B.E.

Seventh Semester Examination, 2009-2010

Digital Signal Processing (EE-407-E)

Note : Attempt any *five* questions.

Q. 1. (a) State and prove the Parseval's energy theorem for discrete time signals.

Ans. For a non-periodic energy signal, such as a single pulse, the total energy in $(-\infty, \infty)$ infinite, whereas the average power i.e., energy per unit times is zero be¬ause 1/T tends to zero as T tends to infinity. Hence, the total energy associated with f(t) is given by,

$$E = \int_{-\infty}^{\infty} f^2(t)\,dt$$

Since, $f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega$, we obtain

$$E = \int_{-\infty}^{\infty} f(t) + \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}\,d\omega\,dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\left[\int_{-\infty}^{\infty} f(t)e^{j\omega t}\,dt\right]d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega).F(-j\omega)\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega).F^{*}(j\omega)\,d\omega$$

$$= \int_{-\infty}^{\infty} \left|F(t)^2\right| dt \text{ joules}$$

$$E = \int_{-\infty}^{\infty} |f(t)|^2\,df = \int_{-\infty}^{\infty} |F(t)|^2\,df$$

This result is called Rayleigh's energy theorem or Parseval's theorem for Fourier transform.

For complex-valued sequences x(n) and y(n)

If $x(n) \xleftrightarrow[N]{DFT} X(k)$ and $y(n) \xleftrightarrow[N]{DFT} Y(k)$

Then, $$\sum_{n=0}^{N-1} x(n)y^{*}(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^{*}(k)$$

If $y(n) = x(n)$, then the above equation reduces to

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2 .$$

Q. 1. (b) Give the different type of signal classifications.

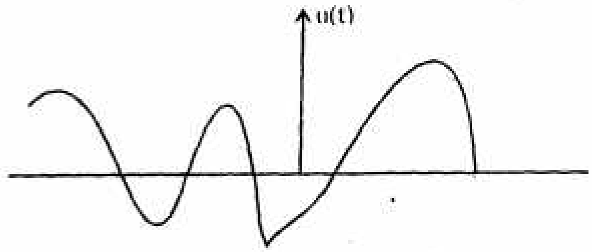
Ans. Signals can be classified based on their nature and characteristics in the time domain. Broadly classified as :

(i) Continuous time signals

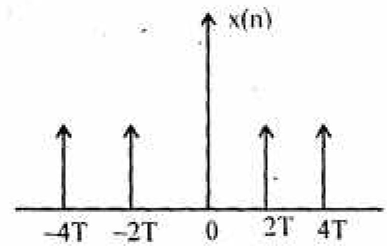
(ii) Discrete time signals.

Continuous time signal is a mathematically continuous function and the function is defined continuously in the time domain.

On the other hand, a discrete time signal is specified only at certain time instants.



Continuous Time Signal



Discrete Time Signal

Both continuous time and discrete time signal are further classified as :

(i) Periodic and aperiodic signal.

(ii) Even and odd signals.

(iii) Energy and power signals and

(iv) Deterministic and non-deterministic signals.

Periodic and Aperiodic Signals : An arbitrary signal x(t) is said to be periodic if it repeat itself after a period of time 'T' where 'T' fundamental period of the signal.

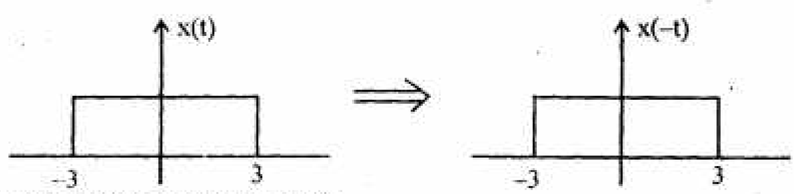
If signal not repeat itself after the fundamental period 'T' then signal is Aperiodic signal.

If, $\boxed{x(t+T)=x(t)}$ then signal is periodic.

If, $\boxed{x(t+T)\neq x(t)}$ then aperiodic signal.

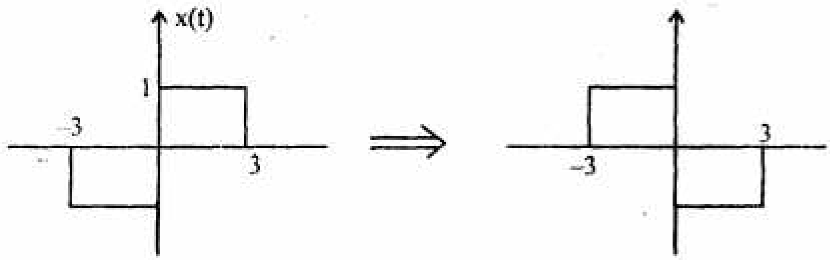
Even & Odd Signals : An arbitrary signal x(t) is said to be 'even', if

$$\boxed{x(-t)=x(t)}$$



An arbitrary signal x(t) is said to be 'odd' if

$$\boxed{x(-t)=-x(t)}$$



Power & Energy Signals : The energy of an arbitrary signal x(t) is given by

$$\boxed{E_X = \int_{-\infty}^{\infty} |x(t)|^2 \, dt}$$

The power of an arbitrary signal x(t) is given by,

$$P_x = \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$T \to \infty$$

Where E_x is finite, then $P_x = 0$

Where P_x is finite, $E_x = \infty$.

Deterministic & Non-Deterministics : Deterministics signals are function of that one completely specified in time. The nature and amplitude of such a signal at any time can be predicted the pattern of the signal is regular and can be characterised mathematically.

A non-deterministic signal is one whose occurrence in random is nature and its pattern is quite irregular. A typical example of a non-deterministics signals is thermal noise in an electrical circuit. Non-deterministics signals are also called random signals.

Q. 2. (a) Explain the causal & non-causal system. What is the physical significance of causal system?

Ans. Causal System & Non-Causal System : A system is said to be causal if the output at any time interval depends only on present and/or past values of the input. Otherwise the system is said to be non-causal or anticipatory. Non-causal system are not physically realize.

Example :

$$\left.\begin{array}{r} y(t) = x(t-2) \\ y(0) = x(-2) \\ y(4) = x(2) \\ y(-4) = x(-6) \end{array}\right\} \text{Causal}$$

System is depends only on present and past values, so the given system is causal system.

$$\left.\begin{array}{r} y(t+2) = x(t+3) \\ y(0+2) = x(0+3) \\ y(3) = x(4) \end{array}\right\} \text{Non-causal}$$

System depends upon past value of the system so, the given system is non-causal system.

Q. 2. (b) What is the relation between z-transform and Fourier transform?

Ans. Relation between Fourier Transform and Z-Transform : Let X(z) be the z-transform for a sequence x(n) which is given by,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

With a ROC that includes the unit circle. If X(z) is sampled at the N equally spaced points on the unit circle.

$$Z_k = e^{j2\pi nk/N},\ k_1 = 0, 1, 2, \ldots\ldots\ldots N\text{-}1 \text{ then}$$

$$X(k) = X(z)\big|_{Z=e^{j2\pi k/N}},\ k = 0, 1, \ldots\ldots, N-1$$

$$= \sum_{-\infty}^{\infty} x(n) e^{-j2\pi k/N}$$

This is identical to the Fourier transform $X\left(e^{j\omega}\right)$ evaluated at the N equally spaced frequencies.

$$\omega_k = 2\pi k / N,\ k = 0, 1, \ldots\ldots N-1$$

If the sequence x(n) has a finite duration of length N, then the z-transform is given by,

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n}$$

Substituting the IDFT relation for x(m), we have

$$X(z) = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j2\pi nk/N}\right] z^{-n}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k) \sum_{n=0}^{N-1}\left(e^{j2\pi k/N} z^{-1}\right)^n$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k) \frac{1-z^{-N}}{1-e^{j2\pi k/N} z^{-1}}$$

$$= \frac{1-z^{-N}}{N}\sum_{k=0}^{N-1} \frac{X(k)}{1-e^{j2\pi k/N} z^{-1}}$$

The above equation is identified to that of frequency sampling form. When this is evaluated over a unit circle, then

$$X\left(e^{j\omega}\right)=\frac{1-e^{-j\omega N}}{N}\sum_{k=0}^{N-1}\frac{X(k)}{1-e^{j(\omega-2\pi k/N)}}$$

Q. 2. (c) Determine the response of the LTI system with difference equation.

$y(n)-2r\cos\theta y(n-1)+r^2y(n-2)=k(n)$ to an excitation $x(n)=a^n u(n)$.

Ans. We know that response of the system,

$$\boxed{H(z)=\frac{Y(z)}{X(z)}}$$

$$\Rightarrow y(n)-2r\cos\theta y(n-1)+r^2y(n-2)=k(n) \qquad \text{...(i)}$$

Taking the z-transform of the equation (ii)

$$=Y(z)-2r\cos\theta z^{-1}y(z)+r^2z^{-2}y(z)=k(z)$$

$$=Y(z)\left[1-2r\cos\theta z^{-1}+r^2z^{-2}\right]=k(z)$$

$$=Y(z)=\frac{k(z)}{\left(1-2r\cos\theta z^{-1}+r^2z^{-2}\right)} \qquad \text{...(ii)}$$

Now given that, $x(n)=a^n u(n)$, taking z-transform,

$$X(z)=\frac{1}{1-az^{-1}}$$

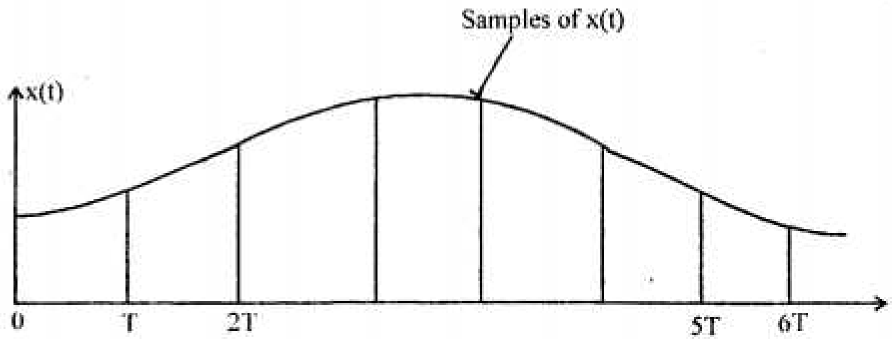
Response :

$$H(z)=\frac{Y(z)}{X(z)}=\frac{\dfrac{k(z)}{\left(1-2r\cos\theta z^{-1}+r^2z^{-2}\right)}}{1/1-az^{-1}}$$

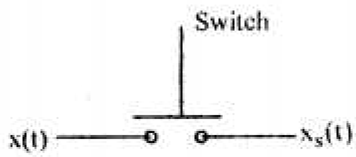
$$\boxed{H(z)=\frac{k(z)\left(1-az^{-1}\right)}{1-2r\cos\theta z^{-1}+r^2z^{-2}}}$$

Q. 3. Explain the frequency domain representation of sampling of any signal. Discuss the discrete time processing of continuous time signals.

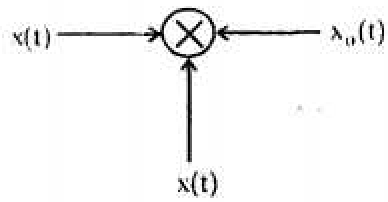
Ans. Sampling of continuous signal is converted into a discrete time. The continuous time signal x(t) must be sampled in such a way that the original signal can be reconstructed from these samples. Otherwise, the sampling process is useless. Let us obtain the condition necessary to faithfully reconstruct the original signal from the samples of that signal. The condition can be easily obtained if the signals are analysed in the frequency domain.



(a)



(b)



(c)

$$\Rightarrow \qquad x_s(t) = x(t)\sum_{n=-\infty}^{\infty} c_n e^{j2\pi f_s t}$$

$$= \sum_{n=-\infty}^{\infty} c_n x(t) e^{j2\pi f_s t}$$

The spectrum of $x_s(t)$ denoted by $X_s(t)$ can be determined by taking the Fourier transform,

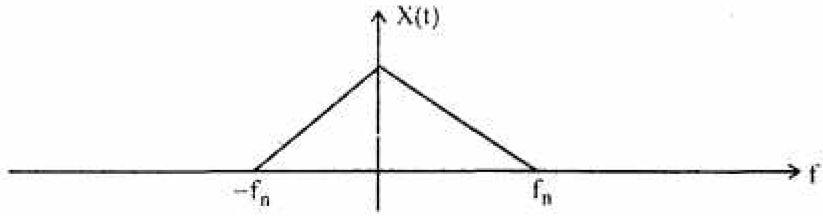
$$X_s(t) = \int_{-\infty}^{\infty} x_s(t) e^{-j2\pi f_s t}\, dt$$

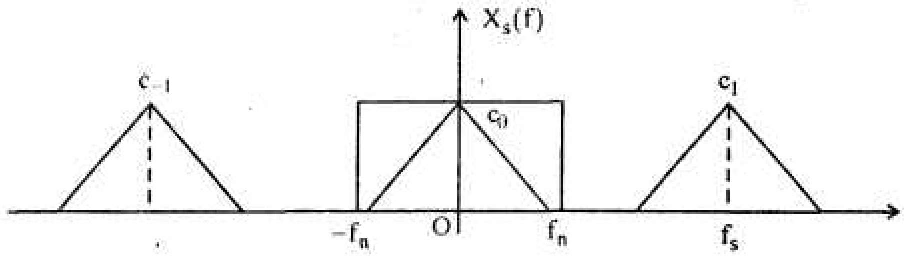
$$X_s(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c_n x(t) e^{j2\pi f_s t} . e^{-j2\pi ft}\, dt$$

$$= \sum_{n=-\infty}^{\infty} c_n \int_{-\infty}^{\infty} x(t) e^{-j2\pi(f-nf_s)t}\, dt$$

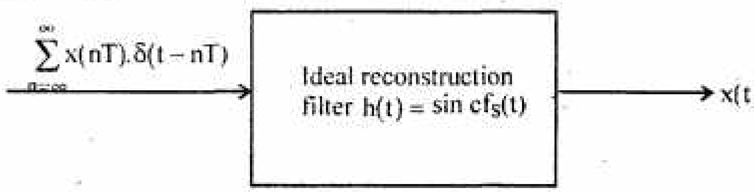
But, $$\int_{-\infty}^{\infty} x(t) e^{-j2\pi(f-nf_s)t} dt = X(f-nf_s)$$

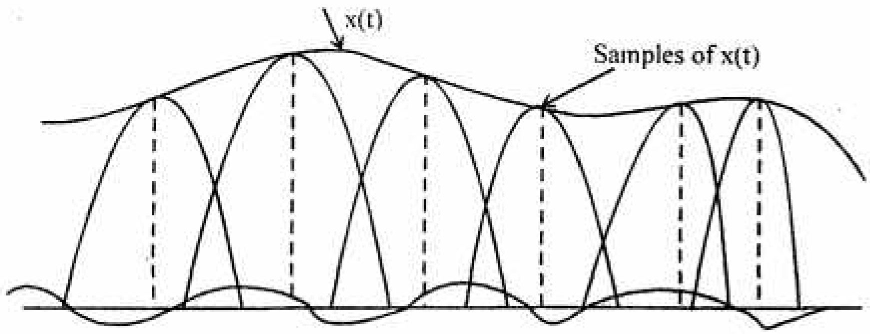
Thus, $$\boxed{X_s(f) = \sum_{N=n}^{\infty} c_n X(f-nf_s)}$$





Signal Reconstruction :





The ideal reconstruction filter used in this method. The input to this filter is the sampled signal x(n+1) and the output. After filter is the reconstructed signal x(t).

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT)\, h(t-nT)$$

$$\boxed{x(t) = \sum_{n=\infty}^{\infty} x(nT) \sin cfs(t-nT)}.$$

Q. 4. (a) Determine the z-transform of the following x(n) with ROC.

$$x(n) = a^n u(n) + b^n u(-n-1)$$

Ans. $x(n) = a^n u(n) + b^n(-n-1)$

Taking the z-transform of above :

$$x(n) = a^n u(n) + b^n(-n-1)$$

$$a^n u(n) \xrightarrow{z} \frac{1}{1-az^{-1}}$$

$$= \frac{z}{z-a} \qquad \text{ROC:} -|z| > |a|$$

$$b^n(-n-1) \xrightarrow{z} \cdot \xrightarrow{\frac{z}{1-bz}}, \qquad \text{ROC:} |z| < \left|\frac{1}{b}\right|$$

Now combining the both,

$$X(z) = \frac{1}{1-az^{-1}} + \frac{z}{1-bz} = \frac{z}{z-a} + \frac{z}{1-bz}$$

$$\boxed{\text{ROC:} |a| < |z| < \frac{1}{|b|}}$$

Q. 4. (b) Discuss the properties of the z-transform. Also give the prove.

Ans. Properties of the z-transform :

(a) Linearity Property :

$$x_1(n) \xrightarrow{z} X_1(z); \qquad \text{ROC} = R_1$$

$$x_2(n) \xrightarrow{z} X_2(z); \qquad \text{ROC} = R_2$$

$$ax_1(n) + bx_2(n) \xrightarrow{z} aX_1(z) + bX_2(z)$$

ROC : Containing $R_1 \cap R_2$

(b) Time Shifting :

$$x(n) \xrightarrow{z} X(z); \text{ROC} = R$$

$$x(n-n_0) \xrightarrow{z} z^{-no} X(z); \text{ROC} = R$$

(c) Time Reversal Property :

$$x(n) \xrightarrow{z} X(z)\,;\, ROC = R$$

$$x(-n) \xrightarrow{z} X\left(z^{-1}\right);\, ROC = \frac{1}{R}$$

(d) Scaling in Z Domain :

$$x(n) \xrightarrow{z} X(z)\,;\, ROC = R$$

$$z_0^n x(n) \xrightarrow{z} X\left(\frac{z}{z_0}\right);\; ROC = |z_0|R$$

(e) Convolution Property :

$$x_1(n) \xrightarrow{z} X_1(z);\;\; ROC = R_1$$

$$x_2(n) \xrightarrow{z} X_2(z);\;\; ROC = R_2$$

$$x_1(n) * x_2(n) \xrightarrow{z} X_1(z).X_2(z)\,;\, ROC : \text{Contain } R_1 \cap R_2.$$

(f) Differentiation and Accumulation :

$$x(n) \xrightarrow{z} X(z)\,; \qquad ROC = R$$

$$x(n) - x(n-1) \to X(z) - z^{-1}X(z)$$

$$= \left(1 - z^{-1}\right)X(z)$$

$$= \frac{z-1}{z}X(z)\,; \qquad ROC : \text{Containing } |z| > 0 \cap R$$

(g) Differentiation in Z Domain :

$$x(n) \xrightarrow{z} X(z)\,;\, ROC = R$$

$$nx(n) \to \frac{-zdX(z)}{dz}\,;\, ROC = R$$

Q. 5. (a) Compare FIR and IIR Filters.

Ans. A discrete-time filter produces a discrete time output sequence y(n) for the discrete-time input sequence x(n). A filter may be required to have a given frequency response or a specific response to an impulse, step or ramp, or simulate an analog system.

Digital filters are classified either as finite duration unit pulse response (FIR) filter or infinite duration unit response (IIR) filters; depending on the form of the unit pulse response of the system.

FIR Filter	IIR Filter
(i) In the FIR system, the impulse response sequence is of finite duration i.e., it has a finite number of non-zero term.	(i) The IIR system has an infinite number of non-zero terms i.e., its impulse response sequence is of infinite duration.
(ii) FIR filters are usually implemented using structures with no feedback (non-recursive structures-all zeros).	(ii) IIR filters are usually number of non-zero terms and is an IIR feedback (recursive structures-poles and zeros).
(iii) FIR filter depends only on the present and past input samples.	(iii) IIR filters depends upon present response is a function of the present and past N values of the excitation as well as past values of the response.

FIR filter have the following advantages over IIR :

(i) They can have an exact linear phase.

(ii) They are always stable.

(iii) The design methods are generally linear.

(iv) They can be realized efficiently in hardware.

(v) The filter start-up transients have finite duration.

FIR filters are employed in filtering problems where linear phase characteristics within the passband of the filter is required. If this is not required, either an IIR or an FIR filters may be employed. An IIR filters has lesser number of parameter. For this reason if some phase distortion is tolerable, an IIR filter is preferable. Also, the implementation of an IIR filter involves fever parameters, less memory requirements and lower computational complexity.

Q. 5. (b) What is finite word length effect in DSP?

Ans. Following are some of the issues connected with finite word length effects :

(i) Quantisation effects in analog-to-digital conversion.

(ii) Product quantisation and coefficient quantisation errors in digital filters.

(iii) Limit cycles in IIR filters, and

(iv) Finite word length effects in fast Fourier transform.

(i) Truncation Error for Sign Magnitude Representation : Whether input number x is positive, truncaction results in reducing the magnitude of the number. Thus, the truncation error is negative and the range is given by

$$-\left(2^{-B}-2^{-L}\right)\le \varepsilon_T \le 0 \qquad \text{...(i)}$$

The largest error occurs when all the discarded bits are one's. When the number x is negative truncation results in reduction of the magnitude only. For example, let the number be $x=-0.375$, that is, in sign magnitude form it is represented as x = 1011 and after truncation of one bit Q(x) = 101. This is equivalent to –0.25 in decimal. But –0.25 greater than –0.375, therefore, the truncation error is positive and its range is,

$$0\le \varepsilon_T \le \left(2^{-B}-2^{-L}\right)$$

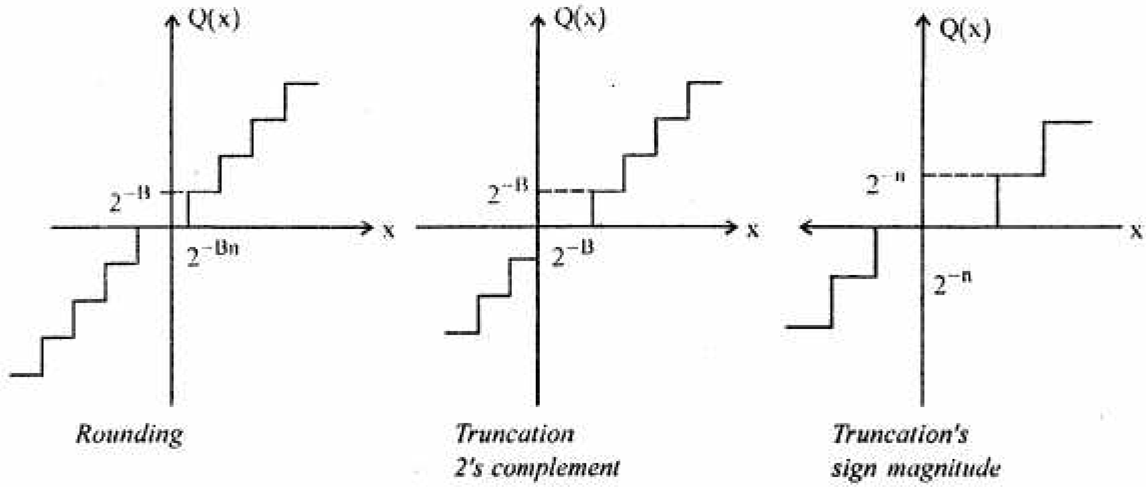
(ii) Truncation Error for Two's Complement Representation : When the input number is positive, truncation results in a smaller number, as in the case of sign magnitude numbers. Hence, the truncation error is negative and its range is same. If the number is negative, truncation of the numbers in two's complement form result in a smaller number and the error is negative. Thus, the complete range of the truncation error for the two's complement representation is,

$$\boxed{-\left(2^{-B}-2^{-L}\right) \le \varepsilon_T \le 0}$$

(iii) Round-off Error for Sign Magnitude and Two's Compplement Representation : The rounding of a binary number involves only the magnitude of the number and is independent of the type of fixed-point binary representation. The error due to rounding may be either positive or negative and the peak value is $\frac{\left(2^{-B}-2^{-L}\right)}{2}$, the round off-error is symmetric about zero and its range is,

$$-\frac{\left(2^{-B}-2^{-L}\right)}{2} \le \varepsilon_R \le \frac{\left(2^{-B}-2^{-L}\right)}{2}$$

ε_R = Round off error.



Q. 6. (a) Explain the design of IIR filter using impulse invariance method.

Ans. IIR Filter Using Impulse Invariance Method : In this technique, the desired impulse response of the digital filter is obtained by uniformly sampling the impulse response of the equivalent analog filter that is,

$$h(n) = ha(nT) \quad \text{...(i)}$$

Where T is the sampling interval. The transformation technique can be well understood by first considering a simple distinct pole case for the analog's filter's system function, as shown below,

$$H_a(s) = \sum_{i=1}^{M} \frac{A_i}{S-P_i} \quad \text{...(ii)}$$

The impulse response of the system specified by equation (ii) can be obtained by taking the inverse Laplace transformation and it will be of the form.

$$h_a(t) = \sum_{i=1}^{M} A_i e^{P_i t} u_a(t) \quad \text{...(iii)}$$

Where $u_a(t)$ is the unit step function in continuous time. The impulse response h(n) of the equivalent digital filter is obtained by uniformly sampling $h_a(t)$, i.e., by applying equation (i)

$$h(n) = h_a(nT) = \sum_{i=1}^{M} A_i e^{P_i nT} u_a(nT) \quad \text{....(iv)}$$

The system response of the digital system of equation (iv) can be obtained by taking the z-transform i.e.,

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

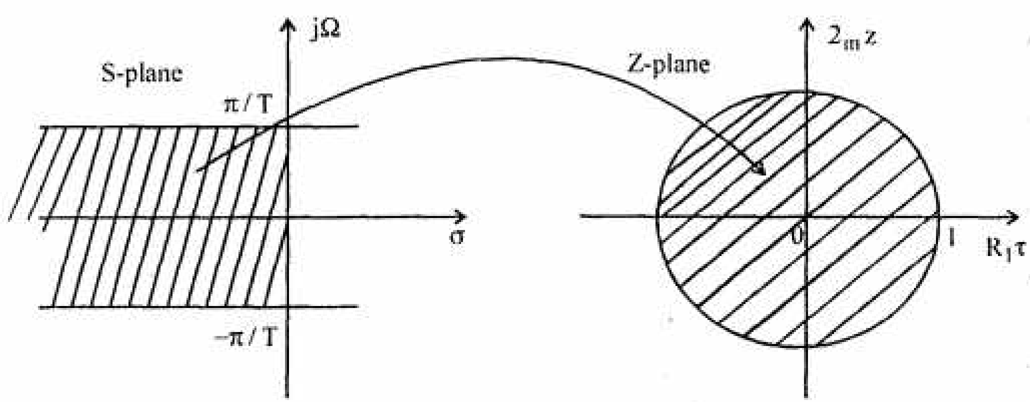
Using equation (iv)

$$H(z) = \sum_{n=0}^{\infty} \left[\sum_{i=1}^{M} A_i e^{P_i nT} u_a(nT) \right] z^{-n} \quad \text{...(v)}$$

Interchanging the order of summation,

$$H(z) = \sum_{i=1}^{M} \left[\sum_{n=0}^{M} A_i e^{P_i nT} u_a(nT) \right] z^{-n} \quad \text{...(vi)}$$

$$H(z) = \sum_{i=1}^{M} \frac{A_i}{1 - e^{P_i T} z^{-1}}$$



Mapping of $z = e^{-S_T}$

Now, by comparing equations (ii) & (vi), the mapping formula for the impulse invariant transformation is given by,

$$\frac{1}{S - P_i} \rightarrow \frac{1}{1 - e^{P_i T} z^{-1}} \quad \text{...(vii)}$$

Equation (vii) shows that the analog pole at $S = P_i$ is mapped into a digital pole at $Z = e^{P_i T}$. Therefore the analog poles and the digital poles are related by the relation.

$$\boxed{z = e^{ST}}$$

Q. 6. (b) Draw and explain the frequency sampling structure for FIR filter.

Ans. Frequency Sampling Method : In this method, a set of samples is determined from the desired frequency response and are identified as discrete Fourier transform (DFT) coefficients. The inverse discrete Fourier transform (IDFT) of this set of samples then gives the filter coefficients. The set of sample points used in this procedure can be determined by sampling a desired frequency response $H_d\left(e^{j\omega}\right)$ at m points $\omega_k = 0$, 1, ... M–1, uniformly spaced around the unit circle.

Type-I Design : The samples are taken at the frequency

$$\omega_k = \frac{2\pi k}{M} = 0, 1, \ldots\ldots M-1$$

The samples of the desired frequency response at these frequency are given by,

$$\tilde{H}(k) = H_d\left(e^{j\omega}\right)\Big|_{\omega=\omega_k}, k = 0, 1 \ldots\ldots M-1$$

$$= H_d\left(e^{j2\pi k/m}\right), k = 0, 1, \ldots\ldots M-1$$

This set of points can be considered as DFT samples, then the filter coefficients h(n) can be computed using the IDFT.

$$h(n) = \frac{1}{M}\sum_{k=0}^{M-1}\tilde{H}(k)e^{j2\pi Mk/n}, n = 0, 1, \ldots\ldots M-1$$

If these numbers are all real, then these can be considered as the impulse response coefficients of an FIR filter. This can happen when all the complex terms appear in complex conjugate pairs and the all the lines can be matched by comparing the exponentials. The matching term $\tilde{H}(k)e^{-j2\pi nk/m}$ as a factor. The matching 9terms that has the exponential $e^{-j2\pi kn/m}$ as a factor. These lines are complex conjugate if $\tilde{H}(0)$ is read and

(i) For m Odd :

$$\tilde{H}(M-k) = \tilde{H}^*(k), k = 1, 2, \ldots\ldots\ldots, (M-1)/2 \qquad \ldots(i)$$

(ii) For m Even :

$$\tilde{H}(m-k) = \tilde{H}^*(k), k = 1, 2, \ldots\ldots m/2-1$$

$$\tilde{H}(m/2) = 0 \qquad \ldots(ii)$$

The desired frequency response $H_d\left(e^{j\omega}\right)$ is chosen such that it satisfies. Equations (i) & (ii) for m odd or even respectively. The filter coefficients can be written as,

$$h(n) = \frac{1}{M}\left[\tilde{H}(0) + 2\sum_{k=1}^{(M-s)/2} R_e\left[\tilde{H}(k)e^{j2\pi nk/m}\right]\right], \text{ m odd}$$

$$h(n) = \frac{1}{m}\left[\tilde{H}(0) + 2\sum_{k=1}^{M/2-1} R_e\left[\tilde{H}(k)e^{j2\pi nk/m}\right]\right], \text{ m even}$$

The system function of the filter can be determined from the filter coefficients h(n).

$$H(z) = \sum_{n=0}^{M-1} h(n)z^{-n}$$

Replacing z by $e^{j\omega}$, we get, $H\left(e^{j2\pi k/m}\right)$

$$= \sum_{n=0}^{M-1} h(n)e^{-j2\pi kn/m}$$

As per the definition of DFT, the above equation is simply the DFT of $\tilde{H}(k)$ using

$$H\left(e^{j2\pi k/m}\right) = \tilde{H}(k) = H_d\left(e^{-j2\pi k/m}\right)$$

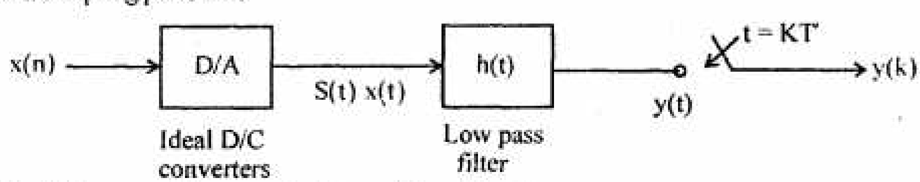
At $\omega = 2\pi k/m$, the value $H\left(e^{j\omega}\right)$ and $H_d\left(e^{j\omega}\right)$ one same and for other values of w, $H\left(e^{j\omega}\right)$ can be thought of as an interpotation of the sampled desired frequency response. $H_d\left(e^{j\omega}\right)$ and/or –m can be varied to get the suitable frequency response $H\left(e^{j\omega}\right)$.

Q. 7. Explain the sampling rate conversion. Explain the down sampling in detail.

Ans. Sampling Rate Conversion : Sampling rate conversion is the process of converting the sequence x(n) which is got from sampling the continuous time signal x(t) with a period T, to another sequence y(k) obtained from sampling x(t) with a period T'.

The new sequence y(k) can be obtained by first reconstructing the original signal x(t) from the sequence x(n) and the sampling the reconstructed signal with a period T'.

Below fig., shows the reconstruction of the original signal with a D/A converter, low-pass filter and resampler with sampling period T'.

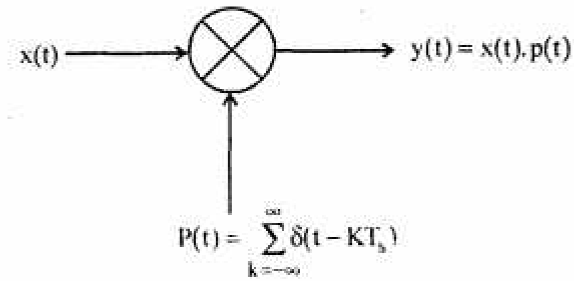


Sampling Theorem : There are two types of sampling theorems :

(i) Baseband sampling theorem

(ii) Bandpass sampling theorem.

(i) Baseband Sampling Theorem : It state that a continuous time signal can be completely represented by and reconvertible from knowledge of its value of sample at points, equally space in time provided the signal is sampled as twice its maximum frequency.



$$\Rightarrow \qquad Y(\omega) = \frac{1}{2\pi}\left[X(\omega) * P(\omega)\right]$$

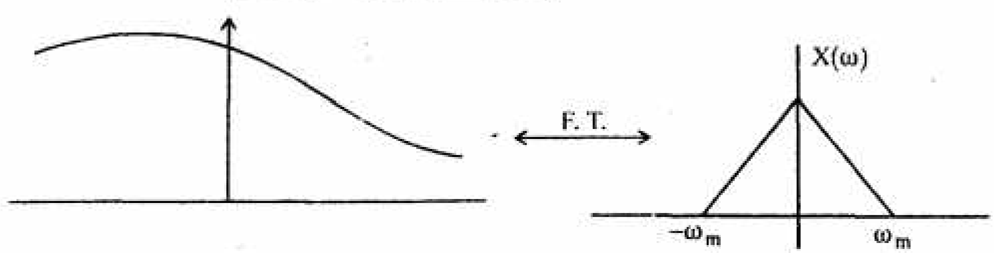
$$\Rightarrow \qquad P(\omega) = \frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

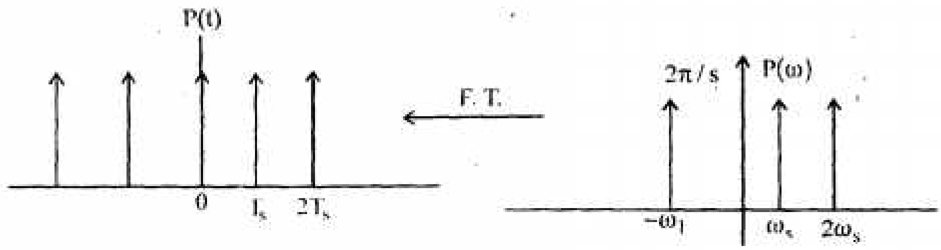
$$Y(\omega) = \frac{1}{2\pi}\left[X(\omega) * P(\omega)\right]$$

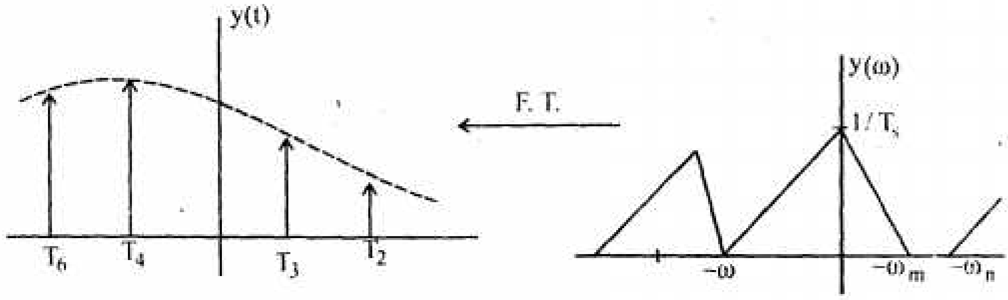
$$= \frac{1}{2\pi}\left[X(\omega) * \frac{2\pi}{T_s}\sum_{k=\infty}^{\infty} \delta(\omega - \omega_s k)\right]$$

$$= \frac{1}{T_s}\left[\sum_{k=\infty}^{\infty} X(\omega - k\omega_s)\right]$$

$$\boxed{Y(\omega) = \frac{1}{T_s}\sum_{k=\infty}^{\infty} X(\omega - k\omega_s)}$$







$$\boxed{Y(\omega) = \frac{1}{T_s}\sum_{k=\infty}^{\infty} X(\omega - \omega_s)}$$

$$\omega_s - \omega_m \geq \omega_m$$

$$\boxed{\omega_s \geq 2\omega_m}$$

(i) Nyquist Rate : $(\omega_s = 2\omega_m)$

Minimum frequency for sampling

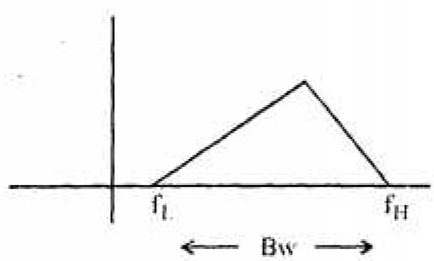
$$\boxed{w_s = 2\omega_m}$$

(ii) Nyquist Interval : (Maximum sampling time)

$$\boxed{T_s(\max) = 2\pi/\omega_s(\min)}$$

$$\boxed{T_s(\max) = 1/f_s(\min)}$$

Band-Pass Sampling : This theorem only applicable on narrowband.



Nyquist rate $= 2.f_H$

Minimum sampling frequency

$$(f_{min}) = \frac{2.f_H}{k}$$

$$K = \text{Integer}\left(\frac{f_H}{Bw}\right)$$

Minimum sampling frequency requirement reduces considerably as K inrease. For higher values of K the ratio f_H / Bw must be very-very high that 5 Bw should be very small as comapre to higher out-off frequency signal.

So, Bandpass sampling theorem is particularly applicable to narrow band signal.

Q. 8. Write short notes on the following :

(a) Applications of DSP

(b) Shift-invariant system

(c) Random and deterministic signals

Ans. (a) Applications of DSP : It is said that digital signal processing technique origin in the seventeenth century when finite difference method, numerical integration method, and numerical interpolation method were developed to solve physicals problems involving continuous variable and functions.

The main reasons for such wide applications are due to the numerous advantages of digital signal processing technique. Some of these advantages are discussed subsequently. Digital-processing of a signal facilitates the sharing of a single processor among a number of signals by time sharing. This reduces the processing cost per signal.

Digital implementation of a system allows easy adjustment of the processor characteristics during processing. Adjustments in the processor characteristics can be easily done by periodically changing the coefficients of the algorithms representing the processor characteristics.

There are various areas in which signal processing is used. To list a few :

(i) Communication system.

(ii) Speech and audio processing system

(iii) Antenna system and

(iv) Radar system.

The various advantages of digital signal processing are :

(i) Computational requirements are less.

(ii) Storage for filter coefficients are less.

(iii) Finite arithmetic effects are less.

(iv) Filter order required in multirate applications are law and

(v) Sensitivity to filter coefficient lengths are less.

(b) Shift Invariant System : Shift invariant system are those system, the system does not change on the shift of any parameter it means; system is invariant on the shifting of any property of system.

The necessary condition for shift invariance system is,

$$\Rightarrow \qquad \boxed{y(n-k) = F[x(n-k)]}$$

Let take an example,

$$y(n) = x(n+1) - 3x(n) + x(n-1)\ ;\ n \geq 0$$

$$\Rightarrow \qquad F[x(n-k)] = x(n-k+1) - 3x(n-k) + x(n-k-1)$$

$$y(n-k) = x(n-k+1) - 3x(n-k) + x(n-k-1)$$

As $\boxed{y(n-k) = F(n(n-k))}$, the system is shift invariant system.

This property show that any shift in the input side the same change in the output side also, means that one not change in the system response, due to this the system is called shift invariant system.

(c) Random and Deterministic Signals : Deterministics signals are functions that are completely specified in time. The nature and amplitude of such a signal at any time can be predicted. The pattern of the signal is regular and can be characterised mathematically. Examples of deterministic signals are :

(i) $x(t) = \alpha t$, this is a ramp whose amplitude increases linearly with time and slope is α.

(ii) $x(t) = A \sin\omega t$, the amplitude of this signal varies sinusoidally with time and its maximum amplitude is A.

For all the signals given above, the amplitude at any time instant can be predicted in advance.

Contrary to this, non-deterministics signal is one whose occurrence is random in nature and its pattern is quite irregular. A typical example of a non-deterministics signal is thermal noise in an electrical circuit. The behaviour of a such a signal is probabilistic in nature and can be analysed only stochastically. Another example which can be easily understand is the number of accidents in an year. One cannot exactly predict what would be the figure in a particular year and this varies randomly. Non-deterministic signals are also called random signals.