Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. Sem - IV Examination - June- 2011

Subject code: 140001

Subject Name: Mathematics-4

Date:02/06/2011 **Time: 10.30 am – 01.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Use residues to evaluate $\int_0^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}$ **05**

(b) 1.Find real and imaginary part of $(-1-i)^7 + (-1+i)^7$. **03**

2.Check whether the following functions are analytic or not at any point: **03**

a. $f(z) = e^{\bar{z}}$ b. $f(z) = 2x + i\,xy^2$

3.Prove that $tan^{-1}z = \frac{i}{2}\log\frac{i+z}{i-z}$ **03**

Q.2 **(a)** 1.Evaluate $\int_c f(z)dz$ where $f(z)$ is defined by **03**

$$f(z) = \begin{cases} 1 & when \quad y < 0 \\ 4y & when \; y > 0 \end{cases}$$ and c is the arc from $z = -1-i$ to $z = 1+i$ along the curve $y = x^3$

2.Evaluate $\int_c \frac{\cos\pi z^2}{(z-2)(z-1)}\,dz$, where c is the circle $|z| = 3$. **03**

(b) 1.Find the Laurent's expansion of **04**

$$f(z) = \frac{7z-2}{(z+1)z(z-2)}$$ in the region $1 < z+1 < 3$.

2.Find bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. Hence find the image of $|z| < 1$. **04**

OR

(b) 1.Show that $u(x,y) = 2x - x^3 + 3xy^2$ is harmonic in some domain and find a harmonic conjugate of $u(x,y)$. **04**

2.Find the image of the semi-infinite strip $x > 0, 0 < y < 2$ when $w = iz + 1$. Sketch the strip and its image **04**

Q.3 **(a)** Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and residue at each pole. Hence evaluate $\int_c f(z)dz$ where c is the circle $|z| = 3$. **05**

(b) Express the function $\frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}$ as a sum of partial fraction, using Lagrange's formula. **05**

(c) State Trapezoidal rule with n=10 and evaluate $\int_0^1 e^x\,dx$. **04**

OR

Q.3 **(a)** Evaluate $\int_0^{\pi} \frac{1}{17-8\cos\theta}\,d\theta$, by integrating around a unit circle. **05**

(b) From the following table, find $f(x)$ using Newton's divided difference formula. **05**

x	1	2	7	8
$f(x)$	1	5	5	4

(c) Evaluate $\int_0^6 \frac{1}{1+x}\,dx$ taking h=1 using Simpson's 1/3 rule.Hence,obtaine an approximate value of $\log_e 7$. **04**

Q.4 **(a)** Employ Stirling's formula to compute $y(35)$ from the following table. **05**

x	20	30	40	50
y	512	439	346	243

(b) With the usual notation, show that **05**

(1).$\Delta = 1 - e^{hD}$ (2).$(1+\Delta)(1-\nabla) = 1$.

(C) Solve:$x+y+z=9, 2x-3y+4z=13, 3x+4y+5z=40$ by Gauss elimination **method**. **04**

OR

Q.4 **(a)** Solve the following system of equation by using Gauss-Seidel method correct up to two decimal places. **05**

$20x+2y+z=30, x-40y+3z=-75,$

$2x-y+10z=30.$

(b) Using Newton's forward formula,find the value of $f(1.6)$ if **05**

x	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.5

(c) Use power method to find the largest eigen value of the matrix **04**

A= $\begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$.

Q.5 **(a)** Find a root of the equation$x^3-4x-9=0$, using the bisection method in four stages. **05**

(b) Using Taylor series method, find correct to four decimal places, the value of $y(0.1)$,given $\frac{dy}{dx}=x^2+y^2$and$y(0)=1$. **05**

(c) Find the positive root of $x=\cos x$ using Newton's method correct to 3 decimal places. **04**

OR

Q.5 **(a)** Apply the fourth order Rung-Kutta method to find $y(0.2)$,given $\frac{dy}{dx}=x+y, y(0)=1$(Take h=0.1) **05**

(b) Develop a recurrence formula for finding $\sqrt{N}$,using Newton-Raphson method and hence compute $\sqrt{27}$ to three decimal places. **05**

(c) Using Euler's method, Find $y(0.2)$given $\frac{dy}{dx}=y-\frac{2x}{y}, y(0)=1$. (Take h=0.1) **04**

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