B. E.

Third Semester Examination, May-2008

Engineering Mechanics (ME-205-E)

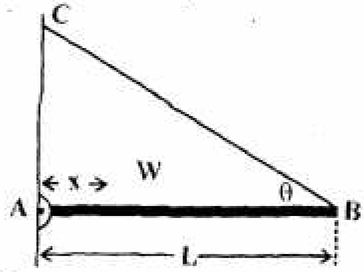
Note : Attempt any ***FIVE*** questions.

Q. 1. (a) A bar AB of negligible weight is pinned at A and supported at B by a thin wire BC. A weight W can move along the bar. Find :

(i) Tension in the wire,

(ii) Horizontal reaction force and

(iii) Vertical reaction force on the bar by the pin at A?

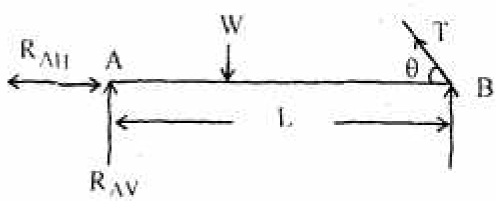


Ans. Let T is tension in wire BC.

Taking moment about A,

$$T\sin\theta \times L = W.x$$

$$\Rightarrow \quad T = \frac{W.x}{L\sin\theta}$$



$$\because \quad \Sigma F_V = 0$$

$$T\sin\theta - W + R_{AY} = 0$$

$$T\sin\theta - W + R_{AY} = 0$$

$$\Rightarrow \quad R_{AV} = W - T\sin\theta$$

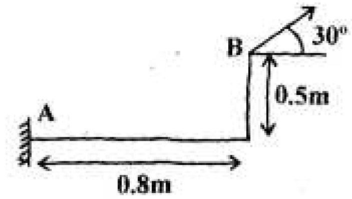
$$= W - \frac{Wx}{L} = W\left[1 - \frac{x}{L}\right]$$

$$\Sigma F_H = 0$$

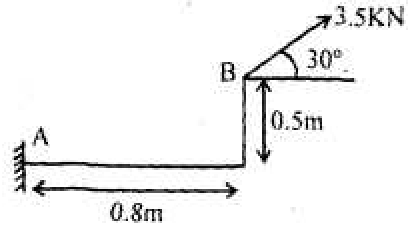
$$R_{AH} = T\cos\theta$$

$$= \frac{Wx}{L} \cot\theta$$

Q. 1. (b) Find the moment of 3.5 kN force about A as shown in fig.



Ans. Moment about A,

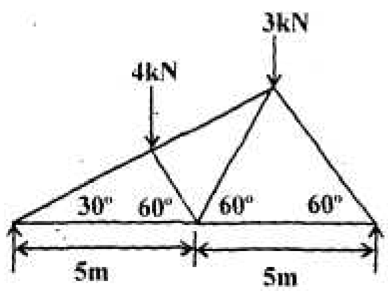


$= 3.5\cos 30 \times 0.5$

$-3.5\sin 30 \times 0.8$

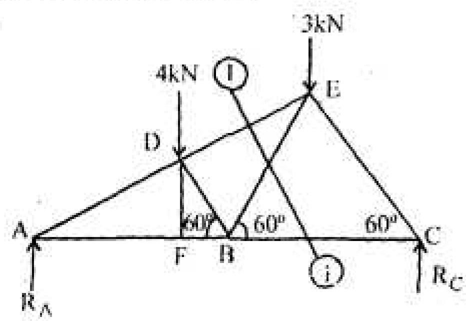
$= 0.1155$ kNM (clockwise)

Q. 2. A truss of 10m span is loaded as shown in fig. Find the forces in the members of the truss by the method of sections.



Ans. Taking moment A,

$$R_c \times = 10 = 3 \times 7.5 + 4 \times AF$$



Now, $AF = AB - FB = 5 - DB\cos 60 = 5 - 5\cos^2 60° = 3.75\ m$

$\Rightarrow$ $R_C = 3.75$kN

$\therefore$ $R_A = 4 + 3 - 3.75 = 3.25$ KN

Taking section (1)-(1) as shown

$\because$ $\Sigma M_E = 0$

$$R_C \times 2.5 - F_{BC} \times 4.33 = 0$$

$$\Rightarrow \quad F_{BC} = 2.16\text{kN (Tensile)}$$

$$\Sigma F_V = 0$$

$$R_C - 3 + F_{BE}\cos 30 - F_{DE}\cos 60 = 0 \quad \text{...(i)}$$

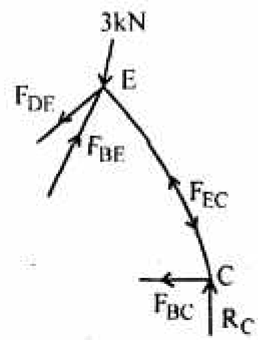
$$\Sigma F_H = 0$$

$$F_{BC} + F_{DE}\sin 60 - F_{BE}\sin 30 = 0 \quad \text{...(ii)}$$

Solving equations (i) & (ii) we get,

$$F_{BE} = 1.73\text{KN (Compressive)}$$

$$F_{DE} = -1.5\text{KN (Tensile)} = 1.5\text{KN (Compressive)}$$



Taking joint C, we get $\quad F_{EC} = \dfrac{R_C}{\sin 60} = 4.33\text{KN (Compressive)}$

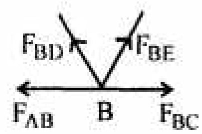
Taking joint A

$$F_{AD}\sin 30 = R_A$$

$$\Rightarrow \quad F_{AD} = 6.5\text{KN (Compressive}$$

$$F_{AB} = F_{AD}\cos 30 = 5.63\text{KN (Ttensile)}$$

Taking joint B



$$F_{BD}\cos 30 = F_{BE}\cos 30$$

$$\Rightarrow \quad F_{BD} = F_{BE} = 1.73\text{KN (Tensile)}$$

Q. 3. (a) From a uniform circular disc of radius 'r' is cut off a circular portion of radius 'r/2'. Find the distance between the centers of the disc and hole if the center of the mass of the remainder is on the circumference of the hole.

Ans. Let O_1 & O_2 are centres of circle of radius r and r/2 respectively.

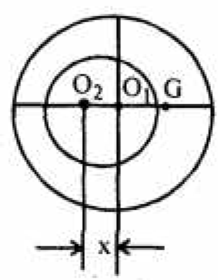
Let $O_1O_2 = x$

'G' is the centre of max. such that

$$O_1G = \left(\frac{r}{2} - x\right) = \frac{A_1x_1 + A_2x_2}{A_1 - A_2}$$

$$\therefore \quad \frac{r}{2} - x = \frac{\frac{\pi}{4}r^2.x}{\pi r^2 - \frac{\pi r^2}{4}}$$

$$\Rightarrow \quad \frac{\pi}{4}r^2x = \frac{\pi r^3}{2} - \pi r^2 x - \frac{\pi r^3}{8} - \frac{\pi r^3}{8} + \frac{2r^2}{4}x$$

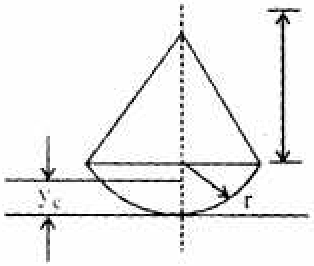


$$\Rightarrow \qquad \pi r^2 x = \frac{3ar^3}{8}$$

$$\Rightarrow \qquad x = \frac{3r}{8}$$

Q. 3. (b) A body consists of a right circular solid cone of height 20 cm and radius 15 cm placed on a solid hemisphere of radius 15 cm of the same material. Find the position of center of gravity.

Ans.
$$y_c = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2}$$



$$V_1 = \frac{2}{3}\pi r^3, \quad V_2 = \frac{\pi}{3} hr^2$$

$$y_1 = r - \frac{3r}{8} = \frac{5r}{8}, \quad y_2 = r + \frac{h}{4}$$

$$\therefore \qquad y_c = \frac{\left(\frac{2\pi}{3} r^3\right)\left(\frac{5r}{8}\right) + \left(\frac{\pi}{3} r^2 h\right)\left(r + \frac{h}{4}\right)}{\frac{2\pi}{3} r^3 + \frac{\pi}{3} r^2 h}$$

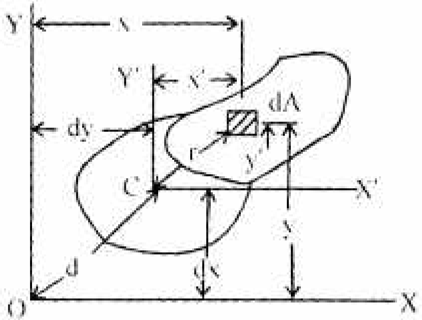
Putting r = 15 cm and h = 20 cm as given, we get

$$y_c = 13.625\,\text{cm}.$$

Q. 4. (a) State and prove the theorems of parallel axis and perpendicular axis.

Ans. Parallel Axis Theorem : It states that moment of Inertia of an area with respect to any axis in its plane is equal to moment of inertia of the area with respect to a parallel centroidal axis plus the product of area and square of distance between the two axes.

Proof : Let x, y be the rectangular coordinate axes through any point O in plane of figure.



Q. 4. (b) Derive an expression for moment of inertia of a quadrant of a circular plate of radius R.

Ans. Consider elemental area dA situated at radius r and angle θ.

$$dA = r\,d\theta\,dr$$

Centroid of dA lies at distance $r\sin\theta$ from x-axis.

Moment of inertia about x-axis.

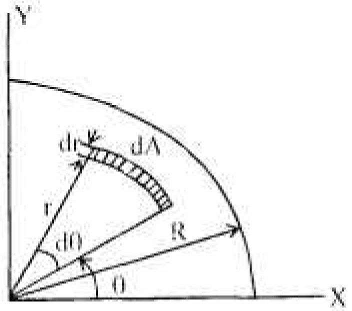
$$dI_x = ((rd\theta)dr)(r\sin\theta)^2$$

$$I_x = \int_{r=0}^{r=R}\int_{\theta=0}^{\theta=\pi/2} r^3 \sin^2\theta\, d\theta\, dr$$

$$= \int_0^R r^3 dr \int_0^{\pi/2}\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$= \int_0^R r^3 dr \times \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= \int_0^R \frac{\pi}{4} r^3 dr = \frac{\pi}{4}\cdot\frac{R^4}{4} = \frac{\pi R^4}{16}$$



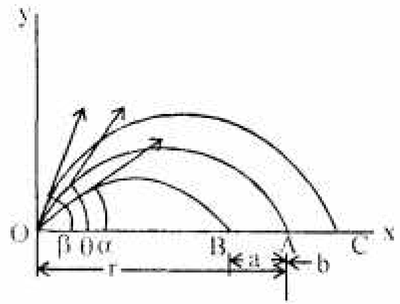
Q. 5. A projectile is aimed at a mark on the horizontal plane through the point of projection and falls 12m short when the angle of projection is 15°, while it overshoots the mark by 24 m when the same angle is 45°. Find the angle of projection to hit the mark. Assume no air resistance. Derive the expressions used.

Ans. Let v be the velocity of projection in each case and θ be the correct angle of projection for desired range r. Let a = 12m when $\alpha = 15°$ and b = 24m when $\beta = 45°$. Then

$$r - a = \frac{v^2 \sin 2\alpha}{g} \quad ...(i)$$

$$r + b = \frac{v^2 \sin 2\beta}{g} \quad ...(ii)$$

$$r = \frac{v^2 \sin 2\theta}{g} \quad ...(iii)$$



of area A as shown. x^1, y^1 be corresponding parallel axes through centroid of area is also called centroidal axes. The moment of inertia of area A about x-axis.

$$I_X = \int y^2 dA$$

Where dA is elemental area at distance y from x-axis.

$$y = dx + y'$$

$$I_x = \int (y' + dx)^2 dA$$

$$= \int (y'^2 + dx^2 + 2y'dx)dA$$

$$= \int y'^2 dA + Adx^2 + 0$$

$$= I_x' + Adx^2 \quad \ldots(i)$$

Similarly $$I_y = Iy' + Ady^2 \quad \ldots(ii)$$

Perpendicular Axes Theorem : It states that polar moment of Inertia i.e. moment of inertia about axis perpendicular to plane passing centroid of figure is equal to sum of moment of inertia about x and y axes parallel to plane passing centroid of figure.

Proof : Polar moment of inertia, $J_0 = \int r^2 dA$

But $$r^2 = x'^2 + y'^2$$

$\Rightarrow$ $$I_0 = \int (x'^2 + y'^2) dA = I_x' + I_y'$$

Subtract (i) from (ii)

$$a + b = \frac{v^2(\sin^2\beta - \sin^2\alpha)}{g} \quad \ldots(iv)$$

Subtract (iii) from (ii) $$b = \frac{v^2(\sin 2\beta - \sin 2\theta)}{g} \quad \ldots(v)$$

From (iv) & (v) $$\frac{v^2}{g} = \frac{a + b}{(\sin 2\beta - \sin 2\alpha)}$$

$$\frac{v^2}{g} = \frac{b}{(\sin 2\beta - \sin 2\theta)}$$

Equating, $$\frac{a + b}{\sin 2\beta - \sin 2\alpha} = \frac{b}{\sin 2\beta - \sin 2\theta}$$

Solving

$\Rightarrow$ $$\theta = \frac{1}{2}\sin^{-1}\left[\frac{a\sin 2\beta + b\sin 2\alpha}{a + b}\right]$$

Putting the values

$$\theta = 20.9^\circ$$

Q. 6. (a) State and explain the D'Alembert's principle for the motion of rotation.

Ans. D'Alembert's principle can be states as: "when internal or external turning moments act on a system having rotation motion, the algebraic jump of all torques acting on the system including the inertia torque, taken in opposite order of angular acceleration is zero."

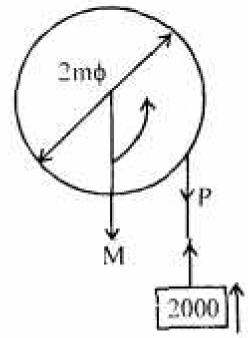
Thus, as per D'Alemberts principle of rotation

$$T - I\alpha = 0$$

Q. 6. (b) A solid cylinder pulley of mass 800 kg, having 0.8 m radius of gyration and 2m diameter, is rotated by an electric motor, which exerts a uniform torque of 60 kN.m. A body of mass 2000 kg is to be lifted by a wire wrapped round the pulley. Find :

(i) Acceleration of the body and
(ii) Tension in the rope.

Ans. Moment of Inertia of Pulley,



$$I_G = MK^2$$

$$= 800 \times (0.8)^2 = 512\ \text{kgm}^3$$

Also, acceleration $\quad a = r\alpha = 1 \times \alpha = \alpha$

Torque $\quad T_1 = I_G\alpha = I_G.a = 512a$

Torque due to tension $\quad T_2 = P \times r = P$

Thus, total torque $\quad = 512a + P = 60000$

$\Rightarrow \quad P = 60000 - 512a$

Equation of motion

$$P - mg = ma$$

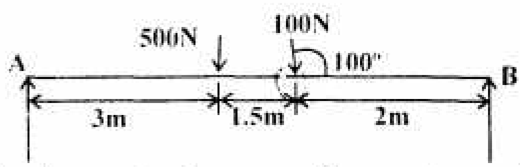
$\Rightarrow \quad 60000 - 512a - 2000 \times 9.8 = 2000 \times a$

$\Rightarrow \quad a = 16.08\ m/s^2$

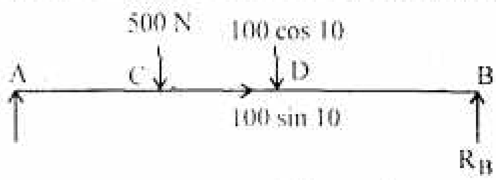
Tension is given by,

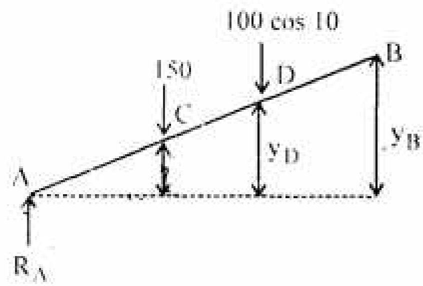
$$P = 2000 \times (9.8 + 16.08) = 50KN$$

Q. 7. Explain the principle of Virtual Work. Solve the beam for reactions at supports show in fig., by virtual work method.



Ans. Principle of virtual work states that if a system of forces acting on a body or a system of bodies be in equilibrium and it the system be imagined to undergo a small displacement consistent with geometrical conditions then algebraic sum of virtual work done by the forces of system is zero.





$$\frac{y_B}{y_C} = \frac{AB}{AC} = \frac{6.5}{3}$$

$$\Rightarrow \qquad y_B = 2.16\, y_C$$

$$\frac{y_D}{y_C} = \frac{AD}{AC} = \frac{4.5}{3}$$

$$\Rightarrow \qquad y_D = 1.5\, y_C$$

As all virtual work principle,

$$-500 \times y_C - 100\cos 10 \times y_D + R_B \times y_B = 0$$

$$\Rightarrow \qquad R_B = \frac{500 + 100\cos 10 \times 1.5}{2.16}$$

$$= 299.8\text{N}$$

$$R_A = 500 + 100\cos 10 - 299.8$$

$$= 298.6\text{N}.$$

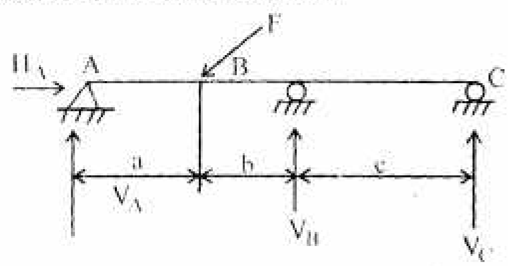
Q. 8. Discuss the following with suitable examples :

(a) Static indeterminacy,

(b) Hamilton principle,

(c) Coriolis forces.

Ans. (a) Static Indeterminancy : A system can be statically indeterminate even though its reactions are determinate. A structure is statically indeterminate when the static equilibrium equations are not sufficient for determining the internal forces and reactions on that structure.



For example, the equilibrium equations for the force system as shown are,

$$V_A - F_V + V_B + V_C = 0$$

$$H_A - F_h = 0$$

$$F_V.a - V_B(a+b) - V_c(a+b+c) = 0$$

There are four unknowns and three equations. Thus, system is said to static indeterminate.

(b) Hamilton Principle : It states that the true evolution $q(t)$ of a system described by N generalised coordinates $q = (q_1, q_2, q_3, \ldots\ldots q_n)$ between two specified states $q_1 = q(t_1)$ and $q_2 = q(t_2)$ at two specified times t_1 and t_2 is an extremum of the action functional,

$$S[q(t)] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\,dt$$

(e) Coriolis Forces : The Coriolis effect is an apparent deflection of moving objects when they are viewed from rotating frame of reference. The Coriolis force is an example of fictitious force because it does not appear when the motion is expressed in an inertial frame of reference. in which the motion of an object is explained by real impressed forces together with inertia. In rotating frame the Coriofis force which depends on the velocity of moving object and centrifugal force which doesn't demand on the velocity of moving object are needed in the equation to correctly describe the motion. The mathematical equation for Coriolis force is,

$$F_c = -2mw \times x$$

Where w = angular velocity

and v = velocity of particle.