B.E.

Seventh Semester Examination, May-2009

Digital Signal Processing (EE-407-E)

Note : Attempt any **five** questions.

Q. 1. (a) Determine the discrete-time fourier transform (DTFT) of $x(n) = \sin\left(\frac{\pi n}{2}\right) u(n)$.

Ans. Given that $x(n) = \sin\left(\frac{\pi n}{2}\right) u(n)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{e^{j\pi n/2} - e^{-j\pi n/2}}{2j}\right) e^{-j\omega n}$$

$$X(e^{j\omega}) = \frac{1}{2j}\left[\sum_{n=0}^{\infty} e^{j\pi n/2}\, e^{-j\omega n} - \sum_{n=0}^{\infty} e^{-j\pi n/2}\, e^{-j\omega n}\right]$$

$$X(e^{j\omega}) = \frac{1}{2j}\left[\sum_{n=0}^{\infty} e^{\left(\frac{j\pi}{2} - j\omega\right)n} - \sum_{n=0}^{\infty} e^{\left(-\frac{j\pi}{2} - j\omega\right)n}\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1 - e^{j\pi/2}\, e^{-j\omega}} - \frac{1}{1 - e^{-j\pi/2}\, e^{-j\omega}}\right]$$

$$X(e^{j\omega}) = \frac{1}{2j}\left[\frac{1 - e^{-j\pi/2}\, e^{-j\omega} - 1 + e^{j\pi/2}\, e^{-j\omega}}{1 - e^{-j\omega}\,(e^{j\pi/2} + e^{-j\pi/2}) + e^{-j2\omega}}\right]$$

$$X(e^{j\omega}) = \frac{1}{2j}\left[\frac{e^{-j\omega}\,(e^{j\pi/2} - e^{-j\pi/2})}{1 - 2\cos\frac{\pi}{2}\, e^{-j\omega} + e^{-j2\omega}}\right]$$

or $$X(e^{j\omega}) = \frac{e^{-j\omega} \sin \pi/2}{1 + e^{-j2\omega}}$$

$$= \frac{e^{-j\omega}}{1 + e^{-j2\omega}}$$ **Ans.**

This is Parseval's theorem for discrete time periodic signals with finite energy which states that energy of a discrete time signal may also be obtained with the help of DTFT.

Q. 2. (b) The discrete time system

$$y(n) = ny(n-1) + x(n)$$

is at rest. Check if the system is linear, time invariant and BIBO stable.

Ans. Input $x(n) = 2\,\delta(n)$ is applied to

$$y(n) = xy(n-1) + x(n)$$

We know that, $\delta(n) = \begin{cases} 1 & \text{for} \quad n = 0 \\ 0 & \text{for} \quad n \neq 0 \end{cases}$

Therefore $x(n) = \begin{cases} 2 & \text{for} \quad n = 0 \\ 0 & \text{for} \quad n \neq 0 \end{cases}$

Also we have $y(0) = y^2\,(0-1) + x(0)$

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Since the system is at rest,

$$y^2(0-1) = y^2(-1) = 0$$

& $\quad x(0) = 2$ as illustrated above.

Hence $\quad y(0) = 2$

Now $\quad y(1) = y^2(1-1) + x(1)$

$$= y^2(0) + x(1)$$

Since, $\quad x(n) = 0$ when $n \neq 0$, above equation becomes

$$y(1) = y^2(0) = 2^2 = 4$$

Similarly we have

$$y(2) = y^2(2-1) + x(2) = y^2(1) \neq (2^2)^2 = 2^4 = 2^{2^2}$$

$$y(3) = 2^{2^3}$$

$$y(4) = 2^{2^4}$$

Similarly $\quad y(n) = 2^{2^n}$

Here as $\quad n \to \infty$

$$y(n) \to \infty$$

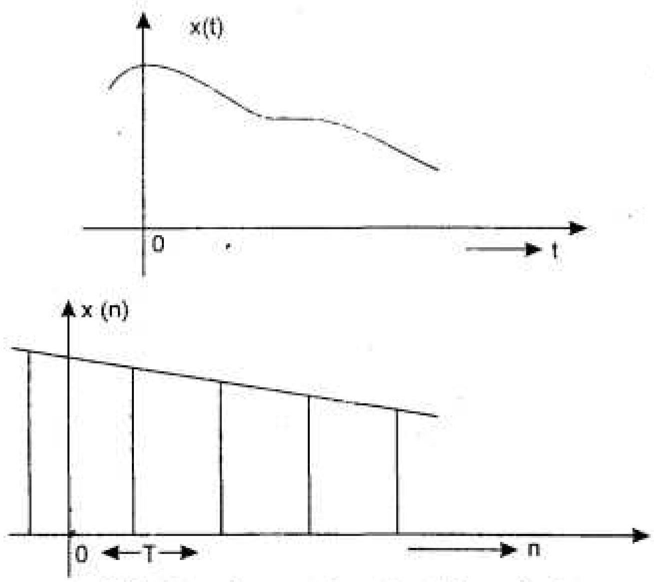
Hence, the input is bounded but output $y(n)$ is not bounded for all n.

Hence, we conclude that the given system is linear, time invariant but BIBO unstable.

Q. 3. (a) Discuss the discrete-time processing of the continuous time signal.

Ans. The best example of DSP of continuous signals is sampling.

Let us consider that $x(t)$ is a continuous time varifying signal. This signal $x(t)$ is sampled at regular interval of time with sampling period T as shown in figure.



DSP of continuous time signal (sampling)

Now, the sampled signal $x(nT)$ is given as

$$x(nT) = x(t)|_{\text{at } t=nT}$$
$$= x(nT), -\infty < n < \infty$$

As a matter of fact, a sampling process can also be interpreted as a modulation or multiplication process.

The continuous time signal $x(t)$ is multiplied by sampling function $s(t)$ which is a series of impulses. The resultant signal will be a discrete-time signal $x(n)$

i.e., $$x(n) = x(t)\ s(t)|_{\text{at } t = nT, -\infty < n < \infty}$$

Q. 3. (b) Determine the response of the system characterized by the impulse response. $h(n) = \left(\frac{1}{2}\right)^n u(n)$ to the input signal $x(n) = u(-n)$.

Ans. Given, Impulse response

$$h(n) = \left(\frac{1}{2}\right)^n u(n), \quad x(n) = u(-n)$$

$$x(0) = u(0)$$
$$x(1) = u(-1)$$
$$x(2) = u(-2)$$
$$x(3) = u(-3) \text{ so on}$$

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$h(0) = \left(\frac{1}{2}\right)^0 u(0) = \quad 1u(0)$$

$$h(0) = x(0) \qquad \ldots(i)$$

$$h(1) = \left(\frac{1}{2}\right)^1 u(1)$$

$$= \frac{1}{2} u(1) \qquad \ldots(ii)$$

Similarly $$h(2) = \left(\frac{1}{2}\right)^2 u(2)$$

$$h(3) = \left(\frac{1}{2}\right)^3 u(3) \text{ and so on.}$$

Hence,

Response of system $= h(n)\ x(n)$

$$= \left(\frac{1}{2}\right)^n u(n)\ u(-n)$$

$$=\left(\frac{1}{2}\right)^0 u(0)\,u(0)+\left(\frac{1}{2}\right)^1 u(1)\,(u(-1)+\left(\frac{1}{2}\right)^2 u(2)\,u(-2)$$

$$=u^n(0)+\frac{1}{2}u(1)\,u(-1)+\frac{1}{4}u(2)\,(-2)\ldots \text{ so on} \quad \textbf{Ans.}$$

Q. 4. (a) Determine the z-transform of $x(n)=na^n\cos(\text{won})\,u(n)$.

Ans. First, calculate transform of $x(n)=\cos\omega_0 n$ for $n\geq 0$

$$\cos\omega_0 n=\frac{1}{2}[e^{j\omega_0 n}+e^{-j\omega_0 n}]$$

The Z-transform is expressed as :

$$X(z)=Z\{x(n)\}$$

$$=\sum_{n=-\infty}^{\infty}x(n)\,z^{-n}$$

For $n\geq 0$, we have

$$Z\{x(n)\}=X(z)=\sum_{n=\infty}^{\infty}x(n)\,z^{-n}$$

For $x(n)=a^n$, we have

$$Z(a^n)=\sum_{n=\infty}^{\infty}a^n z^{-n}=\sum_{n=\infty}^{\infty}(az^{-1})^n$$

or $$Z(a^n)=\frac{1}{1-az^{-1}} \qquad \text{for } |az^{-1}|<1 \quad \text{or} \quad |z|>|a$$

Hence, $$Z[e^{j\omega_0 n}]=Z[e^{(j\omega_0)}n]$$

$$=\frac{1}{1-e^{j\omega_0}\,z^{-1}} \qquad \text{for } |z|>|e^{j\omega_0}|$$

or $$Z[e^{j\omega_0 n}]=\frac{1}{1-e^{j\omega_0}\,z^{-1}} \qquad \text{for } |z|>1$$

Similarly $$Z[e^{-j\omega_0 n}]=\frac{1}{1-e^{-j\omega_0}\,z^{-1}} \qquad \text{for } |z|>|e^{-j\omega_0}|$$

Now, since $$X(z)=Z[\cos\omega_0 n]$$

So $$X(z)=Z\left[\frac{1}{2}(e^{j\omega_0 n}+e^{-j\omega_0 n})\right]$$

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A||inearity property

$$X(z)=\frac{1}{2}Z[e^{j\omega_0 n}]+\frac{1}{2}Z[e^{-j\omega_0 n}]$$

Hence,
$$X(z)=\frac{1}{2}\frac{1}{(1-e^{j\omega_0}z^{-1})}+\frac{1}{2}\frac{1}{(1-e^{-j\omega_0}z^{-1})}$$

$$=\frac{1}{2}\left[\frac{1-e^{-j\omega_0}z^{-1}+1-e^{j\omega_0}z^{-1}}{(1-e^{j\omega_0}z^{-1})(1-e^{-j\omega_0}z^{-1})}\right]$$

$$X(z)=\frac{1}{2}\left[\frac{2-(e^{j\omega_0}+e^{-j\omega_0})z^{-1}}{(1-e^{j\omega_0}z^{-1})(1-e^{-j\omega_0}z^{-1})}\right]$$

$$=\left[\frac{1-\frac{1}{2}(e^{j\omega_0}+e^{-j\omega_0})z^{-1}}{(1-e^{j\omega_0}z^{-1})(1-e^{-j\omega_0}z^{-1})}\right]$$

or
$$X(z)=\frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}=\frac{1-\frac{1}{2}\cos\omega_0}{1-\frac{2}{z}\cos\omega_0+\frac{1}{2}}$$

$$X(z)=\frac{z^2-z\cos\omega_0}{z^2-2z\cos\omega_0+1}=\frac{z(z-\cos\omega_0)}{z^2-2z\cos\omega_0+1}\qquad |z|>1$$

Now, using scaling property

$$Z[xa^n\cos_{\omega_n}(n)u(n)]$$

$$\Rightarrow\qquad \frac{zna^n(zna^n-\cos\omega_0)}{(zna^n)^2-2a^n nz\cos\omega_0+1}$$

$$=\frac{zna^n(za^n n-\cos\omega_0)}{n^2a^{2n}z^2-2a^n nz\cos\omega_0+1}\qquad \textbf{Ans.}$$

Q. 4. (b) Find the inverse z-transform of $x(z)$ where

$$x(z)=\frac{2+z^{-2}+3z^{-4}}{z^2+4z+3};|z|>0$$

Ans. Given
$$X(z)=\frac{2+z^{-2}+3z^{-4}}{z^2+4z+3}\qquad |z|>0$$

It may be observed that $X(z)$ can be written as

$$X(z)=(2z^{-1}+z^{-3}+3z^{-5})X_1(z)$$

$$X_1(z) = \frac{z}{z^2 + 4z + 3}$$

Thus, if $\quad x_1(n) \longleftrightarrow X_1(z)$

Then, by linearity property, and the time surfing property, we get

$$x(n) = 2x_1(n-1) + x_2(n-3) + 3x_1(n-5)$$

Now, $$\frac{X(z)}{z} = \frac{1}{z^2 + 4z + 3} = \frac{1}{(z+1)(z+3)}$$

$$= \frac{c_1}{z+1} + \frac{c_2}{z+3}$$

Where, $$c_1 = \left.\frac{1}{z+3}\right|_{z=-1} = \frac{1}{2}$$

$$c_2 = \left.\frac{1}{z+1}\right|_{z=-3} = -\frac{1}{2}$$

Then, $$X_1(z) = \frac{1}{2} \cdot \frac{z}{z+1} - \frac{1}{2} \frac{z}{z+3} \qquad |z| > 0$$

Since the ROC of $X_1(z)$ is $|z| > 0$, therefore, $x_1(n)$ is a right sided sequence and thus, we get,

$$x_1(n) = \frac{1}{2} [(-1)^n - (-3)^n \, u(n)]$$

Thus, we get, $$x(n) = [(-1)^{n-1} - (-3)^{n-1}] \, u(n-1) + \frac{1}{2} [(-1)^{n-3} - (-3)^{n-3}]$$

$$= u(n-3) + \frac{3}{2} [(-1)^{n-5} - (-3)^{n-5}] \, u(n-5) \qquad \textbf{Ans.}$$

Q. 5. (a) Discuss the design of FIR filter by frequency sampling method.

Ans. Design of Linear Phase FIR Filters Using :

Frequency Sampling Method :

Let us consider that we want to design the FIR filter whose desired frequency response is denoted by $_d(\omega)$.

This frequency response is sampled uniformly at M points. Such frequency samples are given at,

$$\omega k = \frac{2\pi k}{M}, \qquad k = 0, 1, 2 \quad \ldots M-1$$

Such sampled design desired frequency response is discrete fourier transform.

It can be denoted by $H(k)$ i.e.,

$$H(k) = H_d(\omega)|_{\omega = \omega k} \qquad k = 0, 1 \ldots \quad M-1$$

or $$H(k) = H_d\left(\frac{2\pi k}{M}\right)$$

Hence, $H(k)$ is M point DFT, by taking inverse discrete fourier transform (IDFT) of $H(k)$, we get $h(n)$. This $h(n)$ is unit sample response of FIR filter i.e.,

$$h(n) = \frac{1}{M} \sum_{k=0}^{M-1} H(k)\, e^{j2\pi kn/M} \qquad n = 0, 1, \ldots \quad M-1$$

Hence, the unit sample response of FIR filter of length 'M' is obtained using frequency sampling technique.

For the filter (FIR) to be realizable, the coefficients $h(n)$ should be all real.

This is possible if all complex terms appear in complex conjugate pairs.

Let us consider the term $H(M-k)\, e^{j2\pi n(M-k)/M}$ i.e.

$$H(M-k)\, e^{j2\pi n(M-k)/M} = H(M-k)\, e^{j2\pi n} \cdot e^{-j2\pi kn/M}$$

Here, $\quad e^{j2\pi n} = \cos(2\pi n) + j\sin(2\pi n)$

$= 1$ always

Therefore,

$$H(M-k)e^{-j2\pi n(M-j)/M} = H(M-j)\, e^{-j2\pi kn/M}$$

Usually $\quad |H(M-k)| = |H(k)|$

Hence,
$$h(n) = \frac{1}{M}\left\{ H(0) + 2\sum_{k=1}^{P} \operatorname{Re}\left[H(k)\, e^{j2\pi kn/M}\right] \right\}$$

Where,
$$p = \begin{cases} (M-1)/2 \text{ if M is odd} \\ M/2 - 1 \text{ if M is even} \end{cases}$$

Q. 5. (b) Convert the analog filter into a digital filter whose system function is $H(s) = \frac{s+0\cdot 2}{(s+0\cdot 2)^2 + 9}$. Use the impulse invariance method.

Ans.
$$Ha(s) = \frac{(s+0\cdot 2)}{(s+0\cdot 2)^2 + 9}$$

But
$$Ha(s) = \frac{s+a}{(s+a)^2 + b^2}$$

So, $a = 0\cdot 2, \quad b = 3$

Hence
$$H(z) = \frac{1 - e^{-aT}(\cos bT)\, z^{-1}}{1 - 2e^{-aT}(\cos bT)\, z^{-1} + e^{-2aT}\, z^{-2}}$$

Putting values of a and b, we get,

$$H(z) = \frac{1 - e^{-0\cdot 2T}(\cos 3T)\, z^{-1}}{1 - 2e^{-0\cdot 2T}(\cos 3T)\, z^{-1} + e^{-0\cdot 4T}\, z^{-2}}$$

Taking $T = 1$, we have

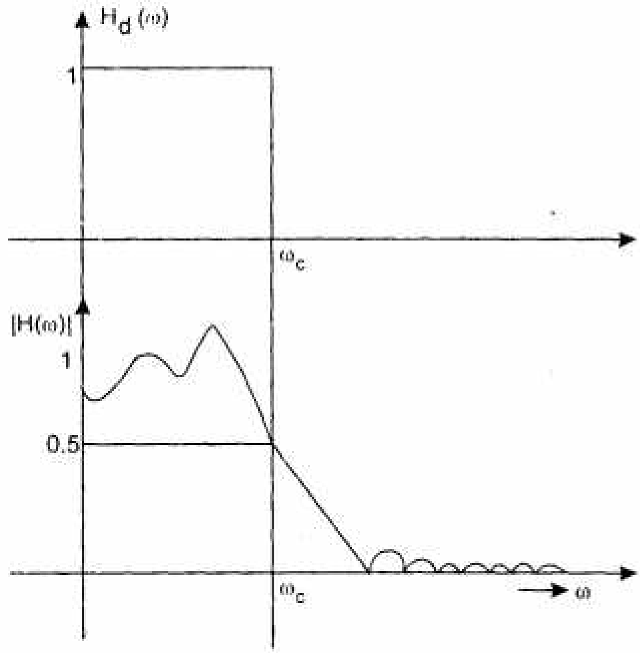
$$H(z) = \frac{1-(0\cdot 8187)(-0\cdot 99)z^{-1}}{1-2(0\cdot 8187)(-0\cdot 99)z^{-1}+0\cdot 6703\,z^{-2}}$$

Simplifying, we get

$$H(z) = \frac{1+(0\cdot 8105)z^{-1}}{1+1\cdot 6210z^{-1}+0\cdot 6703\,z^{-2}} \qquad \textbf{Ans.}$$

Q. 6. (a) Explain the limit cycle oscillation in recursive system.

Ans. Let us consider the example of a low pass filter having desired frequency response $H_d(\omega)$ as depicted in fig. This response has the cut off frequency at ω_c.



Limit cycle oscillations

Hence, different window functions are developed which consists of taper and decays gradually towards zero. This reduces side lobes and hence ringing effect in $H(\omega)$.

Q. 6. (b) Draw and explain the direct form-II structure for IIR filter.

Ans. Direct Form II Realizations for IIR Filters :

Here, since the systems are linear, we can interchange the positions of $H_1(z)$ and $H_2(z)$.

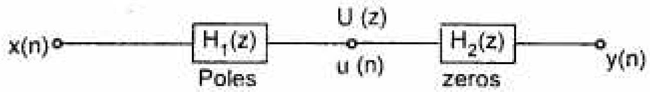
This property will give us the direct form II structure. Hence here, poles of $H(z)$ are realized first and zeros second.

We have, $\quad H(z) = H_1(z)\,H_2(z)$

$$H_1(z) = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

& $$H_2(z) = \sum_{k=0} b_k z^{-k}$$

The cascade connection of $H(z)$ is shown as :



(i) All pole system,

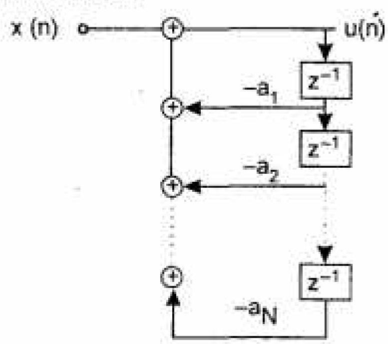
$$H_1(z) = \frac{\text{output}}{\text{input}} = \frac{U(z)}{X(z)}$$

$$\frac{U(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

Hence, $$U(z) = \left[1 + \sum_{k=0}^{N} a_k z^{-k}\right] = X(z)$$

or $$U(z) = X(z) - U(z) \sum_{k=1}^{N} a_k z^{-k}$$

The direct form II realization is shown as :



Similarly (ii) all zero system

$$\frac{u(z)}{U(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

$$y(z) = U(z) \sum_{k=0}^{M} b_k z^{-k}$$

$$y(n) = b_0 u(n) + b_1 u(n-1) + b_2 u(n-2) .... \quad b_n u(n-M)$$

Q. 7. Explain the process of down sampling by decimation in detail.

Ans. Down Sampling by Decimation : The process of reducing the sampling rate of the signal is called decimation.

Let us assume that the discrete-time signal $x(n)$ with spectrum $X(\omega)$ is to be down sampled by integer factor D.

The spectrum $X(\omega)$ is assumed to be non-zero in the frequency interval $0 \le |\omega| \le \pi$ or equivalently

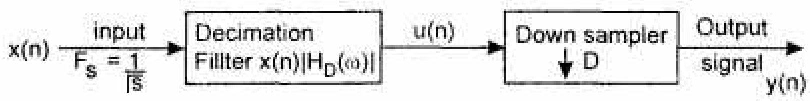
$$F \le F_s|2$$

Furthur, we know that if we reduce the sampling rate simply by selecting every Dth value of signal $x(n)$, the resulting signal would be an alaised version of $x(n)$ with folding frequency of $F_s/2D$.

To avoid alaising, we must reduce the BW of $x(n)$ to

$$F_{\max} = \frac{F_s}{2D} \qquad \text{or} \qquad \omega_{\max} = \frac{\pi}{D}$$

Then, we can down sample by D and thus, can avoid alaising.



$$Fy = \frac{1}{Ty} = \frac{F_s}{D}$$

The input signal/sequence $x(n)$ is passed through decimation filter i.e.,

LPF : This LPF is characterized by the impulse response $h(n)$ and frequency response $H_D(\omega)$.

The LPF ideally satisfies the following condition.

$$H_D(\omega) = \begin{cases} 1, & |\omega| \le \pi/D \\ 0, & \text{elsewhere} \end{cases}$$

Hence, the filter eliminates the spectrum of $X(\omega)$ in the range

$$\pi/D < \omega < \pi$$

Here, only the frequency components of $x(n)$ in the range $|\omega| \le \pi/D$ are of interest, in further processing of the signal.

The output of filter is a sequence $u(n)$ which is given by

$$u(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

$$y(m) = v(mD)$$

Hence, $$y(m) = \sum_{k=0}^{\infty} h(k)\, x(mD - K)$$

Now, $$y(z) = \sum_{m=-\infty}^{\infty} v(m) \left[\frac{1}{D} \sum_{k=0}^{D-1} e^{\frac{-j2\pi mk}{2}} \right] z^{-m/D}$$

Q. 8. Write short notes on the following :

(a) Symmetric FIR filter

(b) Discrete time random signal

(c) Multistage interpolation

Ans. (a) Symmetric FIR Filter : In FIR systems, the impulse response sequence is of finite duration. This means that the impulse response sequence of FIR filters has a finite number of non-zero terms. In other words, if the impulse response of a digital filter is determined for some finite number of sample points, then these filters are known as finite impulse response filters.

Symmetric FIR digital filters can readily be designed to have constant delay as well as prescribed loss specifications. On the other hand, IIR filters have an infinite no. of non-zero terms.

Example. $$h(n)=\begin{cases}2^{n} & \text{for } |n|\le 4\\ 0 & \text{for otherwise}\end{cases}$$

$$h(n)=a^{n}\ u(n)$$

$$\sum_{k=0}^{N} a_k\,y(n-k)=\sum_{k=0}^{M} b_k\,x(n-k)$$

$$y(n)=b_0\ x(n)+b_1\ x(n-1)+b_2(x/n-2)$$

$$b_{M-1}\ x(n-M+1)$$

(b) Discrete Time Random Signal : A discrete time signal is defined only at certain time instants. For discrete time signal, the amplitude between 2 time instants is just not defined for discrete time signals, the independent variable is time n. A discrete time signal is represented as $x(n)$.

A discrete time random signal is one whose occurrence is always random in nature.

The pattern of such a signal is quite irregular. Non-deterministic signals are called random signals.

A typical example is thermal noise generated is an electric circuit. Such a noise signal has parabolostic behaviour.

(c) Multistage Interpolation : Increasing of sampling rate is known as interpolation. An increase in the sampling rate by an integer factor I may be accomplished by interpolating (I-1) new samples between successive values of the signals.

The multistage interpolation process may be accomplished by various methods.

Let $V(m)$ is a signal sequence with sampling rate $F=IF_x$.

Thus, $$V(m)=\begin{cases}x(m/I) & m=0,\pm I,\pm 2I\\ 0 & \text{elsewhere}\end{cases}$$

$$V(z)=z\{V(m)\}$$

$$V(z)=\sum_{m=-\infty}^{\infty} V(m)\,z^{-m}=\sum_{m=-\infty}^{\infty} x(m/I)\,z^{-m}$$

$$V(z)=\sum_{m=-\infty}^{\infty} x(m)\,z^{-mI}$$

or $$V(z)=x(Z^{I})$$

Hence, $$V(\omega y)=X(\omega yI)$$

Where $$\omega y=\frac{\omega s}{I}$$ **Ans.**