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BT-7/D07

STATISTICAL MODELS FOR COMPUTER SCIENCE

Paper–CSE-405

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *five* questions, selecting at least *one* question from each Unit.

UNIT–I

1. (a) If A_1 and A_2 are any two events, prove that

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2).$$

Extend this result to *n* events by using mathematical induction. 10

(b) In a party of five persons, compute the probability that

(i) Two persons have the same birthday (month/day).

(ii) At least two persons have the same birthday (month/day). 10

2. (a) Derive the product law of reliabilities for series systems and product law of unreliabilities for parallel systems. 10

(b) Consider four computer firms A, B, C, D bidding for a certain contract. A survey of past bidding success of these firms on similar contracts shows the following probabilities of winning :

$$P(A) = 0.35, P(B) = 0.15, P(C) = 0.3, P(D) = 0.2.$$

Before the decision is made to award the contract, firm B withdraws its bid. Find the new probabilities of winning for A, C & D. 10

UNIT–II

3. (a) Suppose we are observing the arrival of number of jobs at a computer centre in the interval (0, t). Let the arrival rate be λ which depends upon user population. Show that the probability of K jobs arriving in this interval is given by Poisson PMF

$$e^{-\lambda t}\frac{(\lambda t)^K}{K!}, \quad K = 0, 1, 2, ...\infty \qquad 10$$

(b) Two discrete random variables X and Y have joint PMF given by the following table :

X \ Y	1	2	3
1	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
2	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$
3	$\frac{1}{12}$	$\frac{1}{12}$	0

Compute the probability of each of the following events :

(i) $X \le 1\frac{1}{2}$; (ii) X is odd; (iii) XY is even;

(iv) Y is odd given that X is odd. 10

4. (a) Find Moment Generating Function (MGF) or Z-transform of Geometric distribution and subsequently find its mean and variance using the MGF. 12

(b) The phase X of a sine wave is uniformly distributed over $(-\pi/2, \pi/2)$; that is,

$$f_X(x) = \begin{cases} 1/\pi & -\pi/2 < x < \pi/2 \\ 0 & \text{otherwise} \end{cases}$$

Let Y = sin X and show that

$$f_Y(y) = \frac{1}{\pi} \frac{1}{\sqrt{1-y^2}} \quad -1 < y < 1 .$$ 8

UNIT–III

5. (a) Given that $n \geq 1$ arrivals have occurred in the interval $(0, t)$, the conditional PDF of the arrival times $T_1, T_2, \ldots\ldots T_n$ is given by :

$$f[t_1\ t_2\ \ldots\ldots\ t_n \mid N(t) = n] = \frac{n!}{t^n},$$

$$0 \leq t_1 \leq t_2 \leq \ldots\ldots \leq t_n \leq t.$$ 10

(b) Consider a computer system with Poisson job arrival system at an average rate of 60 per hour. Determine the probability that the time interval between job arrivals is

(i) Longer than 4 minutes.

(ii) Shorter than 8 minutes.

(iii) Between two and six minutes.

6. (a) What are the conditions for a stochastic process to be called wide-sense stationary ? 5

(b) Show that the continuous parameter discrete state random telegraph process $\{x(t) \mid -\infty < t < \infty\}$ with state space I = {–1, 1} is wide sense stationary, if

(i) $P[X(t) = -1] = P[X(t) = 1] = \frac{1}{2}$.

(ii) The number of flips $N(\tau)$ from one value to another, in the duration τ is Poisson distributed with parameter $\lambda\tau$. 15

UNIT–IV

7\. (a) Given a two state Markov chain with transition probability matrix

$$P = \begin{vmatrix} 1-a & a \\ b & 1-b \end{vmatrix} \quad 0 \le a, b \le 1, \ \ |1 - a - b| < 1.$$

Show that the n-step transition probability matrix $P(n) = P^n$ is given by

$$P(n) = \begin{vmatrix} \dfrac{b+a(1-a-b)^n}{a+b} & \dfrac{a-(1-a-b)^n}{a+b} \\ \dfrac{b-b(1-a-b)^n}{a+b} & \dfrac{a+b(1-a-b)^n}{a+b} \end{vmatrix}$$ 15

(b) Describe state classification of a Markov chain explaining Transient, Recurrent and Absorbing states. 5

8\. (a) Assume that a computer system is in one of the three states: busy, idle or undergoing repair, respectively denoted by state 0, 1 and 2. Observing its state at 2 p.m. each day, we believe that the system approximately behaves like a homogeneous Markov chain with transition probability matrix

$$P = \begin{vmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{vmatrix}$$

Prove that the chain is irreducible, and determine the steady state probabilities. 15

(b) Explain the concept of Discrete parameter Birth-Death process. 5
