B.E.

Sixth Semester Examination, 2010

Digital Signal Processing (EE-407-E)

Note : Attempt any five questions.

Q. 1. (a) A system has the input-output relation given by $y(n) = T[x(n)] = nx(n)$. Determine whether the system is :

(i) Memoryless **(ii) Causal**

(iii) Linear **(iv) Ti.ne-in-variant or**

(v) Stable

Ans. $$y(n) = Tx(n) = nx(n)$$

(i) Memoryless : The system is memoryless because no delay or advancement is not there.

(ii) Causal : Causal because output is depends upon the input present value not future value.

(iii) Linear : The system is linear.

(iv) Time-in-Variant : The above system is time invariant because if we change in $y(n)$ cause the same change in $x(n)$.

(v) Stable : The system is unstable because no area of field is defined.

Q. 1. (b) Obtain the Fourier transform of the signal :

$$f(t) = 1/2\,[\delta(t+1) + \delta(t+1/2) + \delta(t-1/2) + \delta(t-1)]$$

Ans. $$f(t) = 1/2[\delta(t+1) + \delta(t+1/2) + \delta(t-1/2) + \delta(t-1)]$$

$$f(t) \xleftrightarrow{F/T} \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt = f(j\omega)$$

By using the property $$\delta(t - t_0) \xleftrightarrow{F/T} e^{-j\omega t_0}$$

$$f(j\omega) = \frac{1}{2}\{e^{j\omega} + e^{j\omega/2} + e^{-j\omega/2} + e^{-j\omega}\}$$

$$= \frac{e^{j\omega} + e^{-j\omega}}{2} + \frac{e^{j\omega/2} + e^{-j\omega/2}}{2}$$

$$\boxed{f(j\omega) = \cos\omega + \cos\omega/2}$$

Q. 2. (a) Define the following elementary signals :

(i) Unit impulse signal **(ii) Unit step signal**

(iii) Unit ramp signal

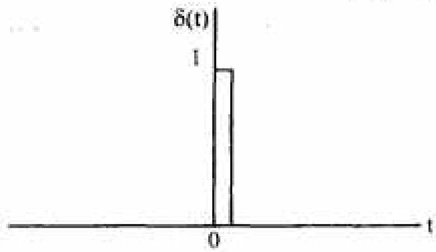
Ans. (i) Unit Impulse Signal : The unit-impulse function is defined as

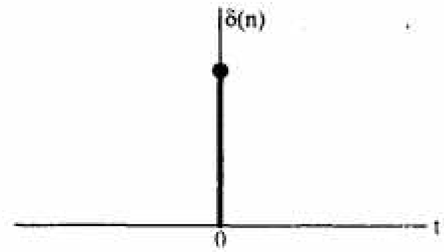
$$\delta(t) = 0,\ t \neq 0$$

$$\delta(t) = 1;\ t = 0$$

$$\delta(n) = 0;\ n \neq 0$$

$$\delta(n) = 1;\ n = 0$$





The area of the impulse function is unity and this area is confined to an infinitesimal interval on the t-axis and concentrated at $t = 0$. The unit impulse system is very useful in continuous time system analysis. It is used to generate the system response providing fundamental information about the system characteristics.

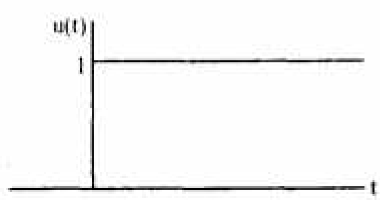
(ii) Unit Step Signal : The integral of the impulse function $\delta(t)$

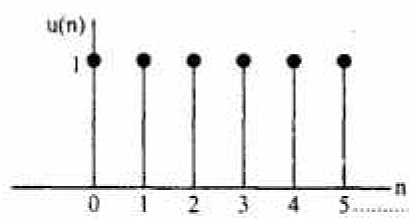
$$\int_{-\infty}^{t} \delta(t)dt = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Since the area of the impulse function is all concentrated at $t = 0$ for any value of $t < 0$, the integral becomes zero.

The integral of the impulse function is also a singularing function and called the unit-step function and is represented as

$$u(n) = \begin{cases} 0 & n > 0 \\ 1 & n > 0 \end{cases}$$





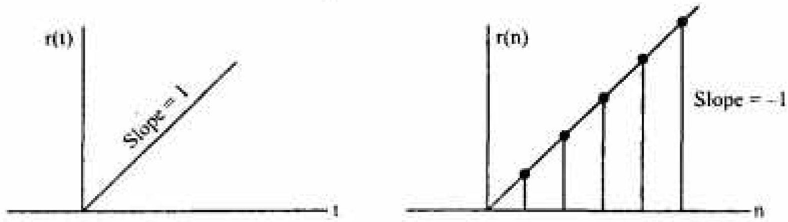
(iii) Unit Ramp Signal : The unit-ramp function $r(t)$ can be obtained by integrating the unit impulse function twice or integrating the unit step function one

$$r(t) = \int_{-\infty}^{t} \int_{-\infty}^{\alpha} \delta(\tau)d\tau d\alpha$$

$$= \int_{-\infty}^{t} u(\alpha)d\alpha$$

$$r(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

$1 \rightarrow$ Step for slope $\qquad r(n) = \begin{cases} 0 & n < 0 \\ 1 & n > 0 \end{cases}$



Q. 2. (b) Discuss the various properties of continuous and discrete time LTI systems in brief.

Ans. LTI System : A time invariant system is one whose input-output relationship does not vary with time. A time-invariant system is also called a fixed system. The condition for a system to be fixed is

$$H[x(t-\tau)] = y(t-\tau) \qquad \text{.... (i)}$$

A time-invariant system satisfies equation (i) for any $x(t)$ and any value of τ.

If $y(t)$ is the response of the system to shifted input is the response of the system to $x(t)$ time shifted by the same amount. In discrete time this property is also referred to as shift-invariance. For a discrete time system the condition for shift-invariance is given by

$$H[x(n-K)] = y(n-K) \qquad \text{.... (ii)}$$

Where K is an integer. A sysfem not satisfy equations (i) and (ii) is said to be time-variant. The system satisfy both linearity and time invariant condition are called linear, time invariant system called simply LTI system.

Linear system is one in which the principle of superposition holds, for a system with two inputs $x_1(t)$ and $x_2(t)$.

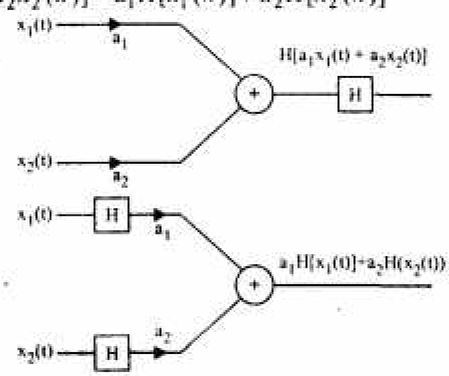
The superposition position is defined as follows

$$H[a_1 x_1(t) + a_2 x_2(t)] = a_1 H[x_1(t) + a_2 H(x_2(t))]$$

Where, a_1 & a_2 are the weights added to the inputs and $H[x(t)] = y(t)$ is the response of the continuous time system to the input $x(t)$. Thus, a linear system is defined as one whose response to the sum of the weight input is same as the sum of the weighted responses.

If a system not satisfy this equation called nonlinear system for discrete time system

$$H[a_1 x_1(n) + a_2 x_2(n)] = a_1 H[x_1(n)] + a_2 H[x_2(n)]$$

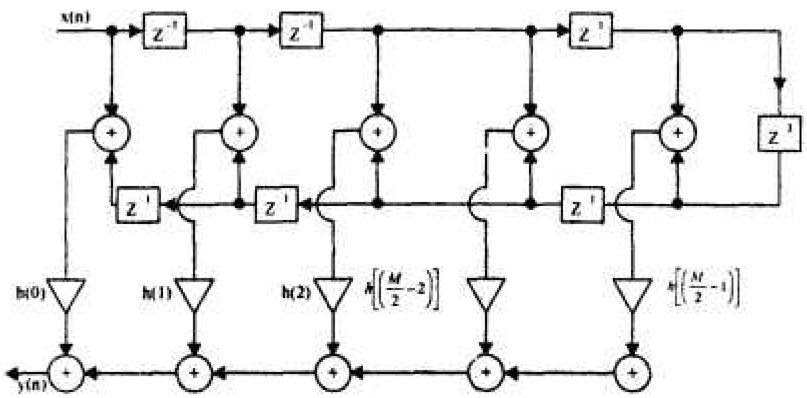


Q. 3. (a) Distinguish between IIR and FIR systems.

Ans.

	FIR	IIR
(i)	The impulse response sequence is of finite duration i.e., it has finite number of non-zero terms. e.g., $x(n) = \{1, 2, 3, -2, 0\}$	It has infinite number of non-zero terms i.e., its impulse response sequence of its infinite duration. e.g., $x(n) = nu(n)$
(ii)	FIR filter are usually implemented using structure with no feedback.	They are usually implemented by using structure having feedback.
(iii)	Non-recursive structure all zeros system.	Recursive structure pole and zeros.

Examples of FIR :



IIR Filter :

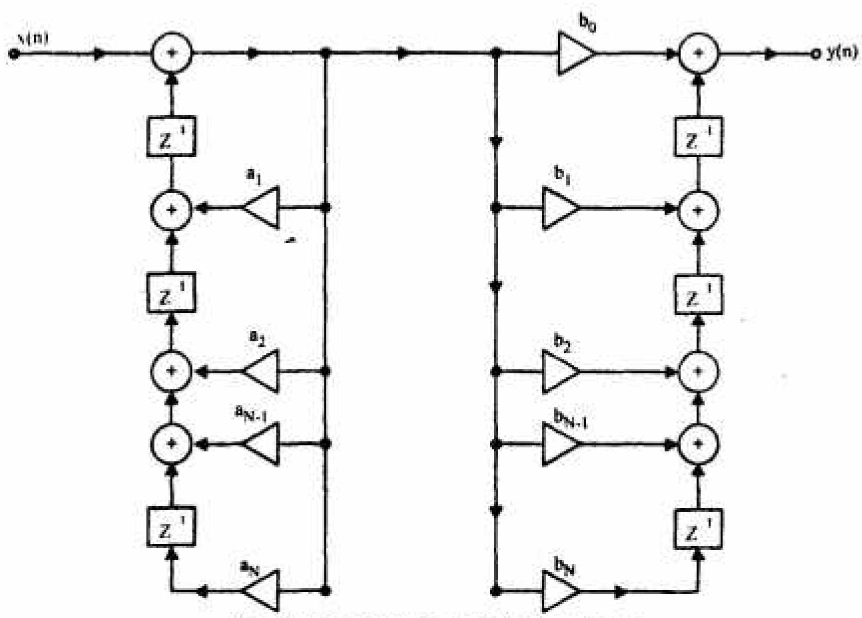
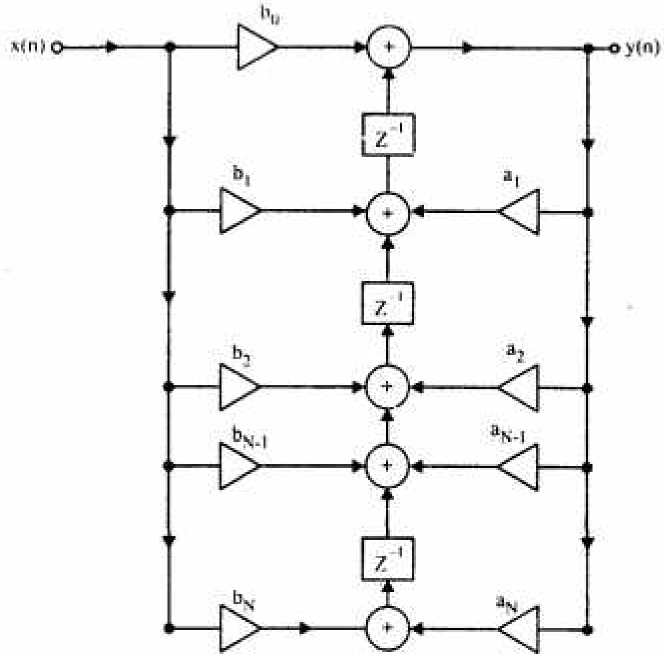


Fig. Transposed direct form I realisation structure

I



Q. 3. (b) Obtain the condition for stability of the filter :

$$H(t) = \frac{a_1 + a_2 z^{-1}}{1 + b_1^{z-1} + b_2 z^{-2}}$$

Ans.

$$H(t) = \frac{a_1 + a_2 z^{-1}}{1 + b_1 z^{-1} + b_2 z^{-2}}$$

$$= \frac{a_1 + \frac{a_2}{z}}{1 + \frac{b_1}{z} + \frac{b_2}{z^2}}$$

$$= \frac{(a_1 z + a_2)}{z^2 + b_1 z + b_2}$$

$$= \frac{a_1 z^2 + a_2 z}{z^2 + b_1 z + b_2} \qquad \text{.... (i)}$$

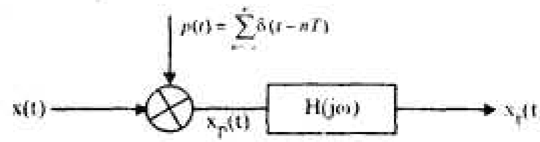
$$= \frac{a_1 z^2 + a_2 z}{(z - P_1)(z - P_2)} \qquad \text{.... (ii)}$$

Where the poles are at $z = P_1$ and z_2

Since $|P_1|, |P_2| < 1$. The system is stable, otherwise unstable.

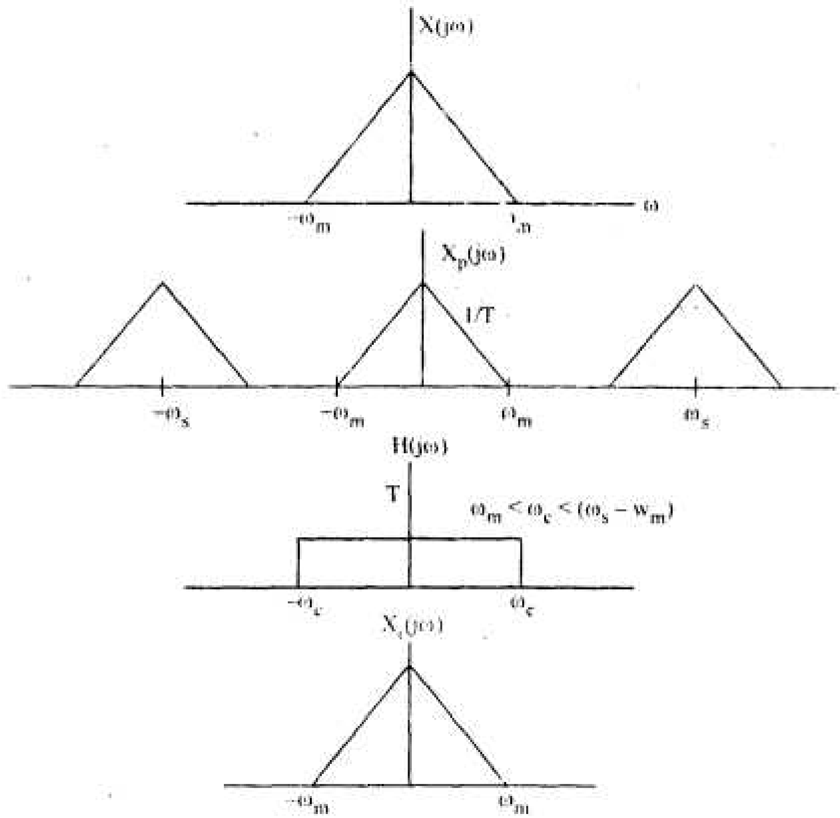
Q. 4. (a) Describe the operation of a sampling circuit.

Ans. Sampling Circuit :



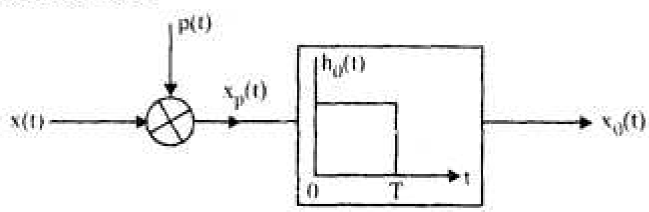
Sampling circuit

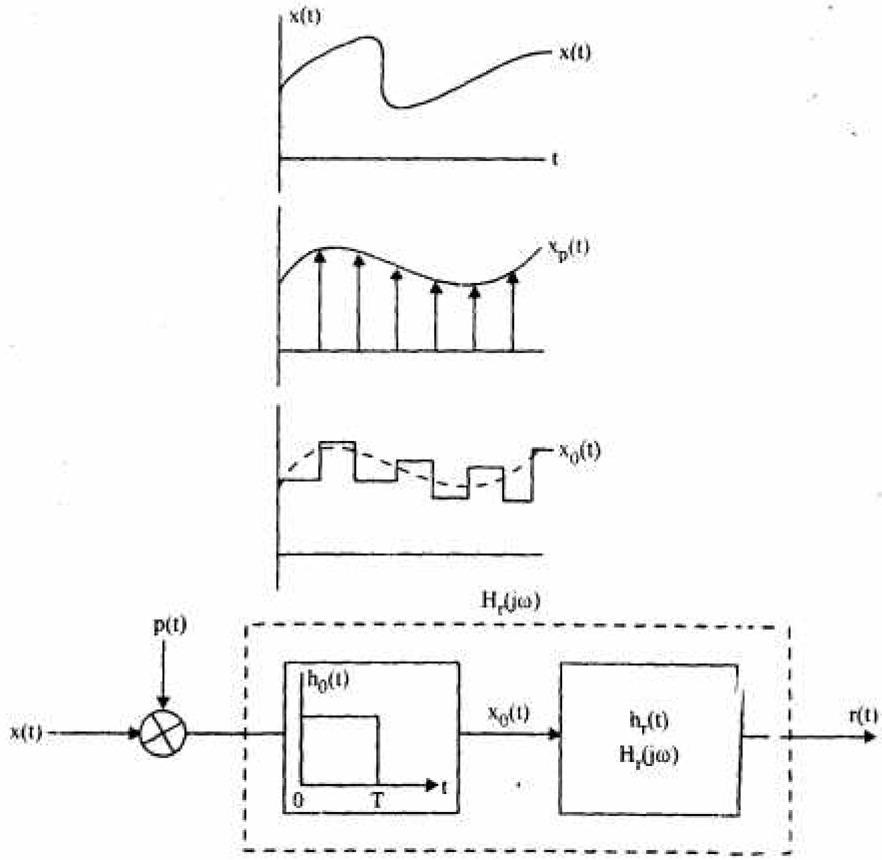
$$x(t) \xleftrightarrow{F/T} X(j\omega)$$



The frequency $2\omega_m$ which under the sampling theorem, must be exceeded by the sampling frequency is commonly referred to as the Nyquistrate.

Sampling with a zero order hold

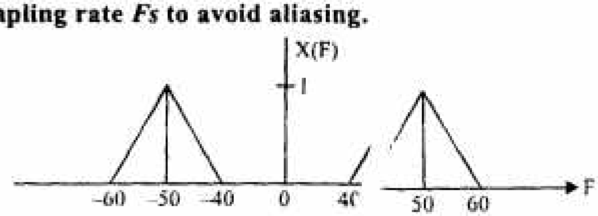




$$H_0(j\omega) = e^{-j\omega T/2}\left\{\frac{2\sin(\omega T/2)}{\omega}\right\}$$

$$H_r(e^{j\omega}) = \frac{e^{j\omega T/2}H(j\omega)}{\dfrac{2\sin(\omega T/2)}{\omega}}$$

Q. 4. (b) Explain the sampling of the bandpass signal whose spectrum is illustrated in figure. Determine the minimum sampling rate *Fs* to avoid aliasing.

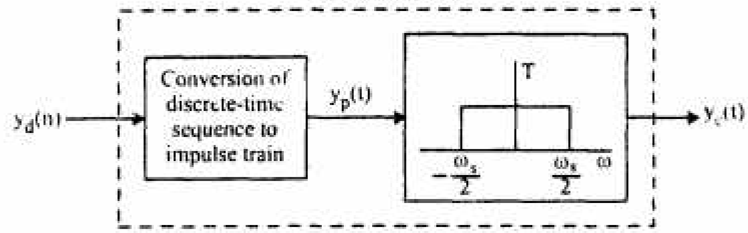


Ans. Sampling Theorem : If the spectrum of the signal is band limited the signal can be reconstructed from its sample, if the sampling rate is greater than or equal to the maximum frequency contained in the original signal.

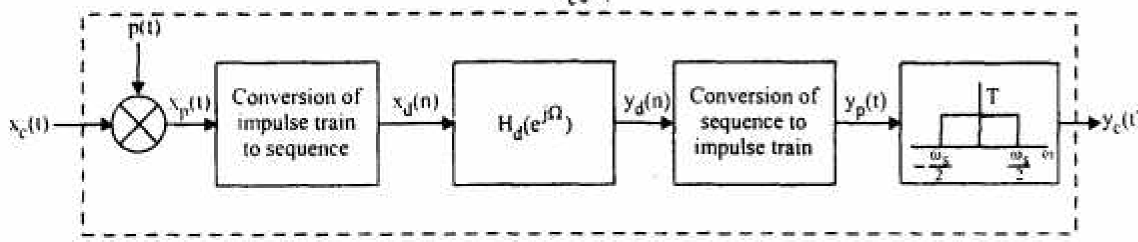
$$\boxed{f_s = 2f_m}$$

$$2f_m = f_N \rightarrow \text{Nyquist frequency}$$

D/C conversion



$H_c(j\omega)$



Sampling rate
$$f_s = 2f_m$$
$$f_m = 60\,\text{Hz}$$
$$f_s = 2 \times 60$$
$$= 120\,\text{Hz}$$

For avoiding aliasing 120Hz sampling rate is required.

Q. 5. (a) Consider a discrete time LTI system whose system function $H(Z)$ is given by :

$$H(Z) = \frac{-Z}{Z - 1/2} \; |Z| > 1/2$$

(i) Find the step response $s(n)$

(ii) Find the output $y(n)$ to the input $x(n) = nu(n)$

Ans.
$$H(Z) = \frac{Z}{Z - 1/2} \; |Z| > 1/2$$

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{Z}{Z - \frac{1}{2}}$$

(i) Step Response : $S(n) \xleftrightarrow{Z} S(Z)$

/

$$S(Z) = X(Z)$$

For unit step

$$S(n) \leftrightarrow \frac{1}{1-Z^{-1}}$$

$$Y(Z) = \frac{Z}{Z-\frac{1}{2}} \times \frac{1}{1-Z^{-1}}$$

$$Y(Z) = \left(\frac{1}{1-\frac{1}{2}Z^{-1}}\right)\left(\frac{1}{1-Z^{-1}}\right)$$

$$Y(n) = \text{Inverse } Z\left\{\left(\frac{1}{1-\frac{1}{2}Z^{-1}}\right)\left(\frac{1}{1-Z^{-1}}\right)\right\}$$

$$\left(\frac{1}{1-\frac{1}{2}Z^{-1}}\right)\left(\frac{1}{1-Z^{-1}}\right) = \frac{A}{1-\frac{1}{2}Z^{-1}} + \frac{B}{1-Z^{-1}}$$

Put $Z^{-1} = 1$

$$\frac{1}{\left(1-\frac{1}{2}\right)} = \frac{A}{1-\frac{1}{2}}$$

$$2 = 2A$$

$$A = 1$$

Put $Z^{-1} = 2$

$$\frac{1}{1-2} = \frac{B}{1-2}$$

$$B = 1$$

$$y(n) = Z^{-1}\left\{\frac{1}{1-\frac{1}{2}Z^{-1}} + \frac{1}{1-Z^{-1}}\right\}$$

$$|Z| > \frac{1}{2}$$

$$y(n) = \left(\frac{1}{2}\right)^n u(n) + (1)^n u(n)$$

$$y(n)=\left\{1+\left(\frac{1}{2}\right)^{n}\right\}u(n)$$

(ii) $x(n)=nu(n)$

$$X(Z)=\frac{Z^{-1}}{(1-Z^{-1})^{2}}$$

$$H(Z)=\frac{Y(Z)}{X(Z)}=\frac{1}{1-\frac{1}{2}Z^{-1}}$$

$$Y(Z)=\left(\frac{1}{1-\frac{1}{2}Z^{-1}}\right)\left(\frac{Z^{-1}}{(1-Z^{-1})^{2}}\right)$$

$$y(n)=Z^{-1}\left\{\frac{1}{\left(1-\frac{1}{2}Z^{-1}\right)}\right\}\left\{\frac{Z^{-1}}{(1-Z^{-1})^{2}}\right\}$$

$$\frac{1}{1-\frac{1}{2}Z^{-1}}\times\frac{Z^{-1}}{(1-Z^{-1})^{2}}=\frac{Z}{Z-\frac{1}{2}}\cdot\frac{Z^{2}}{Z(Z-1)^{2}}$$

$$\frac{Z}{Z-\frac{1}{2}}\cdot\frac{Z}{(Z-1)^{2}}=\frac{A}{Z-\frac{1}{2}}+\frac{B}{Z-1}+\frac{C}{(Z-1)^{2}}$$

Put $Z=\frac{1}{2}$

$$\left(\frac{\left(\frac{1}{2}\right)^{2}}{\left(\frac{1}{2}-1\right)^{2}}\right)=A$$

$$A=\frac{1}{2^{2}}=\frac{1}{4}$$

Put $Z=1$

$$\frac{1}{1-\frac{1}{2}}=B$$

$$B=2$$

$$C=(Z-1)^{2}\frac{d}{dZ}\left\{\frac{1}{(Z-1)}\right\}\Bigg|_{z=1}$$

$$C=(Z-1)^2\left\{\frac{(Z-1)(0)+(1)}{(Z-1)^2}\right\}_{Z=1}$$

$$C=1$$

$$=\frac{1}{4\left(Z-\frac{1}{2}\right)}+\frac{2}{Z-1}+\frac{1}{(Z-1)^2}$$

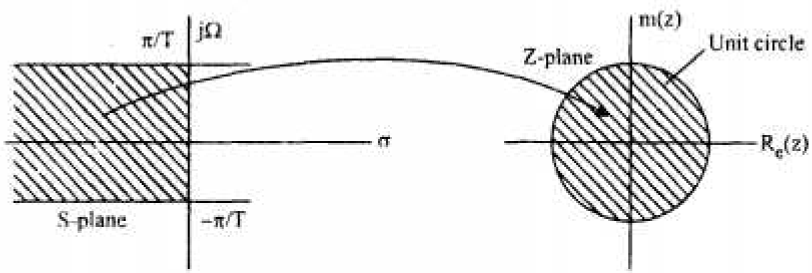
$$=\frac{Z^{-1}}{4\left\{1-\frac{1}{2}Z^{-1}\right\}}+\frac{2Z^{-1}}{1-Z^{-1}}+\frac{Z^{-2}}{(1-Z^{-1})^2}$$

$$=\frac{1}{4}\left(\frac{1}{2}\right)^n u(n)+2u(n)+nu(n) \text{ Ans.}$$

Q. 5. (b) Describe the relationship between Laplace transformation and -Z-transformation.

Ans. $x(t)\xleftrightarrow{L/T}X(S)$ Laplace transform

$x(n)\xleftrightarrow{Z/T}X(Z)$ Z transform



$$Z=e^{ST}$$

$$X(Z)=\sum_{n=-\infty}^{\infty}x(n)Z^{-1} \text{ for digital}$$

$$X(S)=\int_{-\infty}^{\infty}x(t)e^{-St}\,dt \text{ for analog}$$

$$\left.\frac{dy(t)}{dt}\right|_{t=nT}=\frac{y(nT)-y(nT-T)}{T}$$

$$=\frac{y(n)-y(n-1)}{T}$$

Put $S=\frac{1-Z^{-1}}{T}$

$$\frac{d^2y}{dt^2}=\frac{d}{dt}\left\{\frac{dy(t)}{dt}\right\}$$

/

$$S^2 = \left(\frac{1-Z^{-1}}{T}\right)^2$$

$$\boxed{S^i = \left(\frac{1-Z^{-1}}{T}\right)^i}$$

$$\frac{1}{(S+S_i)^m} \rightarrow \frac{(-1)^{m-1}}{(m-1)!}\frac{d^{m-1}}{ds^{m-1}}\left\{\frac{1}{1-e^{-sT}Z^{-1}}\right\}S \rightarrow S_i$$

$$\frac{S+a}{(S+a)^2+b^2} \leftrightarrow \frac{1-e^{-aT}(\cos bT)Z^{-1}}{1-2e^{-aT}\cos bTZ^{-1}+e^{-2aT}Z^{-2}}$$

$$\frac{b}{(S+a)^2+b^2} \rightarrow \frac{e^{-aT}\sin(bT)Z^{-1}}{1-2e^{-aT}(\cos bT)Z^{-1}e^{-2aT}Z^{-2}}$$

$S = P_i$ is mapped into a digital pole at $Z = e^{P_i t}$

$\therefore$ $$Z = e^{ST}$$

$$S = \sigma + j\Omega$$

$$re^{j\omega} = e^{\sigma t}e^{j\Omega T}$$

$$r = e^{\sigma T}$$

$$\omega = \Omega t$$

Q. 6. (a) Convert a analog filter to a digital filter whose system function is given as under :

$$H(S) = \frac{1}{(S+2)^3}$$

Make use of impulse invariant snapping method.

Ans. $$H(S) = \frac{1}{(S+2)^3}$$

Using property of impulse invariant method

$$\frac{1}{(S+S_i)^m} \rightarrow \frac{(-1)^{m-1}}{(m-1)!}\frac{d^{m-1}}{dS^{m-1}}\left\{\frac{1}{1-e^{-ST}Z^{-1}}\right\}$$

$$m = 3$$

$$S_i = 2$$

$$\frac{1}{(S+2)^3} = \frac{(-1)^{3-1}}{(3-1)!}\frac{d^{3-1}}{dS^{3-1}}\left\{\frac{1}{1-e^{-S(1)}Z^{-1}}\right\}$$

$$= \frac{1}{2!}\frac{d^2}{dS^2}\left\{\frac{1}{1-e^{-S}Z^{-1}}\right\}$$

/

$$=\frac{1}{2}\left\{\frac{d}{dS}\left\{\frac{(1-e^{-S}Z^{-1})(0)+Se^{-S}Z^{-1}}{(1-e^{-S}Z^{-1})^2}\right\}\right.$$

$$=\frac{1}{2}\frac{d}{dS}\left\{\frac{Se^{-S}Z^{-1}}{(1-e^{-S}Z^{-1})^2}\right\}$$

$$=\frac{1}{2}\left\{\frac{(1-e^{-S}Z^{-1})^2\{e^{-S}Z^{-1}-S^2e^{-S}Z^{-1}\}+2(1-e^{-S}Z^{-1})Se^{-S}\times Se^{-S}Z^{-1}}{(1-e^{-S}Z^{-1})^4}\right\}$$

$$\frac{1}{(S+2)^3}\to\frac{1}{2}\left\{\frac{(1-e^{-S}Z^{-1})^2\{e^{-S}Z^{-1}-S^2e^{-S}Z^{-1}\}+2\{(1-e^{-S}Z^{-1})^2Se^{-2S}Z^{-1}}{(1-e^{-S}Z^{-1})^4}\right\}$$

Q. 6. (b) What is the principle of designing FIR filter using windows?

Ans. Window Techniques : The desired frequency response of any digital filter is periodic in frequency and can be expanded in Fourier series i.e.,

$$H_d(e^{j\omega})=\sum_{n=-\infty}^{\infty}h_d(n)e^{-j\omega n}$$

$$h(n)=\frac{1}{2H}\int_0^{2\pi}H(e^{j\omega})e^{j\omega n}\,d\omega$$

The Fourier coefficient of the series $h(n)$ are identical to the impulse response of a digital filter.

The infinite duration impulse response can be converted to a finite duration impulse response by truncating the infinite series at $n=\pm N$

Types of Window :

(i) Rectangular Window Function :

$$w_R(n)=\begin{cases}1 & \text{for } |n|\le\frac{M-1}{2}\\ 0 & \text{otherwise}\end{cases}$$

$$w_R(e^{j\omega T})=e^{-j\omega\left(\frac{M-1}{2}\right)T}\frac{\sin\left(\frac{\omega MT}{2}\right)}{\sin\left(\frac{\omega T}{2}\right)}$$

(ii) Hamming Window Function :

$$w_H(n)=\begin{cases}0.54-0.46\cos\frac{2\pi n}{M-1} & 0\le n<M-1\\ 0 & \text{otherwise}\end{cases}$$

$$w_H(e^{j\omega T})=\frac{0.54\sin\left(\frac{\omega MT}{2}\right)}{\sin\left(\frac{\omega T}{2}\right)}-\frac{0.46\sin\left(\frac{\omega MT}{2}-\frac{\omega\pi}{(M-1)}\right)}{\sin\left(\frac{\omega T}{2}-\frac{\pi}{(M-1)}\right)}.-\frac{0.46\sin\left(\frac{\omega MT}{2}+\frac{M\pi}{M-1}\right)}{\sin\left(\frac{\omega T}{2}+\frac{\pi}{M-1}\right)}$$

/

$$w_{Han}(n) = \begin{cases} 0.5 - 0.5\cos\dfrac{2\pi n}{M-1} & 0 \le n \le M-1 \\ 0 & \text{otherwise} \end{cases}$$

(iv) Black Man Window Function :

$$w_B(n) = \begin{cases} 0.42 - 0.5\cos\dfrac{2\pi n}{M-1} + 0.08\cos\dfrac{4\pi}{M-1} & 0 \le n \le M-1 \\ 0 & \text{otherwise} \end{cases}$$

(v) Bartlett Window Function :

$$w_{bart}(n) = \begin{cases} 1+n; & -\dfrac{M-1}{2} < n < 1 \\ 1-n & 1 < n < \dfrac{M-1}{2} \end{cases}$$

(vi) Kaiser Window :

$$w_K(n) = \begin{cases} \dfrac{I_0(\beta)}{I_0(\alpha)} & \text{for } |n| \le \dfrac{M-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

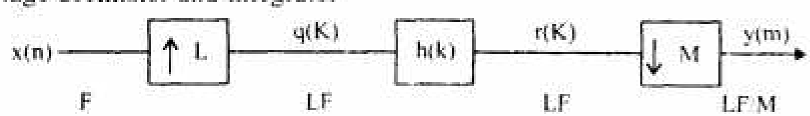
α and β an independent variable determined by Kaiser

$$\beta = \alpha\left\{1 - \left(\frac{2n}{M-1}\right)^2\right\}^{0.5}$$

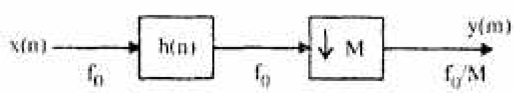
$$I_0(x) = 1 + \sum_{k=1}^{\infty}\left\{\frac{1}{K!}\left(\frac{x}{2}\right)^k\right\}^2$$

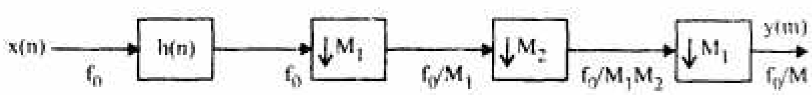
Q. 7. (a) Draw and explain the block diagram of a multistage decimator and integrator.

Ans. Multistage decimator and integrator



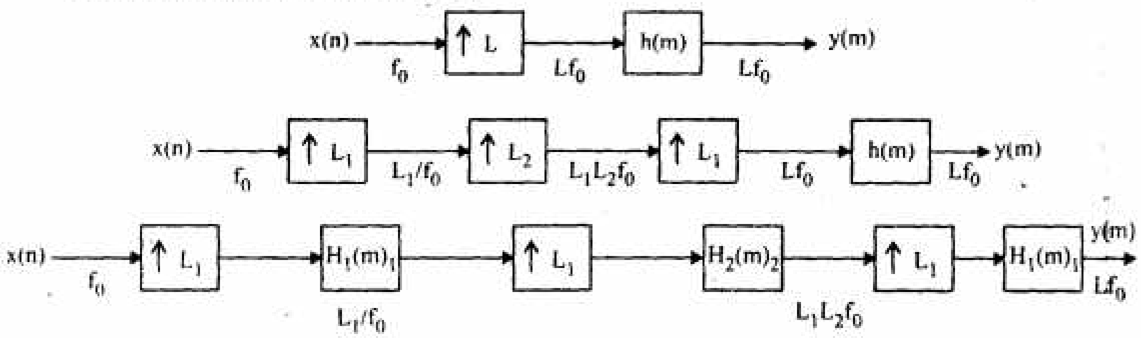
Consider a system for decimating a signal by an integer factor M. Let the input signal sampling frequency be f_0, then the decimated signal frequency will be f_0/M. The decimation factor can be factorised as $M = \prod_{i=1}^{I} M_i$





$$M = M_1 , M_2 \ldots\ldots M_l = \prod_{i=1}^{l} M_i$$

Similarly integrator is shown below



$$L = L_1 , L_2 \ldots\ldots \prod_{i=1}^{l} L_i$$

Q. 7. (b) Give the areas in which signal processing finds its applications.

Ans. Applications finds in various field :

(i) Communication system

(ii) Speech and audio processing system

(iii) Antenna system

(iv) Radar system

(v) Computational requirement

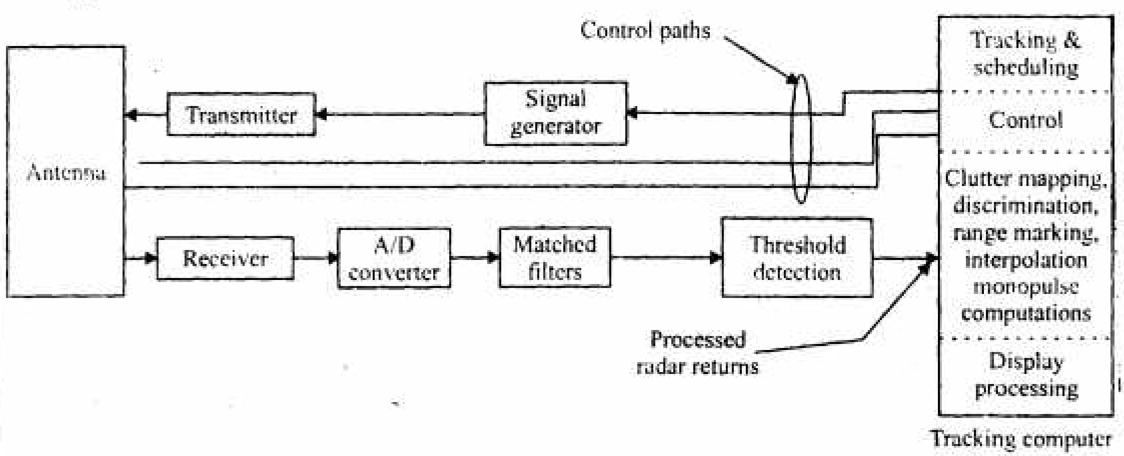
(vi) Storage for filter coefficient

(vii) Finite arithmetic effects are less

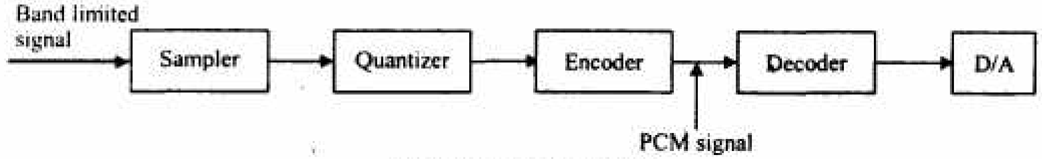
(viii) Finite order required in multirate application are low and

(ix) Sensitivity to filter coefficient length are less.

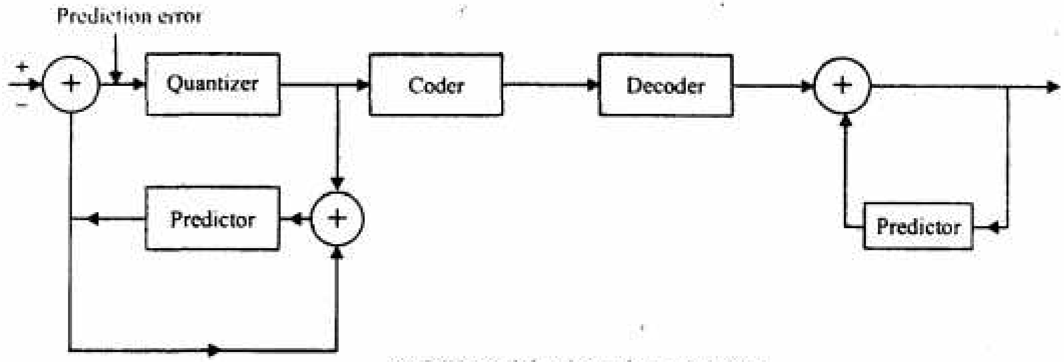
Applications to Radar :



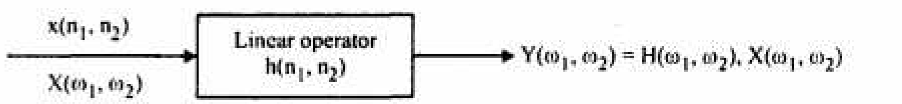
Applications to Image Processing :

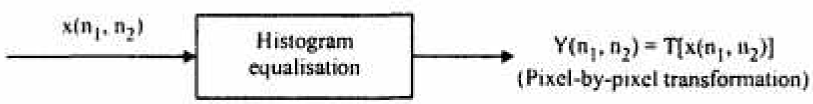


(a) Pulse code modulation



(b) Differential pulse code modulation





2-D signal processing is helpful in processing the images the different processing techniques are image enhancement, image restoration and image coding.

Image enhancement focused mainly on the features of an image. The various feature enhancement are sharpening the image, edge enhancement, filtering, contrast enhancement etc.

Linear filtering emphasises some special regions of the signal. Histogram modification is done on pixel-by-pixel bias and find its application in contrast equalisation or enhancement.

Q. 8. Write short notes on any two of the following :

(a) Finite word length effect in DSP

(b) Application of -Z-Transform

(c) Digital filter banks.

Ans. (a) Finite Word Length Effect in DSP : (i) Quantisation effects in analog-to-digital conversion.

(ii) Product quantisation and coefficient quantisation error in DSP.

(iii) Limit cycle in IIR filter.

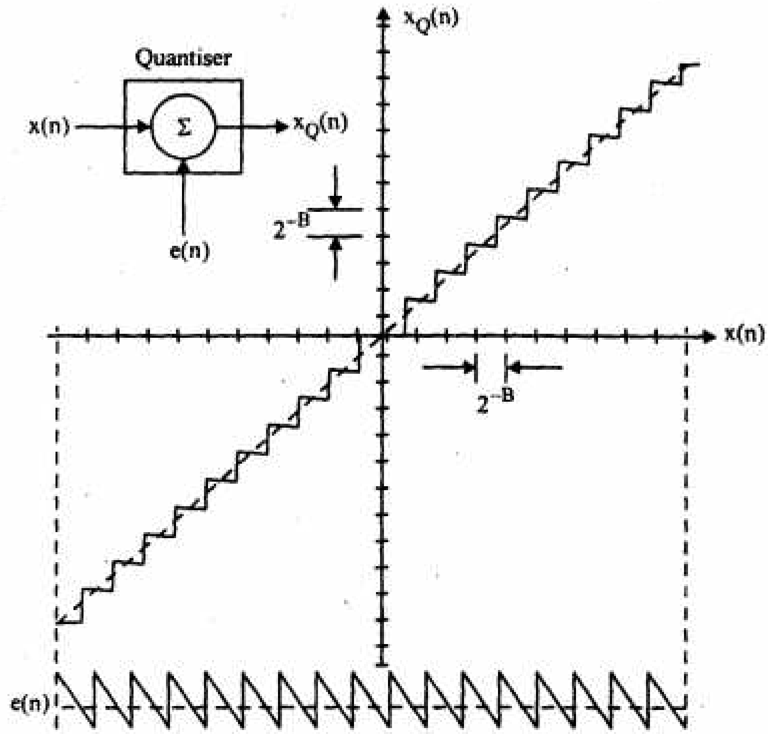
/

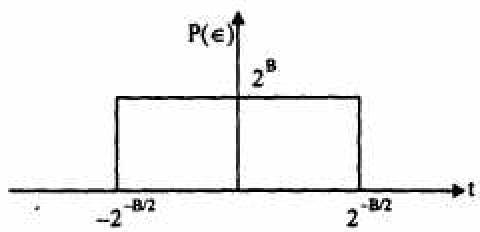
(iv) Finite word length effect in FFT

$$\frac{-2^{-B}}{2} \le e(n) \le \frac{2^{-B}}{2}$$

The quantisation error is assumed to be uniformly distributed over the range by above equation

$$SNR = 10\log \frac{P_x(n) \rightarrow \text{Signal power}}{P_e(n) \rightarrow \text{Quantised noise}}$$





$$P_e(n) = \sigma_e^2 = \int_{-2^{-\beta/2}}^{2^{-\beta/2}} e^2 P(\epsilon) de$$

$$P_e(n) = \frac{2^{-2B}}{12}$$

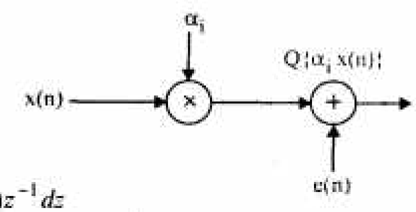
$$SNR = 10\log P_x(n) + 10.8 + 6\,\text{dB}$$

(ii) Product Quantisation :

$$\sigma_e^2 = \frac{2^{-2B}}{12}$$

$$\sigma_{eo}^2 = \sigma_e^2 \sum_{n=0}^{\infty} n^2(n)$$

$$\sigma_{oe}^2 = \frac{\sigma_e^2}{2\pi j} \oint H(z)H(z^{-1})z^{-1}dz$$



(iii) Quantization Error in the Computation of DFT :

$$\sigma_{eq}^2 = 4M\sigma_e^2 = \frac{2^{-2B}}{3} M$$

$$\sigma_{eq}^2 = \frac{2^{-2B}}{3} M = \frac{2^{-2B}}{3} 2^{v} = \frac{2^{-2(B-v/2)}}{3}$$

(b) Applications of Z-Transform :

(i) Response of System with Rational System Function :

$$X(Z) = \frac{N(Z)}{Q(Z)}$$

$$Y(Z) = H(Z)X(Z) = \frac{B(Z)N(Z)}{A(Z)Q(Z)}$$

$$Y(Z) = \sum_{k=1}^{N} \frac{A_k}{1-P_k Z^{-1}} + \sum_{k=1}^{L} \frac{Q_K}{1-q_k Z^{-1}}$$

$$y(n) = \sum_{K=1}^{N} \frac{A_k}{1-P_k Z^{-1}} + \sum_{K=1}^{L} \frac{Q_K}{1-q_K Z^{-1}}$$

$$= \sum_{K=1}^{N} A_K (P_K)^n u(n) + \sum_{K=1}^{L} Q_K (q_K)^n u(n)$$

(ii) Transient and Steady State Response :

$$y_{nr}(n) = \sum_{K=1}^{N} A_K (P_K)^n u(n)$$

$$y_{fr}(n) = \sum_{k=1}^{L} Q_K (q_K)^n u(n)$$

(iii) Causality and Stability :

$$h(n) = 0 \quad n < 0$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$H(Z) = \sum_{n=-\infty}^{\infty} h(n)Z^{-n}$$

$$H(Z) \le \sum_{n=-\infty}^{\infty} |h(n)Z^{-n}| = \sum_{n=-\infty}^{\infty} |h(n)Z^{-n}|$$

$$|H(Z)| \le \sum_{n=-\infty}^{\infty} |h(n)|$$

Stability of the Second Order System :

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) + b_0 x(n)$$

$$H(Z) = \frac{b_0}{1 + a_1 Z^{-1} + a_2 Z^{-2}}$$

$$= \frac{b_0 Z^2}{Z^2 + a_1 Z + a_2}$$

$$P_1, P_2 = -\frac{a_1}{2} \pm \sqrt{\frac{a_1^2 - 4a_2}{4}}$$

$$a_1 = -(P_1 + P_2)$$

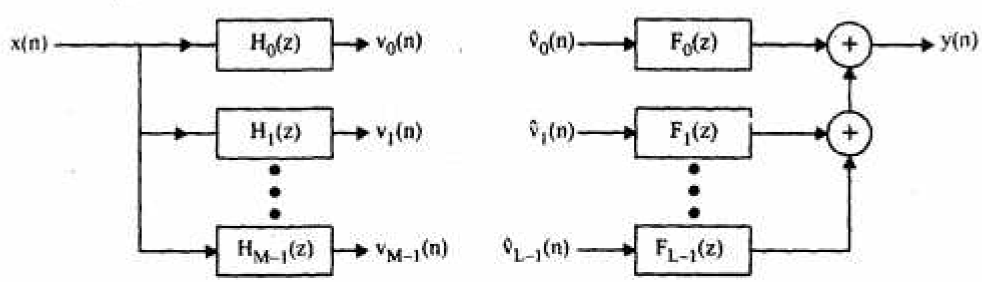
$$a_2 = P_1 P_2$$

$$|a_2| = |P_1 P_2| = |P_1||P_2| < |1$$

$$|a_1| < |a_2|$$

(c) Digital Filter Banks : In some application like spectrum it is required to separate a signal into a set of sub-band signals. There are two application where these sub-band signals have to be combined and represented as a single composite signal. For these application digital filter banks are used. The digital filter banks can be represented as a set of digital bandpass filter that can have either a common input or as a summed output. The signal $x(n)$ can be decomposed into a set of M sub-band signals $V_K(n)$ with the M-band analysis filter bank, each sub-band signal $V_K(n)$ occupies a portion of the original frequency band by filtering process done with the sub fitted $H_K(z)$ also known as the analysis filters.

Similarly the sub-band signals $\hat{y}_K(n)$ which belongs to contiguous frequency bands can be combined to get $y(n)$ by mean of the synthesis filter bank. This can be done by combining various synthesis filter $F_K(z)$



(a) (b)