Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-III Remedial Examination May 2011

Subject code: 130001 **Subject Name: Mathematics-III**

Date: 31-05-2011 **Time: 10.30 am – 01.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as Directed

i) Solve the Euler-Cauchy equation $x^2y'' - 2.5xy' - 2.0y = 0$ **2**

ii) Find the Laplace transforms of $\left[\dfrac{\sin wt}{t}\right]$ **2**

iii) Solve $y' = 2xy$ by power series method. **2**

iv) Find the Fourier cosine and sine transforms of the function **2**

$$f(x) = \begin{cases} k & if\ 0 < x < a \\ 0 & if\ x > a \end{cases}$$

v) Write duplication formula. Use it to find the value of $\left|\frac{1}{4}\right. \left|\frac{3}{4}\right.$ **2**

vi) Write Abel-Liouville formula. Use it to check that the set $\{x, x^2, x\log|x|\}$ is a basis for some third order linear ordinary differential equation. **2**

vii) Obtain $L^{-1}\left\{\log\frac{1}{s}\right\}$ **2**

Q.2 (a) i) Find the order and degree of the differential equation **1**

$$\left[\frac{dy}{dx} + y\right]^{\frac{1}{2}} = \sin x$$

ii) To solve heat equation $\dfrac{\partial u}{\partial t} = c^2\dfrac{\partial^2 u}{\partial x^2}$. How many initial and boundary conditions requires. **2**

iii) Evaluate $\int_{-1}^{1} p_3(x)p_2(x)dx$ **2**

iv) Prove that $\dfrac{\beta(m+1,n)}{\beta(m,n)} = \dfrac{m}{m+n}$ **2**

	(b)	**i)**	Solve $(D^2+a^2)y=\cos ec ax$	**4**
		ii)	Solve $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x^2}$	**3**
			OR	
	(b)	**i)**	Solve $\left(D^4+2a^2D^2+a^4\right)y=\cos ax$	**4**
		ii)	Obtain the second linearly independent solution of $xy''+2y'+xy=0$ given that $y_1(x)=\frac{\sin x}{x}$ is one solution.	**3**
Q.3	**(a)**		Solve $x^3y'''+2x^2y''+2y=10\left(x+\frac{1}{x}\right)$	**4**
	(b)		Solve the initial value problem by method of undetermined coefficients $y'''+3y''+3y'+y=30e^{-x}, y(0)=3, y'(0)=-3, y''(0)=-47,$	**4**
	(c)		Solve the simultaneous equations: Using Laplace transform $\frac{dx}{dt}-y=e^t$, $\frac{dy}{dt}+x=\sin t$ given $x(0)=1, y(0)=0$	**6**
			OR	
Q.3	**(a)**		Solve $x^2y''-4xy'+6y=21x^{-4}$	**4**
	(b)		Solve the nonhomogeneous Euler-Cauchy equation $x^3y'''-3x^2y''+6xy'-6y=x^4\log x$ by Variation of parameters method.	**4**
	(c)	**i)**	Find the Laplace transform of the function $f(t)=\lvert\sin wt\rvert, t\geq 0$	**3**
		ii)	Using Convolution theorem, find the inverse Laplace transform of $\frac{1}{\left(s^2+a^2\right)^2}$	**3**
Q.4	**(a)**		Express $x^4-2x^3+3x^2-4x+5$ in terms of Legendre's polynomials, by using Rodrigue's Formula.	**4**
	(b)	**i)**	Find the generalized Fourier series expansion of $f(x),\ 0<x<3$ arising from the eigenfunction of $y''+\lambda y=0,\ \ 0<x<l;\ \ y'(0)=0,\ \ y'(l)=0$	**4**
		ii)	Obtain the value of $J_{3/2}(x)$	**3**
	(c)		Show that $\int_{-1}^{1}\frac{p_n(x)}{\sqrt{1-2xt+t^2}}dx=\frac{2}{2n+1}t^n$	**3**
			OR	
Q.4	**(a)**		Find Power series solution of the equation $(1-x^2)y''-xy'+py=0$, p is an arbitrary constant.	**4**
	(b)		Find the series solution of $xy''+y'+xy=0$	**7**
	(c)		Find the power series solution of the equation $(x^2+1)y''+xy'-xy=0$ about an ordinary point.	**3**

Q.5 **(a)** Find the Fourier transform of $f(x)=\begin{cases}1-x^2 & if\ |x|<1\\ 0 & if\ |x|>1\end{cases}$ and use it to evaluate $\int_0^\infty \left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}\,dx$ **4**

(b) Find the Fourier series for $f(x)=|\sin x|$ in $-\pi<x<\pi$ **4**

(c) If the string of length L is initially at rest in equilibrium position and each of its points is given the velocity. $u_0\sin\left(\frac{3\pi x}{L}\right)\cos\left(\frac{2\pi x}{L}\right)$ Where $0\le x\le L$ at $t=0$, determine the displacement $u(x,t)$. **6**

OR

Q.5 **(a)** Find half –rang cosine series for $f(x)=\begin{cases}x, & 0<x<\frac{\pi}{2}\\ \pi-x, & \frac{\pi}{2}<x<\pi\end{cases}$ **4**

(b) Find Fourier series for the function $f(x)$ given by **4**

$$f(x)=\begin{cases}1+\frac{2x}{\pi}; & -\pi\le x\le 0\\ 1-\frac{2x}{\pi}; & 0\le x\le\pi\end{cases}$$

Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$

(c) A rod 30 cm long has its end A and B kept 20^0C and 80^0C respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to 0^0C and kept so. Find the resulting temperature function $u(x,t)$ from the end A. **6**

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