R = \frac{D}{2} = \frac{40}{2} = 20\,\text{cm}$$

$$m = 0.0015 \text{ kg/cm}^3$$

$$M = m \times \frac{4\pi R^3}{3}$$

$$= \frac{0.0015 \times 4 \times \pi \times 20^3}{3} \text{ kg.}$$

$$= 50.265 \text{ kg}$$

$$I = \frac{2}{5} MR^2$$

$$= \frac{2}{5} \times 50.265 \times 20^2 \text{ kg/cm}^2$$

$$= 8042.4 \text{ kg/cm}^2$$

$$K = 0.6324 \text{ R}$$

$$K = 0.6324 \times 20$$

$$K = 12.648 \text{ cm}$$

Q. 3. Find the length of a cable stretched between two supports at the same elevation with span length l = 200 ft and sag h = 50 ft, if it is subjected to a vertical load of 4 lb/ft uniformly distributed in the horizontal direction. (Assume that the weight of the cable is either negligible or included in the 4 lb/ft distribution). Find the maximum tension.

Ans. Given : $d_1 = 60 \text{ cm}$

$\therefore$ $r_1 = 30 \text{ cm}$

$d_2 = 24 \text{ cm}$

$\therefore$ $r_2 = 12 \text{ cm}$

$x = 3m = 300 \text{ cm}$

$P_t = 3.75 \text{ kW}$

$N_2 = 300 \text{ rpm}$

$\mu = 0.3$

Safe working tension $= 100 \text{ N/cm width.}$

$b = \text{width of belt in cm}$

$$T_{max} = 100 \times b$$

$$= 100 \text{bN}$$

$$\theta = 150 - 2\alpha$$

$$\sin \alpha = \frac{r_1 - r_2}{x} = \frac{30 - 12}{300} = 0.06$$

$$\alpha = \sin^{-1}(0.06)$$

$$= 3.45^\circ$$

$$\alpha = \sin^{-1}(0.006)$$

$$= 3.45^\circ$$

$$\theta = 180 - 2 \times 3.45^\circ$$

$$= 173.1^\circ$$

$$= 173.1^\circ \times \frac{\pi}{180} \text{ rad.}$$

$$= 3.02 \text{ radians}$$

$$\frac{T_1}{T_2} = e^{\mu \times \nu}$$

$$= e^{0.3 \times 3.02} = e^{0.906} = 2.474$$

$$T_1 = 2.474\, T_2$$

$$P = \frac{(T_1 - T_2) \times v}{1000}$$

$$3 \cdot 75 = \frac{(T_1 - T_2) v}{1000}$$

$$= \frac{\pi d_2 N_2}{60}$$

$$= \frac{\pi \times 24 \times 300}{60}$$

$$= 376 \cdot 9 \text{ cm/sec.}$$

$$= 3.77 \text{ m/sec.}$$

$$3 \cdot 75 = \frac{(T_1 - T_2) \times 3.77}{1000}$$

$$(T_1 - T_2) = \frac{3.75 \times 1000}{3.77}$$

$$= 994.7 \text{ N}$$

$$2.474\, T_2 - T_2 = 994.7$$

$$1.474\, T_2 = 994.7$$

$$T_2 = \frac{994.7}{1.474}$$

$$T_2 = 674.8 \text{ N}$$

$$T_1 = 2.474 \times T_2$$

$$T_1 = 2.474 \times 674.8$$

$$T_1 = 1669.5 \text{ N}$$

$$\boxed{T_{max} = T_1}$$

$$100 \times b = 1669.5$$

$$b = \frac{1669.5}{100}$$

$$\boxed{b = 16.7\,\text{cm}}$$

Q. 4. (a) Derive the relation between second moments of area and the products of area.

Ans. Given, Weight of ladder $w = 850\,\text{N}$

Length of ladder $L = AB = 6\,\text{m}$

Angle made by ladder with horizontal,

$$\alpha = 65^\circ$$

$$W_1 = 750 N$$

$$L_1 = 4\,\text{m}$$

$$L_2 = L - L_1 = 6 - 4 = 2\,\text{m}$$

Let μ = co-efficient of friction between the ladder and floor. Verticle wall is smooth and hence there will be no force of friction between the ladder and verticle wall.

Let AB is the ladder and G is the middle point of the ladder at which the weight 850 N is acting. The man of weight 750 N is standing at ε. At this position, the ladder is at the point of sliding. This means that ladder at A will be start moving towards right.

Hence, a force of friction $F_A = \mu R_A$ will be acting towards left.

R_A = Normal reaction at A

R_B = Normal reaction at B

$$R_A = 850 + 750$$

$$= 1600\,\text{N}$$

$$R_B = F_A = \mu R_A = \mu \times 1600$$

$$= 1600\mu\text{N}$$

$$B_C = AB \sin 65^\circ$$

$$= 6 \cos 65^\circ$$

$$= 6 \times 0.4220$$

$$= 2.5357\,\text{cm}$$

$$A_D = \frac{A_C}{2}$$

$$= \frac{2.5357}{2} = 1.267\,\text{m}$$

$$A_H = A\varepsilon \cos 65^\circ$$

$$= (AB - B\varepsilon)\cos 65^\circ$$

$$R_B \times R_C = 850 \times A_D + 750 \times A_H$$

$$1600\mu \times 5.437 = 850 \times 1.267 + 750 \times 0.8452$$

$$\mu = \frac{1710.85}{8699.24}$$

$$\boxed{\mu = 0.199}$$

Q. 4. (b) Define the following :

(i) Principal Axes **(ii) Moment of Inertia**

Ans. The product of mass & velocity of a body is known as momentum of the body.

The product of mass moment of inertia & angular velocity of a rotating body is knwon as moment of momentum or angular momentum.

Let d_m = Mass of elementary mass

r = Radius of mass 'd_m'

ω = Angular velocity of body

v = Linear velocity of mass 'd_m' $= \omega \times r$

$= d_m \times v$

$= d_m \times \omega_r$

Moment of momentum of elementary mass 'd_m' about 0.

$= \text{Momentum} \times \text{radius}$

$= (d_m \times \omega_r) \times r$

$= d_m \times \omega_r^2$

$\int d_m \times \omega_r^2 d_m$

But moment of momentum is also known as angular momentum.

Q. 5. Explain why equilibrium of a concurrent force system is guaranteed by having $\Sigma(F_y)_i = 0$, $\Sigma(M_d)_i = 0$, and $\Sigma(M_e)_i = 0$. Axes d and e are not parallel to the xz plane. Moreover the axes are oriented so that the line of action of the resultant force cannot intersect both the axes.

Ans. Force at $C = 4000\text{ N}$

$B = 2500\text{ N}$

Moment at $D = 2000\text{ Nm}$

Distance $AC = 1\text{ m}$

$BC = 1.5\text{ m}$

$CD = 0.8\text{ m}$

$BD = 0.7\text{ m}$

Resultant of the System : This means to find the resultant of all the forces & also the point at which the resultant is acting.

There are 2 verticle forces only.

Hence resultant $R = 4000 - 2500$

$= 1500\text{ N}$ acting downward

The point at which the resultant is acting is obrained by taking moments about point A

$$A = 4000 \times 1$$
$$= 4000\,\text{Nm}$$

Moment at $D = 2000\,\text{Nm}$

Moment at resultant force (R) about A

$$= 1500 \times x$$
$$1500 \times x = 250$$
$$x = \frac{250}{1500} = 0.166\text{ m}$$

Q. 6. A high speed land racer is moving at a speed of 100 m/sec. The resistance to motion is primarily due to the aerodynamic drag, which for this speed can be approximated as 0.2 V^2 N with V in m/sec. If the vehicle has a mass of 4000 kg what distance will the vehicle coast before its speed is reduced to 70 m/sec ?

Ans. Given,

Initial Velocity of body $u = 0$

$$g = 980\,\text{m/s}^2$$
$$h = ut + \frac{1}{2}gt^2$$
$$= 0 + \frac{1}{2}gt^2$$
$$h_1 = \frac{1}{2}g(t-1)^2$$
$$h_2 = \frac{1}{2}g(t-2)^2$$
$$h - h_1 = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$$
$$= \frac{1}{2}g[t^2 - (t-1)^2] = \frac{1}{2}g[t^2 - (t^2 + 1 - 2t)]$$
$$= \frac{1}{2}g(2t-1)$$

Distance covered in the last but one second,

$$= h_1 - h_2$$
$$= \frac{1}{2}g(t-1)^2 - \frac{1}{2}g(t-2)^2$$
$$= \frac{1}{2}g[(t-1)^2 - (t-2)^2]$$
$$= \frac{1}{2}g[t^2 + 1 - 2t - (t^2 + 4 - 4t)]$$
$$= \frac{1}{2}g(2t-3)$$

$$\frac{\frac{1}{2}g(2t-1)}{\frac{1}{2}g(2t-3)} = \frac{4}{3}$$

$$\frac{(2t-1)}{(2t-3)} = \frac{4}{3}$$

$$t = 4.5\,\text{sec}$$

$$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 4.5^2\ \text{m}$$

$$h = 99.225\,\text{m}$$

Q. 7. A particle moves with a constant speed of 1.5 m/sec along a path given by $x = y^2 - \ln y$. Give the acceleration vector of the particle in terms of rectangular components when the particle is at the position $y = 3$ m. How many g's of acceleration is the particle subjected to ?

Ans. Given, Height of tower = 100 m

Initial velocity of Ist particle,

$$\mu_1 = 0$$

Height from the foot of the tower at which the 2 particles meet = 30 m

$$S_1 = 100 - 30$$

$$= 70\,\text{m}$$

$$t = \text{time}$$

$$\mu_2 = \text{initial velocity}$$

$$s = ut + \frac{1}{2}gt^2$$

$$s = 0 \times t + \frac{1}{2} \times 9{\cdot}80t^2$$

$$t = \sqrt{\frac{70}{4.9}}$$

$$\boxed{t = 3.78\,\text{sec}}$$

Case of 2nd particle $\quad s = ut - \frac{1}{2}gt^2$

$$s_2 = ut - \frac{1}{2}9.80t^2$$

$$30 = \mu \times 3.98 - 4.90 \times 3.78^2$$

$$= 3.78\mu - 70$$

$$\mu = \frac{100}{3.78}$$

$$\boxed{\mu = 26.45\,\text{m/sec}}$$