Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-IV Remedial Examination Nov/ Dec. 2010

Subject code: 140001 **Subject Name: Mathematics-4**

Date: 27 / 11/2010 **Time: 03.00 pm – 06.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** State the necessary and sufficient condition for a function to be analytic and prove the necessary condition. **05**

(b) Attempt the following. **09**

1. Show that if c is any nth root of unity other than unity itself, then $1+c+c^2+-----+c^{n-1}=0$.
2. Show that $u(x,y)=2x-x^3+3xy^2$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$.
3. Discuss the convergence of $\sum_{n=0}^{\infty}\frac{(2n)!}{(n!)^2}(z-3i)^n$ and also find the radius of convergence.

Q.2 **(a)** Attempt the following.

1. Define bilinear transformation. Also find the bilinear transformation which maps the points $z=1,\ i,-1$ into the points $w=i,\ 0,\ -i$. Hence find the image of $|z|<1$. **05**
2. Evaluate $\int_C \bar{z}.dz$. Where C is the right-half of the circle $|z|=2$. Hence show that $\int_C \frac{dz}{z}=\pi i$. **03**

(b) Attempt the following. **06**

1. Find the image of infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ under the transformation $w=\frac{1}{z}$. Also show the region graphically.
2. Define residue at simple pole and find the sum of residues of the function $f(z)=\frac{\sin z}{z\cos z}$ at its poles inside the circle $|z|=2$.

OR

(b) Attempt the following. **06**

1. State Cauchy's integral formula and hence evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}dz$; where C is the circle $|z|=3$.

2. Expand $f(z) = -\frac{1}{(z-1)(z-2)}$ in the region

a) $|z| < 1$ b) $1 < |z| < 2$ c) $2 < |z| < \infty$

Q.3 **(a)** State and prove Cauchy Goursat theorem. **05**

(b) Attempt the following. **09**

1. Write the two Laurent series expansion in powers of z that represent the function $f(z) = \frac{1}{z^2(1-z)}$ in certain domains, and also specify domains.
2. Perform the five iterations of the bisection method to obtain a root of the equation $f(x) = \cos x - xe^x = 0$.
3. Compute the real root of $f(x) = x - 2\sin x = 0$, correct up to 6 decimal places using Secant method, starting from $x_0 = 2,\ x_1 = 1.9$.

OR

Q.3 **(a)** Using an indentation along a Branch cut show that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$. **05**

(b) Attempt the following. **09**

1. State Cauchy's residue theorem and evaluate $\int_C \frac{5z-2}{z(z-1)} dz$. Where C is the circle $|z| = 2$.
2. Use power method to find the largest of Eigen values of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$. Perform four iterations only.
3. Find a real root of the $x^3 + x - 1 = 0$ correct up to six places of decimal places.

Q.4 **(a)** Compute $\cosh(0.56)$ using Newton's forward difference formula and also estimate the error for the following table. **05**

x	0.5	0.6	0.7	0.8
$f(x)$	1.127626	1.185465	1.255169	1.337435

(b) Attempt the following. **09**

1. Solve the following linear system of equations by Gauss elimination method

$$8x_2 + 2x_3 = -7$$
$$3x_1 + 5x_2 + 2x_3 = 8$$
$$6x_1 + 2x_2 + 8x_3 = 26$$

2. State Trapezoidal rule with n=10 and evaluate $\int_0^1 e^{-x^2} dx$.
3. Evaluate $\int_1^3 \sin x\, dx$ using Gauss Quadrature of five points. Compare the result with analytic value.

OR

Q.4 **(a)** State Newton's divided difference interpolation formula and compute $f(9.2)$ from the following data. **05**

x_j	8.0	9.0	9.5	11.0
$f(x)_j$	2.079442	2.197225	2.251292	2.397895

(b) Attempt the following. **09**

1. Solve the following linear system of equations by Gauss-Seidel

$$10x_1 + x_2 + x_3 = 12$$
$$2x_1 + 10x_2 + x_3 = 13$$
$$2x_1 + 2x_2 + 10x_3 = 14$$

2. The speed, v meters per second, of a car, t seconds after it starts, is show in the following table.

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.60	10.08	18.90	21.60	18.54	10.26	4.50	4.5	5.4	9.0

Using Simpson's $1/3$ rule, find the distance travelled by the car in 2 minutes.

3. State Simpson's $3/8$ -rule and evaluate $\int_0^1 \frac{dx}{1+x^2}$ taking $h = \frac{1}{6}$.

Q.5 **(a)** Explain the Euler's method to find Numerical solution of $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$. **05**

(b) Attempt the following. **09**

1. Use Runge–Kutta second order method to find the approximate value of $y(0.2)$ given that $\frac{dy}{dx} = x - y^2$ & $y(0) = 1$ & $h = 0.1$

2. Use Euler method to find $y(1.4)$ given that $\frac{dy}{dx} = xy^{\frac{1}{2}}$, $y(1) = 1$

3. Use Taylor's series method to solve $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$. Also find $y(0.03)$.

OR

Q.5 **(a)** Derive the Newton Raphson iterative scheme by drawing appropriate figure. **05**

(b) Attempt the following. **09**

1. Use Runge-Kutta fourth order method to find $y(1.1)$, given that $\frac{dy}{dx} = x - y$, $y(1) = 1$ and $h = 0.05$.

2. Find the Lagrange interpolating polynomial from the following data

x	0	1	4	5
$f(x)$	1	3	24	39

3. Set up a Newton iteration for computing the square root x of a given positive number c and apply it to $c=2$

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