Seat No.: ________ **Enrolment No.**____________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEM-III Examination-Dec.-2011

Subject code: 130001 **Date: 22/12/2011**

Subject Name: Mathematics-III

Time: 2.30 pm -5.30 pm **Total marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Follow usual conventions.**

Q. 1 (a) Do as directed :

1 Verify that $y=e^x$ (a cos x + b sin x) is a solution of $y'' + 2y' + 2y = 0$, where a and b are constatnts. (2)

2. Solve : $9yy' + 4x = 0$ (2)

3. Verify that the functions $x^{-1/2}$ and $x^{3/2}$ form a basis of solutions of $4x^2 y'' - 3y = 0$; and solve it when $y(1)=3$, $y'(1) = 2.5$ (3)

(b) Do as directed :

1. Find the Laplace Transform of (2)

$$f(t) = 0, \quad 0 \le t < 2$$
$$= 3, \quad \text{when } t \ge 2$$

2. Find the general solution of $(D^2+1)\, y = 0$. (2)

3. Show that cos mx and sin nx are orthogonal on $-\pi \le x \le \pi$, where m and n are integers. (3)

Q. 2 (a) Do as directed :

1. Show that $u = \sin 9t \sin (1/4)\, x$ is a solution of a one dimensional wave equation. (2)

2. Determine if x=1 is a regular singular point of $(1-x^2)\, y'' - 2xy' + n(n+1)\, y = 0$, where n is a constant. (2)

3. Show that $\Gamma(m+1) = m!$, where Γ is the Gamma function and m is positive integer. (3)

(b) Do as directed :

1. Solve the IVP : $xy' + y = 0$, $y(2) = -2$. (2)

2. Solve the Bernoulli equation $y' + y \sin x = e^{\cos x}$ (3)

3. Solve : $y'' + 4y' + 4y = 0$, $y(0)=1$, $y'(0)=1$ (2)

OR

(b) Do as directed :

1. Test for exactness and solve : $[(x+1)e^x - e^y]\, dx - x e^y\, dy = 0$, $y(1)=0$ (3)

2. Find the general solution : $16y'' - 8y' + 5y = 0$ (2)
3. Solve : $(x^2D^2 - 3xD + 4)\, y = 0$, $y(1)=0$, $y'(1)=3$ (2)

Q. 3 (a) Solve the non-homogeneous equations (7)
1. $y'' - 3y' + 2y = e^x$
2. $y'' + y = \sec x$.

(b) Do as directed :
1. Define the terms : Laplace Transform of f(t) , and its Inverse Transform. (2)
2. Using the Beta and Gamma functions evaluate the integral (3)

$$\int_{-1}^{1} (1-x^2)^n \, dx, \text{ where n is a positive integer.}$$

3. Find the Laplace Transform of $\cos^2(at)$, where a is a constant. (2)

OR

Q. 3 (a) Do as directed :
1. Find the general solution : $y''' - 3y'' + 3y' - y = 4e^t$ (4)
2. Solve : $y''' - y'' + 100y' - 100y = 0$, $y(0)=4$, $y'(0)=11$, $y''(0)= -299$ (3)

(b) Do as directed :
1. Find the Laplace Transform of $f(t)=\sinh(\omega t)$, $t \geq 0$ (3)
2. Find the Laplace Transform of (4)
$(5s^2 + 3s - 16)/\{(s-1)(s-2)(s+3)\}$

Q. 4 (a) Do as directed :
1. Solve the IVP using the Laplace Transform : $y'' + 4y = 0$, $y(0)=1$, $y'(0)=6$. (3)
2. Find the Inverse Laplace Transform of $(6+s)/(s^2+6s+13)$, use Shifting theorem. (4)

(b) Do as directed
1. Find a power series solution in powers of x of $y' + 2xy = 0$ (4)
2. Derive the Legendra Polynomials $P_0(x)=1$ and $P_2(x)=(1/2)(3x^2-1)$ from the Rodrigue's formula. (3)

OR

Q. 4 (a) Do as directed :
1. Applying the Binomial theorem to $(x^2-1)^n$ and differentiating n times – or by any other method - derive the Rodrigue's formula (3)

$$P_n(x) = 1/(2^n\, n!)\; d^n / dx^n \; [\,(x^2-1)^n\,]$$

2. Find a series solution of $y'' + y = 0$ near $x=0$ (4)

(b) Do as directed :
1. Solve by Frobenious method at x=0 : $x(x-1)y'' + (3x-1)y' + y = 0$ (4)
2. What is the Bessel's function $J_\upsilon(x)$ of the first kind? Write the formula. And show that $J_0'(x) = -J_1(x)$ (3)

Q. 5 (a) Find the Fourier series expansions of

1 $f(x)=x, \quad -\pi \le x \le \pi, \; f(x+2\pi) = f(x)$ (3)

2 $f(x)=x^2, \; -2 \le x \le 2$ (4)

(b) Derive the one dimensional wave equation that governs small vibrations of an elastic string. Also state physical assumptions that you make for the system. (7)

OR

Q. 5 (a) Do as directed

1 Define the terms : Fourier Transform and its Invertse. Give details. (3)

2 Find the Fourier Transform of e raised to $-ax^2$, where $a>0$ (4)

(b) Derive the expression for the Laplacian operator in cylindrical coordinates from its expression in rectangular coordinates. (7)

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