Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-III Regular / Remedial Examination December 2010

Subject code: 130001 **Subject Name: Mathematics – 3**

Date: 11 /12 /2010 **Time: 10.30 am – 01.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as directed. **14**

a) Solve : $xy' = y^2 + y$

b) Find a second order homogeneous linear differential equation for which the functions x^2, $x^2 \log x$ are solutions.

c) Find the convolution of t and e^t.

d) Evaluate : $\int_0^1 x^4 \left[\log\left(\frac{1}{x}\right)\right]^3 dx$

e) Solve : $y'' + 2y' + 2y = 0,\quad y(0) = 1,\quad y\left(\frac{\pi}{2}\right) = 0$.

f) Find $L^{-1}\left\{\frac{1}{\left(s+\sqrt{2}\right)\left(s-\sqrt{3}\right)}\right\}$.

g) Compute : $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$

Q.2 **(a)** Using the method of variation of parameters find the general solution of the differential equation $\left(D^2 - 2D + 1\right)y = 3x^{\frac{3}{2}} e^x$. **05**

(b) Attempt all. **09**

1) Solve the initial value problem $y' - \left(1 + 3x^{-1}\right)y = x + 2$, $y(1) = e - 1$.

2) Find the orthogonal trajectories of the curve $y = x^2 + c$.

3) Find a basis of solution for the differential equation $x^2 y'' - xy' + y = 0$, if one of its solutions is $y_1 = x$.

OR

(b) Attempt all. **09**

1) Solve : $y' + \frac{1}{3} y = \frac{1}{3}(1 - 2x)x^4$.

2) Solve the initial value problem $L\frac{dI}{dt} + RI = 0,\quad I(0) = I_0$, where R ,L and I_0 being constants.

3) Prove that $\int_0^1 \frac{xdx}{\sqrt{1 - x^5}} = \frac{1}{5}\beta\left(\frac{2}{5}, \frac{1}{2}\right)$.

Q.3 **(a)** Using Laplace transforms solve the initial value problem $y'' + y = \sin 2t$, $y(0) = 2, y'(0) = 1$. **05**

(b) Find the Fourier cosine series of the periodic function $f(x)=x;(0<x<L), p=2L$. Also sketch $f(x)$ and its periodic extension. **05**

(c) Using the method of undetermined coefficient, find the general solution of the differential equation $y''+2y'+10y=25x^2+3$. **04**

OR

Q.3 **(a)** Find the Fourier series of the periodic function $f(x)=\pi\sin\pi x$, $(0<x<1)$, $p=2L=1$. **05**

(b) Solve the initial value problem $y''+4y=8e^{-2x}+4x^2+2$, $y(0)=2$, $y'(0)=2$. **05**

(c) Find the complex Fourier series of the function $f(x)=x$, $(0<x<2\pi)$, $p=2L=2\pi$. **04**

Q.4 **(a)** Find a series solution of the differential equation $x^2y''+x^3y'+(x^2-2)y=0$ by Frobenious method. **06**

(b) Find the Laplace Transforms of **04**

1) $t^2\sin\pi\, t$ 2) $e^t u(t-2)$

(c) Find the inverse Laplace Transformation of **04**

1) $\dfrac{se^{-2s}}{s^2+\pi^2}$ 2) $\log\dfrac{s+a}{s+b}$

OR

Q.4 **(a)** Attempt all. **06**

1) Express $f(x)=x^3+x+1$ in terms of Legendre's polynomials.

2) Show that $\int_{-1}^{1}P_m(x)P_n(x)dx=0,\ if\,m\neq n.$

(b) Find the general solution of the equation $(x^2D^2-2xD+2)y=x^3\cos x$. **04**

(c) State Convolution theorem and use to evaluate $L^{-1}\left\{\dfrac{1}{(s^2+\omega^2)^2}\right\}$. **04**

Q.5 **(a)** Using the method of separation of variables, solve the partial differential equation $u_{xx}=16u_y$. **06**

(b) Show that $\displaystyle\int_0^\infty \frac{\cos x\omega+\omega\sin x\omega}{1+\omega^2}d\omega=\begin{cases}0; if x<0\\ \dfrac{\pi}{2}; if x=0\\ \pi e^{-x}; if x>0\end{cases}$ **05**

(c) Prove that $J_1'(x)=J_0(x)-\dfrac{1}{x}J_1(x)$. **03**

OR

Q.5 **(a)** Using Laplace transform, find the solution of the initial value problem $x\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial t}=xt$, $u(x,0)=0; if x\geq 0$, $u(0,t)=0, if t\geq 0$. **06**

(b) Find the Fourier Transforms of the Function $f(x)=\begin{cases}xe^{-x}; if x>0\\ 0; if x<0\end{cases}$. **05**

(c) Show that $P_n(-x)=(-1)^nP_n(x)$. Hence find $P_n(-1)$. **03**

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