B.E.

Sixth Semester Examination, Dec.-2009

Digital Signal Processing (EE-407-E)

Note : Attempt any *five* questions. All questions carry equal marks.

Q. 1. (a) Define the fourier transform of a time function and explain under what conditions it exits.

Ans. Fourier transform of a time function of the repetition period T becomes infinity, i.e., $T \to \infty$, the wave f(t) will become non-periodic, the separation between two adjacent harmonic components will be zero, i.e., $\omega_0 = 0$.

The exponential form of the fourier series given $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$ to a periodic waveform such as single pulses or single transients by making a few changes.

Where $$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t}\,dt$$

To the limit, for a single pulse

$T \to \infty$, $$\omega_0 = 2\pi / T \to d\omega \text{ (a small quantity)}$$

$$1/T = \omega_0 / 2\pi \to d\omega / 2\pi$$

Furthermore, the n^{th} harmonic in the fourier series in $n\omega_0 \to nd\omega$. Here n must tend to infinity as ω_0 approaches zero, so that the product is finite i.e., $n\omega_0 \to \omega$

$$C_n = \frac{d\omega}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-j\omega t}\,dt$$

& $$f(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\left[\int_{-\infty}^{\infty} f(t) e^{-j\omega t}\,dt\right] e^{j\omega t}$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\,dt \quad ...(i)$$

It is called the FT of f(t), substituting for f(t) above,

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega) e^{-j\omega t}\,d\omega$$

Or equivalently

$$f(t) = \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t}\,df \quad ...(ii)$$

Q. 1. (b) Obtain the fourier transform of the signal $f(t) = e^{-a|t|}\cos\omega_0 t$

Ans. $$f(t) = e^{-a|t|}\cos\omega_0 t$$

$\Rightarrow \quad \int_{-\infty}^{\infty} e^{-a|t|} \cos\omega_0 t$

$\Rightarrow \quad \int_{-\infty}^{0} e^{at} \cos\omega_0 t\, e^{-j\omega_0 t} dt + \int_{0}^{\infty} e^{-at} \cos\omega_0 t e^{-j\omega_0 t} dt$

$\Rightarrow \quad \int_{-\infty}^{0} e^{at} \frac{\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)}{2} e^{-j\omega_0 t} dt + \int_{0}^{\infty} e^{-at}$

$\left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right) e^{-j\omega_0 t}\, dt$

$$= \frac{1}{2}\left\{\int_{-\infty}^{0} e^{at}\left(1 + e^{-2j\omega_0 t}\right)dt + \int_{0}^{\infty} e^{-at}\left(1 + e^{-2j\omega_0 t}\right)dt\right\}$$

$$= \frac{1}{2}\left\{\left[\int_{-\infty}^{0} e^{at}dt + \int_{-\infty}^{0} e^{(a-2j\omega_0)t}dt\right]^2 + \left[\int_{0}^{\infty} e^{-at}dt + \int_{0}^{\infty} e^{-(a+2jw_0)t}\, dt\right]\right\}$$

$$= \frac{1}{2}\left\{\left(\frac{e^{at}}{a}\right)_{-\infty}^{0} + \left[\frac{e^{(a-2j\omega_0)t}}{(a-2j\omega_0)}\right]_{-\infty}^{0} + \left(\frac{e^{-at}}{-a}\right)_{0}^{\infty} - \left[\frac{e^{-(a+2j\omega_0)t}}{(a+2j\omega_0)}\right]_{0}^{\infty}\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{a} + \frac{1}{a-2j\omega_0} + \left(-\frac{1}{a}\right) - \frac{1}{a+2j\omega_0}\right\}$$

$$= \frac{1}{2}\left\{\frac{a+2j\omega_0 - a + 2j\omega_0}{(a-2j\omega_0)(a+2j\omega_0)}\right\}$$

$$= \frac{1}{2}\left\{\frac{4j\omega_0}{a^2 + 4\omega_0^2}\right\}$$

$$\Rightarrow \quad \frac{2j\omega_0}{a^2 + 4\omega_0^2}$$

Q. 1. (c) Determine the energy of the signal $x(t) = 10\left[\frac{\sin 10\pi t}{10\pi t}\right]$

Ans. $$x(t) = 10\left[\frac{\sin 10\pi t}{10\pi t}\right]$$

$$E = 10\int_{-\infty}^{\infty}\left|10\frac{\sin 10\pi t}{10\pi t}\right|^2 dt$$

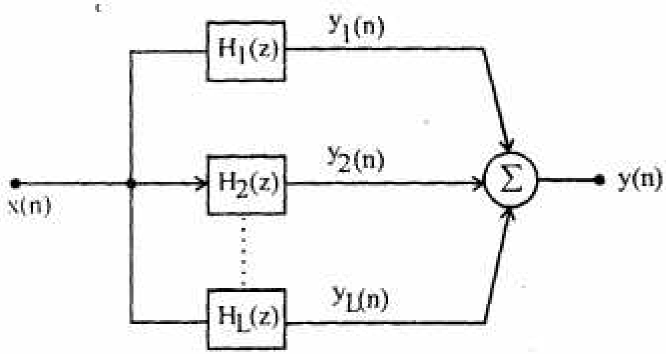
$$= 100\int_0^{\infty} \left[\frac{\sin^2 10\pi t}{100\pi^2}\right]_0^{\infty}$$

$$E = \frac{10}{\pi}.$$

Q. 2. (a) Discuss briefly the frequency response of LTI systems.

Ans. Frequency Response of LTI Systems :

Parallel Connection :



If there are L number of linear time invariant systems in the time domain connected in parallel, the impulse response h(n) of the resultant system.

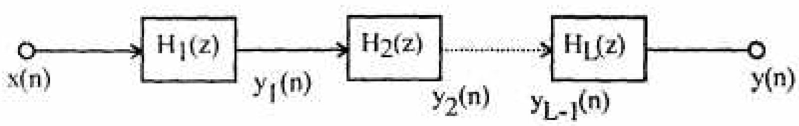
$$h(n) = \sum_{k=1}^{L} h_k(n)$$

Where $h_k(n)$; $k = 1, 2, \ldots\ldots L$ is the impulse response of the individual system.

$$H(z) = \sum_{k=1}^{L} H_k(z) = H_1(z) + H_2(z) + \ldots\ldots\ldots\ldots + H_L(z) \qquad \ldots(i)$$

Where $z = e^{j\omega}$. Here $H_k(e^{j\omega})$ is the frequency response to the impulse response $h_k(n)$.

Cascade Connection :



If the L linear time invariant systems are connected in cascade, the impulse response of the overall system.

$$h(n) = h_1(n) * h_2(n) * \ldots\ldots\ldots h_L(n)$$

Using the convolution property of the z-transform,

$$H(z) = H_1(z)H_2(z) ------ H_L(z)$$

Hence, $$H(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega}) ------- H_L(e^{j\omega}) \quad ...(iii)$$

Here we observe that the cascade connection involves convolution of the impulse response in the time domain and multiplication of the frequency response in the frequency domain.

Q. 2. (b) Determine the impulse response h(n) for the system described by the second-order difference equation,

$$\mathbf{y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)}$$

Ans. $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

Taking z-transform of both sides,

$$y(z) - 4z^{-1}y(z) + 4z^{-2}y(z) = x(z) - z^{-1}x(z)$$

$$\therefore \quad y(z)\left[1 - 4z^{-1} + 4z^{-2}\right] = x(z)\left[1 - z^{-1}\right]$$

$$\frac{y(z)}{x(z)} = H(z) = \frac{1 - z^{-1}}{1 - 4z^{-1} + 4z^{-2}}$$

$$\therefore \quad H(z) = \frac{z^2 - z}{z^2 - 4z + 4}$$

$$\therefore \quad \frac{H(z)}{z} = \frac{z - 1}{z^2 - 4z + 4}$$

The roots of denominator as under

$$\frac{H(z)}{z} = \frac{z-1}{(z-2)^2}$$

In partial fraction expansion form

$$\frac{H(z)}{z} = \frac{A_1}{z-2} + \frac{A_2}{(z-2)^2}$$

$$A_1 = \frac{d}{dz}\left[(z-2)^2 . \frac{z-1}{(z-2)^2}\right]_{z=2}$$

$$A_1 = 1$$

& $$A_2 = (z-2)^2 . \frac{z-1}{(z-2)^2}\bigg|_{z=2}$$

/

$\therefore$ $$A_2 = 1$$

$$\frac{H(z)}{z} = \frac{1}{z-2} + \frac{1}{(z-2)^2}$$

Or $$H(z) = \frac{z}{z-2} + \frac{z}{(z-2)^2}$$

Standard z-transform pairs

$$\alpha^2 u(n) \xleftrightarrow{z} \frac{z}{z-\alpha}$$

$\therefore$ $$z^{-1}\left\{\frac{z}{z-2}\right\} = z^n u(n)$$

& $$na^n u(n) \xleftrightarrow{z} \frac{az}{(z-\alpha)^2}$$

Second term of equation of H(z)

$$\frac{z}{(z-2)^2} = \frac{1}{2} \cdot \frac{2z}{(z-a)^2}$$

Therefore, $$z^{-1}\left\{\frac{1}{2} \times \frac{2z}{z-2}\right\} = \frac{1}{2} \cdot 2^n u(n)$$

$$h(n) = n2^n u(n) + \frac{1}{2} n(2)^n u(n).$$

Q. 3. (a) A band limited continuous time signal x_a (t) is sampled at a sampling frequency $F_s \geq 2B$. Determine the Energy Ed of the resulting discrete-time signal x(n) as a function of the energy of the signal Ea and the sampling period T = $1/F_s$.

Ans. $$E_a = \int_{-\infty}^{\infty} |xa(t)|^2 \, dt$$

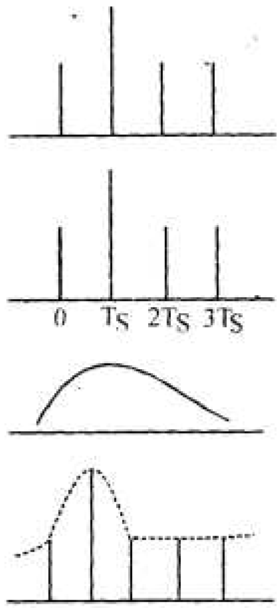
$$E_d = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$x(n) = \sum_{n=-\infty}^{\infty} x_a(t)\delta(t - nT_s)$$

$$n = 1$$

$$x(n) \approx x(nT)$$

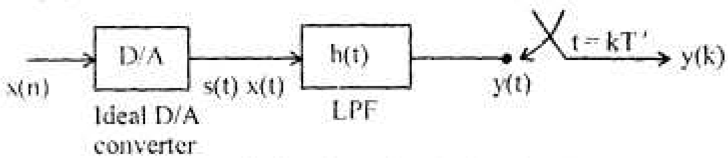
$$= x_a(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



Q. 3. (b) Describe the sampling rate conversion by use of linear approximation method.

Ans. Sampling Rate Conversion by Use of Linear Approximation Method :

Sampling rate conversion is the process of converting the sequence x(n) which is got from sampling the continuous time signal x(t) with a period of T, to another sequence y(k) obtained from sampling, x(t) with a period T'.



Let M be the integer sampling rate reduction factor for the signal x(n).

$$T'/T = M$$

$$F' = 1/T'$$

$$= 1/MT$$

$$= F/M$$

Let the signal x(n) a full band signal with non-zero in the frequency range.

$$-F/2 \le f \le F/2, \text{ where } \omega = 2\pi fT$$

$$\left|X\left(e^{j\omega}\right)\right| \ne 0, \ |\omega| = |2\pi fT| \le 2\pi FT/2 = \pi$$

$$H\left(e^{j\omega}\right) = \begin{cases} 1 & |\omega| \le 2\pi F'T/2 = \pi/M \\ 0 & \text{Otherwise} \end{cases}$$

Let the impulse response of the filter be h(n), then the filtered output $\omega(n)$

$$\omega(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

& $w(e^{j\omega})$ is the spectrum of $\omega(n)$. The decimated signal y(m)

$$y(m) = w(Mm)$$

Since $\omega'(n)$ take values only for m = 0, ±M, ±2M – y(z).

$$y(z) = \sum_{m=\infty}^{\infty} \omega'(m) z^{-m/M}$$

Where $\omega' = 2\pi fT$, $\omega = \pi / M$

$$y(e^{j\omega'}) = (1/M) \times (e^{j\omega/M}), \text{ for } |\omega| \le \pi$$

$H_d(e^{j\omega})$ at M points ω_k, k = 0, 1, ... M-1 uniformly spaced around the unit circle. The samples are taken at the frequency.

$$\omega_k = \frac{2\pi k}{M}, = 0, 1, \text{-------} M-1 \qquad ...(i)$$

The samples of the desired frequency response at these frequencies are given by

$$H(k) = H_d(e^{j\omega})\Big|_{\omega=\omega_k}, k = 0, 1 \text{-------}, M-1$$

$$H_d(e^{j2\pi k/M}) \quad k = 0, 1 \text{ --------- } M-1 \qquad ...(ii)$$

$$h(n) = \frac{1}{M} \sum_{k=0}^{M-1} H(k) e^{j2\pi nk/M}, n = 0, 1, \text{..............}, M-1 \qquad ...(iii)$$

If these numbers are all real, then these can be considered as the impulse response coefficients of an FIR filter.

Q. 4. (a) Determine x(n) for $X(z) = \dfrac{(z-0.5)}{z(z-0.8)(z-1)}$

Ans.

$$X(z) = \frac{(z-0.5)}{z(z-0.8)(z-1)}$$

$$= \frac{A}{(z)} + \frac{B}{(z-0.8)} + \frac{C}{z-1}$$

$$= A(z-0.8)(z-1) + B(z-1)z + Cz(z-0.8)$$

$$z - 0.5 = B\left(-\frac{1}{5}\right)\frac{4}{5}$$

$$\boxed{B = -\frac{15}{8}}$$

$$\boxed{A = -\frac{5}{8}}$$

$$\boxed{C = \frac{5}{2}}$$

$$X(z) = \frac{5}{8} - \frac{15}{8} \times \left(\frac{4}{5}\right)^{n-1} z(z-1) + \frac{5}{2} z(z-1)$$

Q. 4. (b) Explain the different properties of ROC of z-transform.

Ans. Important Properties of the ROC for the z-transform :

(i) X(z) converges uniformly if and only if the ROC of the z-transform X(z) of the sequence includes the unit circle. The ROC of X(z) consists of a ring in the z-plane centered about the origin. That is the ROC of the z-transform of x(n) has values of z for which $x(n)r^{-n}$ is absolutely summable.

$$\sum_{n=-\infty}^{\infty} \left|x(n)r^{-n}\right| < \infty$$

(ii) The ROC does not contain any poles.

(iii) When x(n) is of finite duration, then the ROC is the entire z-plane, except possibly z = 0 and/or $z = \infty$.

(iv) If x(n) is a right-sided sequence, the ROC will be not include infinity.

(v) It x(n) is a left-sided sequence, the ROC will not include z = 0. However, if $x(n) = 0$ for all n > 0, the ROC will include z = 0.

(vi) If x(n) is two-sided, and if the circle $|z| = r_0$ is in the ROC, then the ROC will consist of a ring in the z-plane that includes the circle $|z| = r_0$. That is, the ROC includes the intersection of the ROC's of the components.

(vii) If x(z) is rational, then the ROC extends to infinity, i.e., the ROC is bounded by poles.

(viii) If x(n) is causal, than the ROC includes $z = \infty$.

(ix) If x(n) is anti-causal, then the ROC includes z = 0.

Q. 5. (a) Explain the process of windowing using illustrations.

Ans. Window Techniques Using Illustration : The desired frequency response of any digital filter is periodic in frequency and can be expanded in a fourier series, i.e.,

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n} \qquad \text{...(i)}$$

Where,

$$h(n) = \frac{1}{2\pi}\int_0^{2\pi} H(e^{j\omega})e^{j\omega n}d\omega \quad ...(ii)$$

The fourier coefficients of the series h(n) are identical to the impulse response of a digital filter. First, the impulse response is of infinite duration and second, the filter is non-causal and unreliable. No finite amount of delay can make the impulse response realisable. Hence the filter resulting from a fourier series representation of $H(e^{j\omega})$ is an unreliable IIR filter.

The finite duration impulse response can be converted to a finite duration impulse response by truncating the infinite series at $n = \pm N$. This results in undesirable oscillations in the passband and stopband of the digital filter. These undesirable oscillations can be reduced by using a set of time-limited weighting functions, w(n), referred to as window functions, to modify the fourier coefficients. The windowing technique is illustrated.

A major effect of windowing is that the discontinuities in $H(e^{j\omega})$ are converted into transition bands between values an either side of the discontinuity. The width of these transition bands depends on the width of the main lobe of $W(e^{j\omega})$. A secondary effect of windowing is that the ripples from the side lobes of $W(e^{j\omega})$ produces approximation errors for all ω.

Q. 5. (b) Convert the analog filter to digital filter whose system function is $H(S) = \frac{1}{(S+0.2)^2+16}$ using bilinear transformation.

Ans. $$H(S) = \frac{1}{(S+0.2)^2+16}$$

By bilinear transformation

Let $T_S = 2$ sec.

$$\Rightarrow \frac{1}{\left[\left(\frac{z-1}{z+1}\right)+0.2\right]^2+16}$$

$$\Rightarrow \frac{1}{[(z-1)+(0.2)(z+1)]^2+16}$$ **Ans.**

Q. 6. (a) Explain coefficient quantisation.

Ans. Coefficient Quantisation : The realisation of digital filters in hardware or software has some limitation s due to the finite word length of the registers that are available to store these filter coefficients. Since the coefficients stored in these registers are either truncated or rounded-off, the system that is realised using these coefficients is not accurate. The location of poles and zeros of the resulting system will be different from the original locations and consequently the system may have a different frequency response than the one desired.

There are two approaches for the analysis and synthesis of digital filters with quantised coefficients. In the first approach, the quantization error is treated as a statistical quantity. In second approach, each filter is studied separately and the quantised coefficients can be optimised to minimize the maximum weighted difference between the ideal and actual frequency responses.

Let the T.F. of the coefficient-quantized digital filter being realized.

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} \quad \text{...(i)}$$

If $\bar{a}_k$ and $\bar{b}_k$ are the filter coefficients of unquantised filter and α_k and β_k are the error quantities.

$$a_k = \bar{a}_k + \alpha_k \text{ and } b_k = \bar{b}_k + \beta_k \quad \text{...(ii)}$$

The error due to quantisation of the filter coefficient is,

$$e(n) = y'(n) - y(n) \quad \text{...(iii)}$$

$$e(n) = \sum_{k=0}^{M} b_k x(n-k) - \sum_{k=0}^{M} \bar{b}_k x(n-k) \sum_{k=0}^{M} a_k y'(n-k) + \sum_{k=1}^{N} \bar{a}_k y(n-k) \quad \text{...(iv)}$$

Equations (i) & (iii)

$$e(n) = \sum_{k=0}^{M} \beta_k x(n-k) - \sum_{k=1}^{N} \bar{a}_k \left[y'(n-k) - y(n-k) - \sum_{k=1}^{N} a_k y'(n-k) \right]$$

Equation (ii)

$$e(n) = \sum_{k=0}^{M} \beta_k x(n-k) - \sum_{k=1}^{N} \bar{\alpha}_k e(n-k) - \sum_{k=1}^{N} \alpha_k e(n-k) - \sum_{k=1}^{N} \alpha_n y(n-k) \quad \text{...(iv)}$$

Neglecting the second order quantities i.e.

$$\sum_{k=1}^{N} \alpha_k e(n-k) \text{ equation (iv)}$$

$$e(n) = \sum_{k=0}^{M} \beta_k x(n-k) - \sum_{k=1}^{N} \bar{\alpha}_k e(n-k) - \sum_{k=1}^{N} \alpha_k y(n-k) \quad \text{...(v)}$$

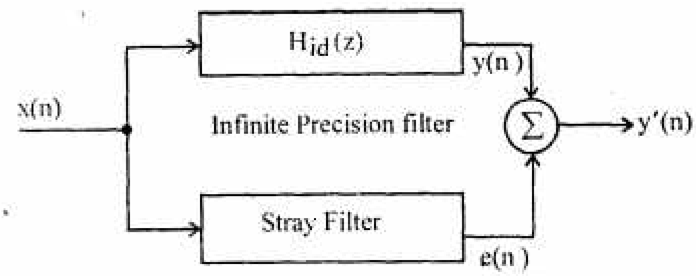
Taking z-transform of equation (v)

$$E(z) = X(z) \sum_{k=0}^{M} \beta_k z^{-k} - E(z) \sum_{k=1}^{N} \bar{\alpha}_k z^{-k} - y(z) \sum_{k=1}^{N} \alpha_k z^{-k}$$

$$E(z)\left[1+\sum_{k=1}^{N}\bar{\alpha}_k z^{-k}\right] = X(z)\sum_{k=0}^{M}\beta_k z^{-k} - y(k)\sum_{k=1}^{N}\alpha_k z^{-k} \quad \text{...(vi)}$$

The output y(z) of the ideal filter is given by

$$Y(z) = H_{ideal}(z).X(z)$$



Using equation (vi)

$$E(z)\left[1+\sum_{k=1}^{N}\bar{\alpha}_k z^{-k}\right] = X(z)\left[\sum_{k=0}^{M}\beta_k z^{-k} - H_{ideal}(z)\sum_{k=1}^{N}\alpha_k z^{-h}\right]$$

$$E(z) = X(z)\left[\sum_{k=0}^{M}\beta_k z^{-k} - H_{ideal}(z)\sum_{k=1}^{N}\alpha_k z^{-k}\right]$$

Therefore,

$$E(z) = X(z)\left[\frac{\sum_{k=0}^{M}\beta_k z^{-k} - H_{ideal}(z)\sum_{k=1}^{N}\alpha_k z^{-k}}{1+\sum_{k=1}^{N}\bar{\alpha}_k z^{-k}}\right] \quad \text{...(vii)}$$

From equations (iv), (vi) & (vii), the output of the actual filter

$$y'(z) = X(z)\left[H_{ideal}(z) + \frac{\sum_{k=0}^{M}\beta_k z^{-k} - H_{ideal}(z)\sum_{k=1}^{N}\alpha_k z^{-k}}{1+\sum_{k=1}^{N}\bar{\alpha}_k z^{-k}}\right]$$

That is,

$$y'(z) = \left[H_{ideal}(z)X(z) + E(z)\right] \quad \text{...(viii)}$$

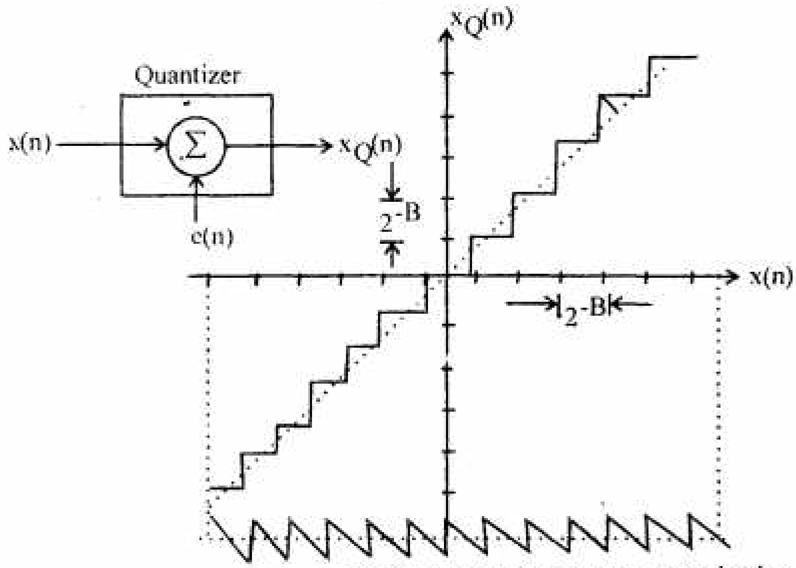
Q. 6. (b) What are the effects of finite word length in digital filters.

Ans. Effects of Finite Word Length in Digital Filters :

The various effects of quantization that arise in digital signal processing following are some of the issues connected with finite word length effects :

1. Quantisation effects in analog-to-digital conversion.
2. Product quantisation and coefficient quantisation errors in digital filters.
3. Limit cycles in IIR filters and
4. Finite word length effects in FFT.

(i) Quantisation Effects in Analog-to-Digital Conversion of Signals :



Two's complement number quantization

This quantiser sounds the sampled signal to the nearest quantised output level. The difference between the quantised signal amplitude $x_Q(n)$ and the actual signal amplitude x(n) is called the quantisation error In(n).

That is $$e(n) = x_Q(n) - x(n) \quad \text{...(i)}$$

The process of quantisation, the range of values for this quantisation error is

$$-\frac{2^{-B}}{2} \le e(n) \le \frac{2^{-B}}{2} \quad \text{...(ii)}$$

$$\text{SNR} = 10\log\frac{P_{x(n)}}{P_{e(n)}} \quad \text{...(iii)}$$

Where $P_{x(n)}$ is the signal power and $P_{x(n)}$ is the power of the quantised noise.

(ii) Coefficient Quantisation Effects in Direct

(a) Form Realisation of IIR Filters :

Let the transfer function of the coefficient quantised digital filter being realised be,

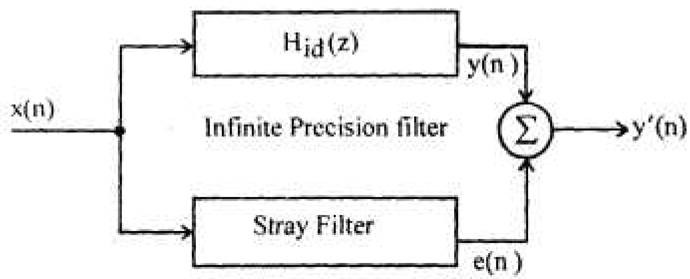
$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} \quad \ldots(i)$$

Y(z) is the Ideal filter

$$Y(z) = H_{ideal}(z).X(z)$$

$$E(z) = X(z)\left[\frac{\sum_{k=0}^{M} \beta_k z^{-k} - H_{ideal}(z)\sum_{k=1}^{N} \alpha_k z^{-k}}{1+\sum_{k=1}^{N} \bar{\alpha}_k z^{-k}}\right]$$

$$Y'(z) = \left[H_{ideal}(z)X(z) + E(z)\right]$$



(b) Direct form Realisation of FIR Filters : The statistical bounds on the error in the frequency response due to coefficient quantisation.

$$H(e^{j\omega}) = e^{-j\omega(M-1)/2}\left\{h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{(M-3)/2} h(n)\cos\left[\left(\frac{M-1}{2} - n\right)\omega\right]\right\}$$

$$= e^{j\phi(\omega)} M(\omega)$$

For a linear phase FIR filter,

$$h(n) = h(M-1-n)$$

$$e^{j\phi(\omega)} = e^{-j\omega(M-1)/2}$$

$$\left|E(e^{j\omega})\right| \le M\frac{2^{-B}}{2}$$

$$\sigma^2_{E(\omega)} = E(e^{j\omega})^2 = \frac{2^{-2B}}{12}\left[1 + 4\sum_{n=1}^{\frac{M-1}{2}} \cos^2(\omega n)\right]$$

$$W(\omega) = \left\{ \frac{1}{2M-1} \left[1 + 4 \sum_{n=1}^{\frac{M-1}{2}} \cos^2(\omega n) \right] \right\}^{1/2}$$

(c) Limit Cycles in IIR Filters : The amplitudes of such oscillations are much more serious in nature than zero input limit cycle oscillations. Consider a causal all-pole second order IIR digital filter implemented using two's complement arithmetic with a sounding of the sum of the products by a single quantiser. The difference equation describing the system is given by,

$$\tilde{y}(n) = Q_R\left[-a_1\tilde{y}(n-1) - a_2\tilde{y}(n-2) + x(n)\right]$$

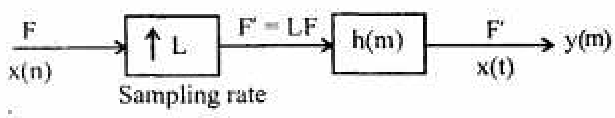
(d) Finite Word Length Effects in FFT : In the computation of FFTs, each butterfly computation involves one complex-valued multiplication, equivalently, four real multiplications. The quantisation errors introduced in each butterfly propagate to the output. The variance of the total quantisation error at the output is,

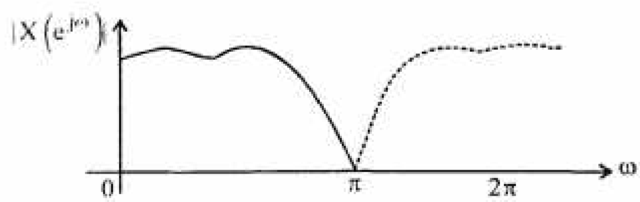
$$\sigma_{eq}^2 = 4(M-1)\sigma_e^2 \approx \frac{2^{-2B}}{3} M \qquad ...(i)$$

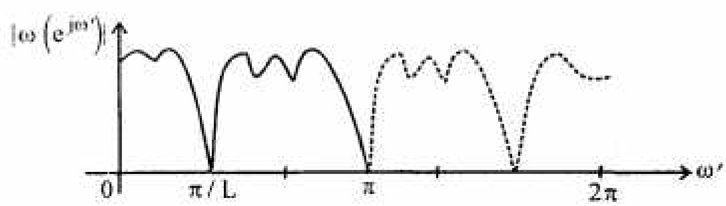
The FFT algorithm does not really reduce the number of multiplications required to compute a single point of the DFT.

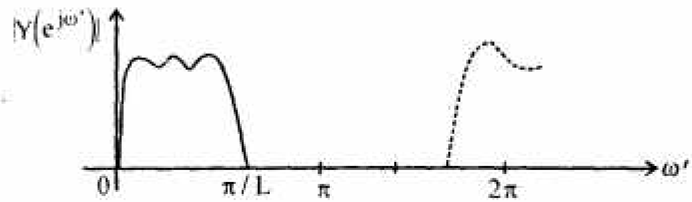
Q. 7. (a) Explain the interpolation process with an example.

Ans. Interpolation Process : The process of increasing the sampling rate of a single in interpolation. (sampling rate expansion).









Interpolation of x(n) by a factor L

Example : Assume L be an integer interpolating factor of the signal x(n), then

$$T'/T = \frac{1}{L}$$

The sampling rate is given by

$$\begin{aligned} F' &= 1/T' \\ &= L/T \\ &= LF \end{aligned} \quad ...(i)$$

Interpolation of a signal x(n) by a factor L refers to the process of interpolating factor, of the sampling x(n), then.

The signal $w(m)$ is got by interpolating L–1 samples between each pair of the sample of x(n).

$$w(m) = \begin{cases} x(m/L), & m = 0, \pm L \pm 2L......... \\ 0 & \text{othewise} \end{cases} \quad ...(ii)$$

The z-transform of the signal w(m) is given by

$$\begin{aligned} w(z) &= \sum_{m=-\infty}^{\infty} w(m) z^{-m} \\ &= \sum_{m=-\infty}^{\infty} x(m) z^{-mL} \\ &= X(z^{L}) \end{aligned} \quad ...(iii)$$

$$Z = e^{j\omega}$$

$$W(e^{j\omega}) = X(e^{j\omega L}) \quad ...(iv)$$

Where $\omega' = 2\pi f T'$

$$H(e^{j\omega}) = \begin{cases} G, & |j\omega'| \le 2\pi f T'/2 = \pi / L \\ 0, & \text{Otherwise} \end{cases} \quad ...(v)$$

Where G is the gain of the filter and it should be L in the passband.

$$y\left(e^{j\omega'}\right) = H\left(e^{j\omega'}\right) \times \left(e^{j\omega' L}\right)$$

$$= \begin{cases} G \times \left(e^{j\omega' L}\right); & |\omega'| \le \pi / L \\ 0 & \text{Otherwise} \end{cases} \qquad \text{...(vi)}$$

The output signal y(m) is given by

$$y(m) = \sum_{k=-\infty}^{\infty} h(m-k)\, w(k)$$

$$= \sum_{k=-\infty}^{\infty} h(m-k)\, x(k/L)\text{; k/L an integer}$$

The entire process of interpolation by a factor L. The interpolation process is also known as up sampler.

Q. 7. (b) Explain filter banks with equal and non-equal filter passbands.

Ans. Filter Banks with Equal Passbands : If a two-channel QMF filter bank is inserted between the down sampler and up sampler of another two-channel. QMF filter, then the resultant is four-channel QMF filter. Because of its tree structure, it is also called as tree structured filter bank. The relationships between the two-level filter bank and its equivalent structure is given by

$$H_0(z) = H_L(z) H_{10}\left(z^2\right)$$

$$H_1(z) = H_L(z) H_{11}\left(z^2\right)$$

$$H_2(z) = H_H(z) H_{10}\left(z^2\right)$$

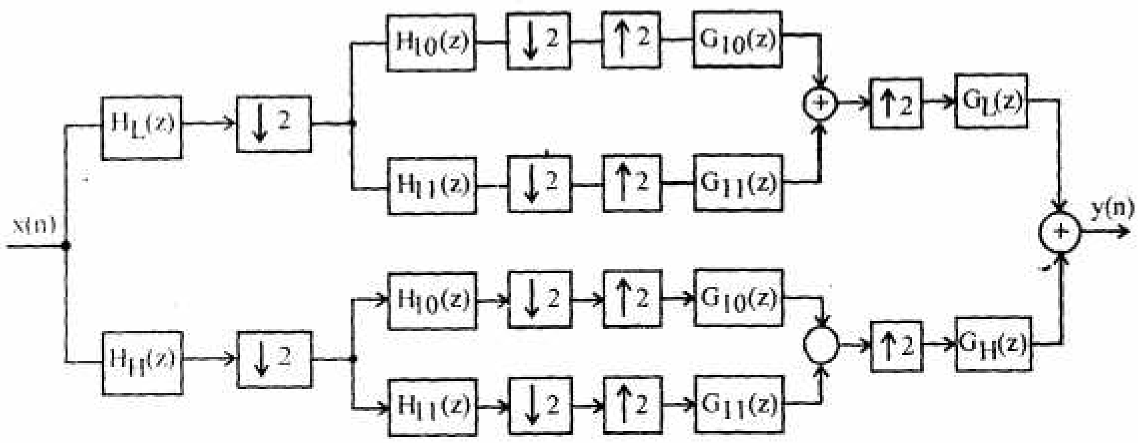
$$H_3(z) = H_H(z) H_{11}\left(z^2\right)$$

$$G_0(z) = G_L(z) G_{10}\left(z^2\right)$$

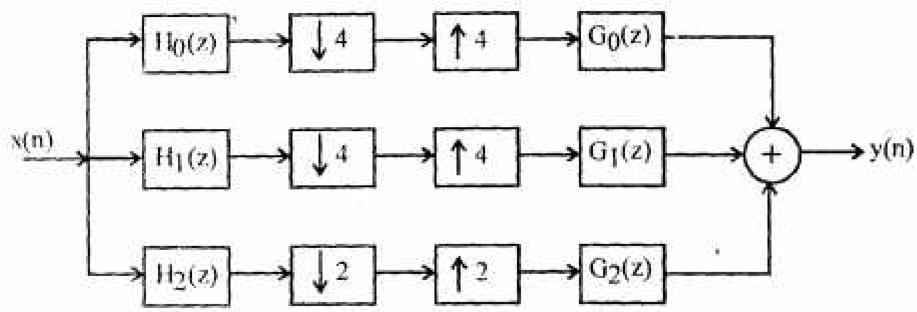
$$G_1(z) = G_L(z) G_{11}\left(z^2\right)$$

$$G_2(z) = G_H(z) G_{10}\left(z^2\right)$$

$$G_3(z) = G_H(z) G_{11}\left(z^2\right)$$



(a) Two-level four-channel QMF filter bank



(b) Equivalent three-channel QMF filter bank realization.

On similar lines, A four-channel QMF filter bank from a three-channel QMF filter bank. These structures come under the class of non-uniform QMF filter bank.

Q. 8. Write short note on any two of the following :

(a) Energy and Power theorems

(b) Properties of time invariant system

(c) Frequency domain representation of sampling.

Ans. (a) Energy Signal : A signal x(t) is called an energy signal if its total energy over the internal $(-\infty, \infty)$ in finite, that is,

$$E_x = \underset{T\to\infty}{\mathrm{Lt}} \int_{-T}^{T} |x(t)|^2 \, dt < \infty$$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt < \infty$$

If the signal x(t) contains finite signal energy, i.e., $0 < E_x < \infty$, then x(t) is called an energy signal.

For a digital signal, the energy is defined by

$$E = \sum_{n} |x(n)|^2$$

As $n \to \infty$, the energy of periodic signals becomes infinite, whereas the energy of aperiodic pulse like signals have a finite value. So, these a periodic pulse logic signals are called energy signals.

Power Signal : The average power of a signal x(t) over a single period $(t_1, t_1 + T)$ is given by

$$P_{av} = \frac{1}{T}\int_{t_2}^{t_1+T} |x(t)|^2 dt \text{ where x(t) is a complex periodic signal}$$

A signal f(t) is called a power signal if the average power is expressed by

$$P_x = \lim_{t \to \infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2$$

$$P_{av} = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt \text{ for a continuous periodic signal with period T.}$$

Or

$$P_{av} = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2 \text{, for a digital signal with } x(n) = 0 \text{ for } n < 0$$

is equal to a finite value, i.e., equal to the average power over a single period. If $x(t)$ is bounded P_{∞} is finite. Hence every bounded and periodic signal is a power signal.

(b) Properties of Time Invariant System : The output y(t) in continuous-time linear time invariance systems is expressed as a convolution integral i.e.,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) \otimes h(t)$$

Similarly, the output y(n) in descrete-time LTI system is expressed as a convolution system i.e.,

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = x(n) \otimes h(n)$$

From above, it is evident an LTI system is completely characterized by its impulse response. The linear time invariance system have a number of properties not exhibited by other systems.

(i) Commutative property of linear-time invariance

(ii) Distributive property of LTI system

(iii) Associative property of LTI system

(iv) Static and dynamic LTI systems

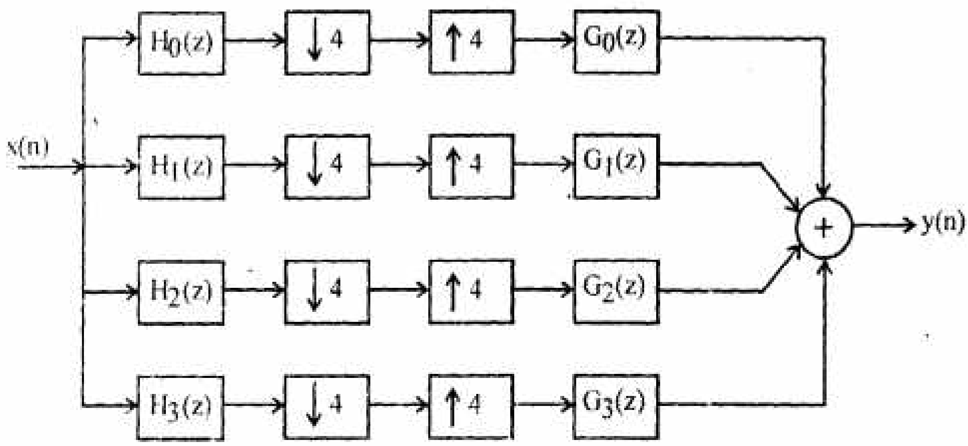
(v) Invertibility of LTI systems

(vi) Causability of LTI systems

(vii) Stability of LTI systems

(viii) Unit-step response of LTI systems.

(c) Frequency Sampling Method : In this method, a set of samples is determined from the desired frequency response and are identified as discrete fourier-transform (DFT) coefficients. The set of sample points used in this procedure can be determined by sampling a desired frequency response.



Equivalent Representation

The analysis filter $H_k(z)$ which are in cascade has the following characteristics :

(i) One filter has a single passband and single stopband.

(ii) Another filter has two passbands and two stopbands.

Filter Banks with Unequal Passbands : If a two channel QMF filter bank is introduced in one sub-band channel of another two-channel QMF filter bank, the resultant structure has filters with unequal passbands.

$$H_0(z) = H_L(z)H_L(z^2)$$

$$H_1(z) = H_L(z)H_H(z^2)$$

$$H_2(z) = H_H(z)$$

$$G_0(z) = G_L(z)G_L(z^2)$$

$$G_1(z) = G_L(z)G_H(z^2)$$

$$G_2(z) = G_H(z).$$