

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-III • EXAMINATION – SUMMER 2013****Subject Code: 130001****Date: 01-06-2013****Subject Name: Mathematics-III****Time: 02.30 pm - 05.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** Do as directed **14**
- (a) Find the Laplace transform of $t^2 \sin 2t$
 - (b) Evaluate $\beta\left(\frac{7}{2}, \frac{5}{2}\right)$
 - (c) Solve $e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$
 - (d) Find general solution of $y''' - y = 0$
 - (e) Define Convolution and unit step function.
 - (f) Discuss singularities of $x^3(x-1)y'' - 3(x-1)y' + xy = 0$
 - (g) Express $f(x) = 3x^2 + 3x + 1$ in terms of Legendre polynomial
- Q.2** (a) By power series method solve $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ **07**
- (b) (i) Prove that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ **04**
- (ii) Define Gamma function and Evaluate $\int_0^\infty x^6 e^{-2x} dx$ **03**
- OR**
- (b) (i) Solve $x \frac{dy}{dx} + (1+x)y = x^3$ **03**
- (ii) Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$ **04**
- Q.3** (a) Define Laplace transform and find Laplace transform of **07**
- (i) $t^3 + e^{-3t} + t^{1/2}$ **07**
- (ii) $e^{-2t} \sin^2 2t$
- (b) Find inverse laplace transform of **07**
- (i) $\frac{3s^2 + 2}{(s+1)(s+2)(s+3)}$ **07**
- (ii) $\frac{s^3 + 2s^2 + 2}{s^3(s^2 + 1)}$
- OR**
- Q.3** (a) State and prove convolution theorem **07**
- (b) Find inverse laplace transform of **07**
- (i) $\log\left(\frac{s+1}{s-1}\right)$ **07**
- (ii) $\frac{e^{-4s}(s+2)}{s^2 + 4s + 5}$
- Q.4** (a) (i) Using method of variation of parameter solve $y'' - 2y' + y = e^x x^{\frac{3}{2}}$ **04**
- (ii) Solve $(D^2 + D - 6)y = e^{2x} \sin 3x$ **03**
- (b) (i) Find the Fourier series of $f(x) = x + |x|$, where $x \in (-\pi, \pi)$ **04**
- (ii) find Fourier series of $f(x) = x^3$ where $x \in (-\pi, \pi)$ **03**

OR

- Q.4 (a)** (i) Solve $x^2 y'' + 4xy' - 4y = \sin(\ln x)$ **04**
(ii) Using method of undetermined coefficients solve $y'' - 2y' + y = e^x + x$ **03**
- (b)** Solve using Frobenius method $2x(1-x)y'' + (1-x)y' + 3y = 0$ **07**
- Q.5 (a)** Using method of separation of variables solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$; **07**
 $u = 0, \frac{\partial u}{\partial x} = 1 + e^{-3y}$ when $x = 0$
- (b)** A rod 30cm long has its end A and B kept 20°C and 80°C respectively until steady state condition prevail. The temperature at each end is suddenly reduce to 0°C and kept so. Find the resulting temperature function $u(x, t)$ from end A. **07**
- OR**
- Q.5 (a)** Find the Fourier transform of e^{-ax^2} where $a > 0$. **07**
- (b)** Find Fourier cosine integral of $f(x) = \begin{cases} x & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$ **07**
