B.E.

Seventh Semester Examination, Dec-2008

Digital Signal Processing (EE-407-E)

Note : Attempt any five questions.

Q. 1. (a) State and prove the power theorem. **10**

Ans. Power Theorem : Signals having infinite energy, but finite average power, are called power signals. The average power dissipated by a voltage $f(t)$ applied across a 1-ohm resistor is defined as the average power. The average power P, therefore may be expressed as;

$$\therefore \qquad P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 \, dt \qquad \text{.... (i)}$$

Equation (i) represents the mean square value of the signal $f(t)$ and hence, the average power P is same as the mean square value of $f(t)$. The ratio of power P_1 and P_2, is defined in terms of decibel;

$$\therefore \qquad \alpha = 10 \log_{10}\left(\frac{P_2}{P_1}\right)$$

V_2 and V_1 are rms voltage corresponding to power P_2 and P_1.

Parseval's Power Theorem : The theorem defines the power of a signal in terms of its Fourier coefficients i.e., in terms of amplitudes of the harmonic components present in the signal.

Let us consider a function $f(t)$, we know that

$$|f(t)|^2 = f(t) f^*(t)$$

The power of the signal $f(t)$ over a cycle is given by

$$\therefore \qquad P = \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 \, dt = \frac{1}{T} \int_{-T/2}^{T/2} f(t) f^*(t) \, dt$$

Replacing $f(t)$ by its exponential Fourier series;

$$\therefore \qquad P = \frac{1}{T} \int_{-T/2}^{T/2} f^*(t) \left[\sum_{n=-\infty}^{\infty} F_r e^{jn\omega_o t} \right] dt$$

Where, $\omega_o = \dfrac{2\pi}{T}$

Interchanging the order of integration and summation.

$$\therefore \qquad P = \frac{1}{T} \sum_{n=-\infty}^{\infty} F_n \int_{-T/2}^{T/2} f^*(t) e^{jn\omega_o t} \, dt$$

The integral in the above expression is equal to TF_n^*

$$\therefore \qquad P = \sum_{n=-\infty}^{\infty} F_n F_n^*$$

$$P = \sum_{n=-\infty}^{\infty} |F_n|^2$$

This equation is known as Parseval's Power Theorem.

Q. 1. (b) Define the Fourier transform of discrete time signal. Also state and prove its any four properties. **10**

Ans. Fourier Transform of Discrete Time Signal : The Fourier transform of a finite-energy discrete-time signal $x(n)$ is defined as :

$$\therefore \quad X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \quad \text{.... (i)}$$

$X(\omega)$ represents the frequency content of the signal $x(n)$

Properties of Fourier Transform :

(i) Symmetry : If $\quad f(t) \Leftrightarrow F(j\omega)$

Then, $\quad F(jt) \Leftrightarrow 2\pi f(-\omega)$

Proof :

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} . d\omega$$

$$2\pi f(-t) = \int_{-\infty}^{\infty} F(j\omega') e^{-j\omega' t} . d\omega'$$

Where the dummy variable ω is replaced by ω'.

Now, if t is replaced by ω, we have

$$2\pi f(-\omega) = \int_{-\infty}^{\infty} F(j\omega') e^{-j\omega'\omega} d\omega'$$

Now, ω' is replaced by (t) $\quad \therefore \quad 2\pi f(-\omega) = \int_{-\infty}^{\infty} F(jt) e^{-j\omega t} . dt = T[F(jt)]$

Therefore $\quad F(jt) \Leftrightarrow 2\pi f(-\omega)$

If $f(t)$ is an even function, $\quad f(t) = f(-t)$

Hence $\quad T[F(jt)] = 2\pi f(\omega)$

(ii) Scaling : If $\quad f(t) \Leftrightarrow F(j\omega)$

Then, $\quad f(at) \Leftrightarrow \frac{1}{|a|} \frac{F(j\omega)}{a}$

Proof : If $a > 0$, then the transform of $f(at)$ is

$$T[f(at)] = \int_{-\infty}^{\infty} f(at) e^{-j\omega t} . dt \quad \text{... (i)}$$

Putting $x = at$, we have $dx = a dt$, put in equation (i)

$$T[f(at)] = T[f(x)] = \int_{-\infty}^{\infty} f(x) e^{\frac{-j\omega x}{a}} \frac{dx}{a} = \frac{1}{a} F\left(\frac{j\omega}{a}\right).$$

If a < 0, then $$T[f(at)] = -\frac{1}{a} F\left(\frac{j\omega}{a}\right)$$

$$f(at) \Leftrightarrow \frac{1}{|a|} F\frac{(j\omega)}{a}$$

Hence proved

(iii) Time Convolution : If $$x(t) \Leftrightarrow X(j\omega)$$

$$h(t) \Leftrightarrow H(j\omega)$$

Then, $$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \Leftrightarrow Y(j\omega) = X(j\omega) H(j\omega)$$

Proof : $$T[y(t) = Y(j\omega)] = \int_{-\infty}^{\infty} e^{-j\omega t} \left[\int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \right] dt$$

$$= \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau) e^{-j\omega t} . dt \right] d\tau$$

Putting $a = t - \tau$, then $t = a + \tau$ and $da = dt$

$\therefore$ $$Y(j\omega) = \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h(a) e^{-j\omega(a+\tau)} . da \right] . d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) e^{-j\omega t} \int_{-\infty}^{\infty} h(a) e^{-j\omega a} . da$$

$$= X(j\omega) H(j\omega)$$

(iv) Frequency Shifting (Modulation) :

If $$f(t) \Leftrightarrow F(j\omega)$$

Then $$f(t) e^{j\omega_o t} \Leftrightarrow F(j\omega - j\omega_o)$$

Proof : The transform of $f(t) e^{j\omega_o t}$ is by definition

$$T[f(t) e^{j\omega_o t}] = \int_{-\infty}^{\infty} f(t) e^{j\omega_o t} e^{-j\omega t} . dt$$

$$= \int_{-\infty}^{\infty} f(t) e^{-j(\omega - \omega_o) t} . dt$$

$$= F(j\omega - j\omega_o)$$

$$f(t) e^{j\omega_o t} \Leftrightarrow F(j\omega - j\omega_o)$$

Q. 2. (a) Give the detail classification of discrete time systems. **10**

Ans. Classification of Discrete Time Systems : The discrete time systems may be classified as :

(i) Static and dynamic systems (ii) Linear and non-linear systems

(iii) Time-variant and time-invariant systems (iv) Casual and non-casual systems

(i) Static and Dynamic Systems : The output of a static system at any specific time depends on the input at that particular time. It does not depend on past or future values of the input. The output of a dynamic system, on the other hand at any specified time depends on the inputs at that specific time and at other times. Such systems have memory or energy storage elements.

(ii) Linear and Non-linear Systems : A linear system is one in which the principle of superposition holds. For a system with two inputs $x_1(t)$ and $x_2(t)$, the superposition is defined as

$$H[a_1x_1(t)+a_2x_2(t)]=a_1H[x_1(t)]+a_2H[x_2(t)] \quad \text{.... (i)}$$

$a_1, a_2 \rightarrow$ weights added to the inputs.

and $H(x(t))=y(t)$ is the response of the continuous-time system to the input $x(t)$. Thus, a linear system is defined as one whose response to the sum of the weighted inputs is same as the sum of the weighted responses. If a system does not satisfy equation (i), then the system is non-linear. For a discrete-time system, the condition for linearity is given as :

$$H[a_1x_1(n)+a_2x_2(n)]=a_1H[x_1(n)]+a_2H[x_2(n)]$$

(iii) Time-variant and Time-invariant Systems : A time-invariant system is one whose input-output relationship does not vary with time. A time-invariant system is also called a fixed system. The condition for a system to be fixed is

$$\therefore \quad H[x(t-\tau)]=y(t-\tau) \quad \text{.... (ii)}$$

A time-invariant system satisfies equation (a), for any $x(t)$. Equation (a) states if $y(t)$ is the response of the system to any input $x(t)$, then the response of the system to the time shifted input is the response of the system to $x(t)$ time shifted by the same amount. A system not satisfying equation (a), is said to be time-variant. The systems satisfying both linearity and time invariant conditions are called linear, time-variant systems.

(iv) Casual and Non-casual Systems : The response of the casual system to an input does not depend on future values of that input, but depend only on the present and past values of the input. This type of system is called "casual system." If the response of the system to an input depends on the future values of that input, then the system is "non-casual."

(v) Stable and Unstable System : A system is said to be bounded-input, bounded-output (BIBO) stable, if every bounded input produces a bounded output. Thus, a BIBO stable system will have a bounded output for any bounded input so that its output does not grow unreasonably large. The system not satisfying the above conditions are unstable.

Q. 2. (b) Determine the response of the system characterized by the impulse response.

$$h(n)=\left(\frac{1}{2}\right)^n u(n)$$

to the input signal $x(n)=2^n u(n)$

Ans. $$h(n)=\left(\frac{1}{2}\right)^n u(n)$$

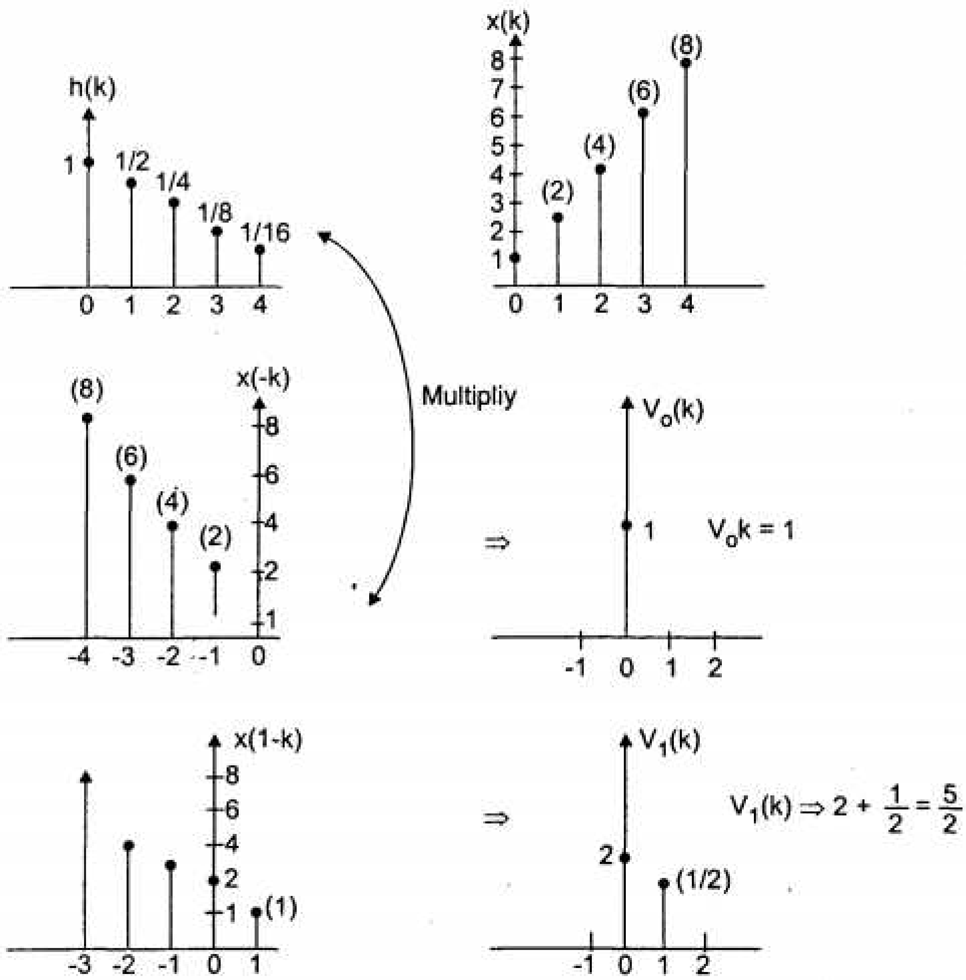
$$x(n) = 2^n u(n) \qquad u(n) = \begin{bmatrix} 1, & n \geq 0 \\ 0, n \leq 0 & \end{bmatrix}$$

$$h(n) = \left(\frac{1}{2}\right)^0 .1 + \left(\frac{1}{2}\right)^1 u(n) + \dots\dots\dots$$

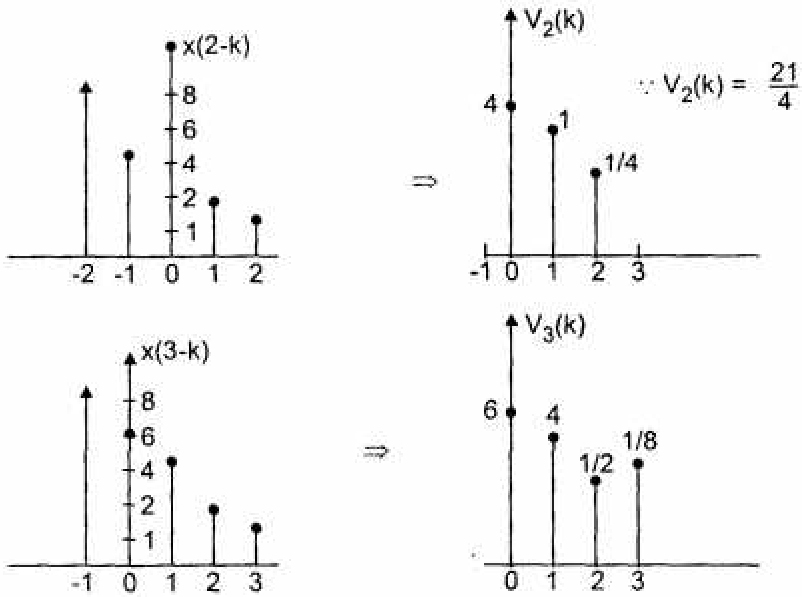
$$h(n) = 1 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots\dots\dots$$

$$h(n) = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots\dots\dots$$

$$x(n) = 1 + 2 + 4 + 6 + 8 + \dots\dots\dots\dots$$

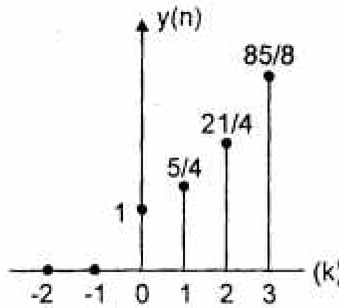


$\therefore$



$$V_3(k) = 6 + 4 + \frac{1}{2} + \frac{1}{8}$$

$$= \frac{48 + 32 + 4 + 1}{8} = \frac{85}{8}$$



$$y(n) = \{\underset{\uparrow}{1},\ 5/4,\ 21/4,\ 85/8\}$$

Ans.

Q. 3. Determine all possible signals that can have the following z-transform. **10**

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

Ans.
$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

I

$$\therefore \qquad X(z) = \frac{Y(z)}{H(z)}$$

$$\begin{array}{r|l}
 & 1 + 1.5z^{-1} + 1.75z^{-2} + \ldots\ldots\ldots\ldots \\
\hline
1 - 1.5z^{-1} + 0.5z^{-2} & 1 \\
 & 1 - 1.5z^{-1} + 0.5z^{-2} \\
 & - \quad + \qquad - \\
\hline
 & 1.5z^{-1} - 0.5z^{-2} \\
 & 1.5z^{-1} - 2.25z^{-2} + 0.75z^{-3} \\
 & - \qquad + \qquad - \\
\hline
 & 1.75z^{-2} - 0.75z^{-3} \\
 & 1.75z^{-2} - 2.855z^{-3} + 0.875z^{-4} \\
 & - \qquad + \qquad - \\
\hline
 & 2.105z^{-3} - 0.875z^{-4} \ldots\ldots\ldots\ldots
\end{array}$$

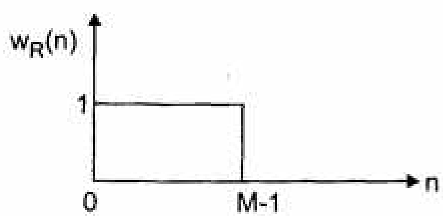
Therefore, $X(z) = 1 + 1.5z^{-1} + 1.75z^{-2} + \ldots\ldots\ldots$

Taking inverse z-transform, we obtain $x(n)$

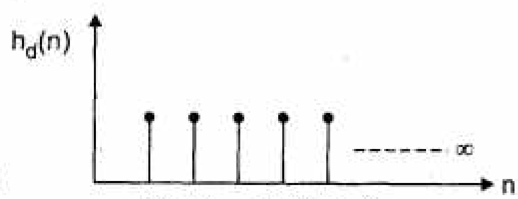
$$\therefore \qquad x(n) = [\underset{\uparrow}{1}, \ 1.5, \ 1.75, \ldots\ldots\ldots\ldots]$$

Q. 5. (a) Explain the design of FIR filters using window technique. What is Gibbs phenomenon?

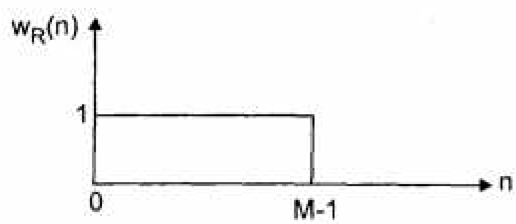
Ans. FIR Filter Using Window Technique : Let us discuss the design of FIR filter using rectangular window. Figure (a) shows the rectangular window. It is denoted $w_R(n)$. Its magnitude is L for the range $n = 0$ to $M-1$. Now, let $h_d(n)$ be the impulse response having infinite duration. If $h_d(n)$ is multiplied by $W_R(n)$ then a finite impulse response is obtained as shown in figure (b). This means that we will get only limited pulse of $h_d(n)$; not all (∞) pulse.



(a) Rectangular window



(b) (x) ⇐ Multiplication



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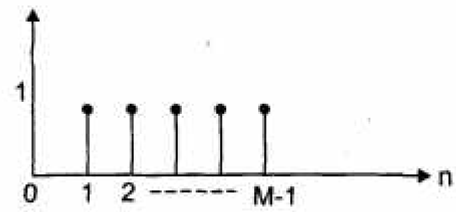


Truncation Process

$$u(n) = h_d(n) \times w_R(n)$$

Since, we are truncating the input sequence by using a window, this process is called as truncation process. Since the shape of window function is rectangular, it is called as rectangular window.

The rectangular window is represented as $w_R(n)$ and it is expressed as

$$\therefore \qquad w_R(n) = \begin{bmatrix} 1, \text{for } n = 0, 1, \ldots\ldots\ldots N-1 \\ 0, \text{ elsewhere} \end{bmatrix}$$

Gibb's Phenomenon : Let us consider the example of a lowpass filter having desired frequency response $H_d(\omega)$ as shown in given figure.

Q. 5. (b) What is zero input limit cycle in IIR filters? Explain it with one example. 10

Ans. Zero Input Limit Cycle : When a stable IIR digital filter is excited by a finite input sequence, that is constant, the output will ideally decay to zero. However, the non-linearities due to the finite-precision arithmetic operations often cause periodic oscillations to occur in the output. Such oscillations in recursive systems are called zero input limit cycle oscillations.

Consider a first order IIR filter with difference equations

$$\therefore \qquad y(n) = x(n) + \alpha y(n-1) \qquad \text{.... (i)}$$

Now, let us assume $\alpha = 1/2$ and the data register length is 3-bits plus a sign bit. If the input is

$$\therefore \qquad x(n) = \begin{bmatrix} 0.875 & \text{for } n = 0 \\ 0 & \text{otherwise} \end{bmatrix}$$

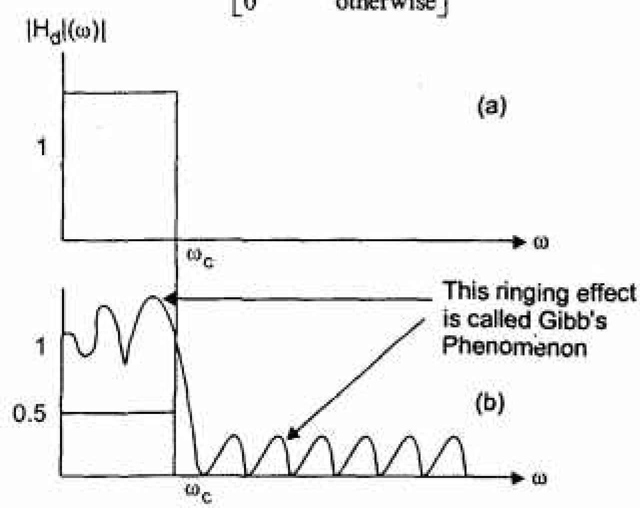


Fig. (a)

In above figure (a), oscillations or ringing take place near band-edge (ω_c) of the filter. These oscillations or ringing is generated because of sidelobes in the frequency response $W(\omega)$ of the window function. This oscillatory behaviour near the band edge of the filter is known as "Gibb's Phenomenon."

And rainding applied after the arithmetic operation then table (a), illustrates the limit cycle behaviour. Here $\theta[.]$ represents the rounded operation. From table (a), we can find that for $n \geq 3$, the output remains constant and gives 1/8 as steady output causing limit cycle behaviour.

When $\alpha = -1/2$ we can table (b) that the output oscillates between 0.125 and −0.125.

(Table a)

n	$x(n)$	$y(n-1)$	$\alpha y(n-1)$	$\theta[\alpha y(n-1)]$	$y(n) = x(n) + \theta[\alpha y(n-1)]$
0	0.875	0.0	0.0	0.000	7/8
1	0	7/8	7/16	0.100	1/2
2	0	1/2	1/4	0.010	1/4
3	0	1/4	1/8	0.001	1/8
4	0	1/8	1/16	0.001	1/8
5	0	1/8	1/16	0.001	1/8

Fig. (b) Zero Input limit cycle oscillation

Table (b)

n	$x(n)$	$y(n-1)$	$\alpha y(n-1)$	$\theta[\alpha y(n-1)]$	$y(n) = x(n) + \theta[\alpha y(n+1)]$
0	0.875	0	0	0.000	7/8
1	0	7/8	−7/16	1.100	−1/2
2	0	−1/2	1/4	0.010	1/4
3	0	1/4	−1/8	1.001	−1/8
4	0	−1/8	1/16	0.001	1/8
5	0	1/8	−1/16	1.001	−1/8
6	0	−1/8	1/16	0.001	1/8

Q. 6. (a) Explain the backward difference methods for IIR filter design. **10**

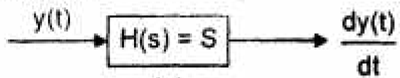
Ans. Backward Difference Method for IIR Filters : One of the simplest method of digitizing an analog filter into a digital filter is to approximate the differential equation by an equivalent difference equation. For the derivative $\frac{dy(t)}{dt}$ at time $t = nT$, we substitute the backward difference $\frac{y(nT) - y(nT - T)}{T}$

$\therefore$ Thus,

$$\frac{dy(t)}{dt}\bigg|_{t=nT} = \frac{y(nT) - y(nT - T)}{T}$$

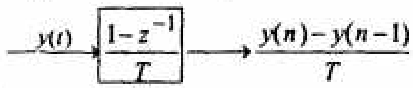
$$= \frac{y(n) - y(n-1)}{T}$$

Where T represents the sampling interval and $y(n) \equiv y(nT)$. We know Laplace transform of $SY(s)$, which can be represented as :



(a)

The Z-transform of $\frac{y(n) - y(n-1)}{T}$ is $\frac{(1 - z^{-1})y(z)}{T}$, which can be represented as :



(b)

Comparing figure (a) and (b), for analog and digital domains.

$\therefore$
$$S = \frac{1 - z^{-1}}{T}$$

Hence,
$$H(z) = H(S)\Big|_{S = \frac{1 - z^{-1}}{T}}$$

$\therefore$
$$Z = \frac{1}{1 - ST} = \frac{1}{1 - J\Omega T} = \frac{1 + j\Omega T}{1 + \Omega^2 T^2}$$

$$= \frac{1}{1 + \Omega^2 T^2} + \frac{j\Omega T}{1 + \Omega^2 T^2} = x + jy$$

Hence, backward difference method is given by

$$S = \frac{1 - z^{-1}}{T}$$

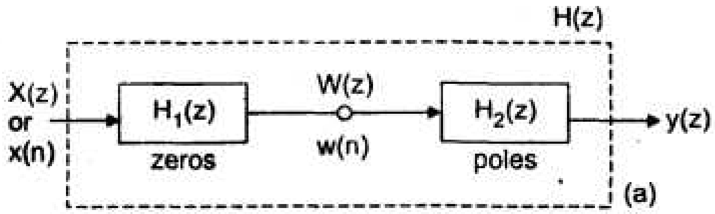
Q. 6. (b) Draw and explain the direct form-I and direct form-II structures for IIR filters. **10**

Ans. Direct Form-I and Direct Form-II Structure :

Direct Form-I : The digital system structure determined directly from given equation (a)

$$\therefore \quad H(z)=\frac{Y(z)}{X(z)}=\frac{\sum_{k=0}^{N} b_K z^{-K}}{1-\sum_{K=1}^{N} a_K z^{-K}} \quad \text{... (i)}$$

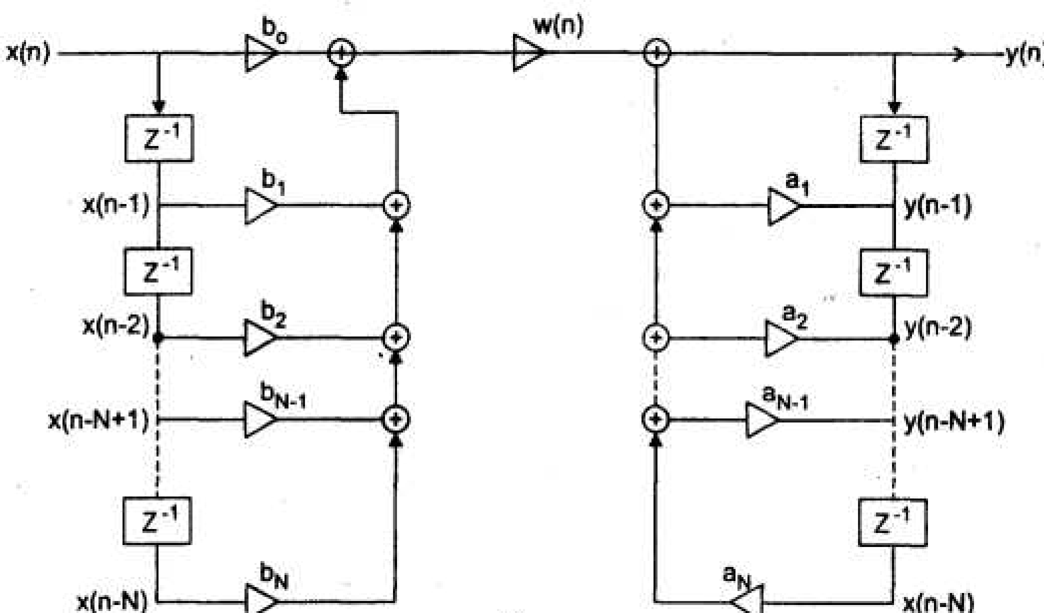
In this case, the system function is divided into two parts connected in cascade, the first part containing only the zeros, followed by the part containing only the poles. A possible IIR system direct form I realisation is shown in figure (a), in which $w(n)$ represents the output of the first part and input to the second.



$$w(n)=\sum_{K=0}^{N} b_K x(n-K) \qquad \therefore \ Y(z)=Y(z)\sum_{K=1}^{N} a_K z^{-K}+W(z)$$

$$Y(n)=\sum_{K=1}^{N} a_K y(n-K)+w(n)$$

Taking z-transform, we get $W(z)=X(z)\sum_{K=0}^{N} K_k z^{-K}$, $\quad \therefore \ Y(z)=\dfrac{W(z)}{1-\sum_{K=1}^{N} a_K z^{-K}}$



(a)

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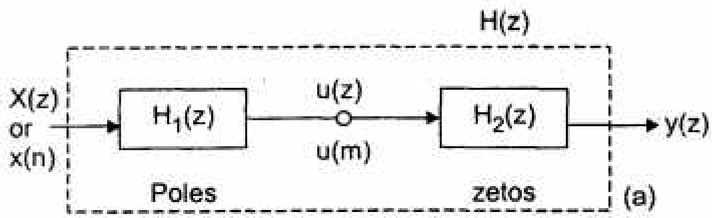
Direct-form-I realisation of an N^{th} order difference.

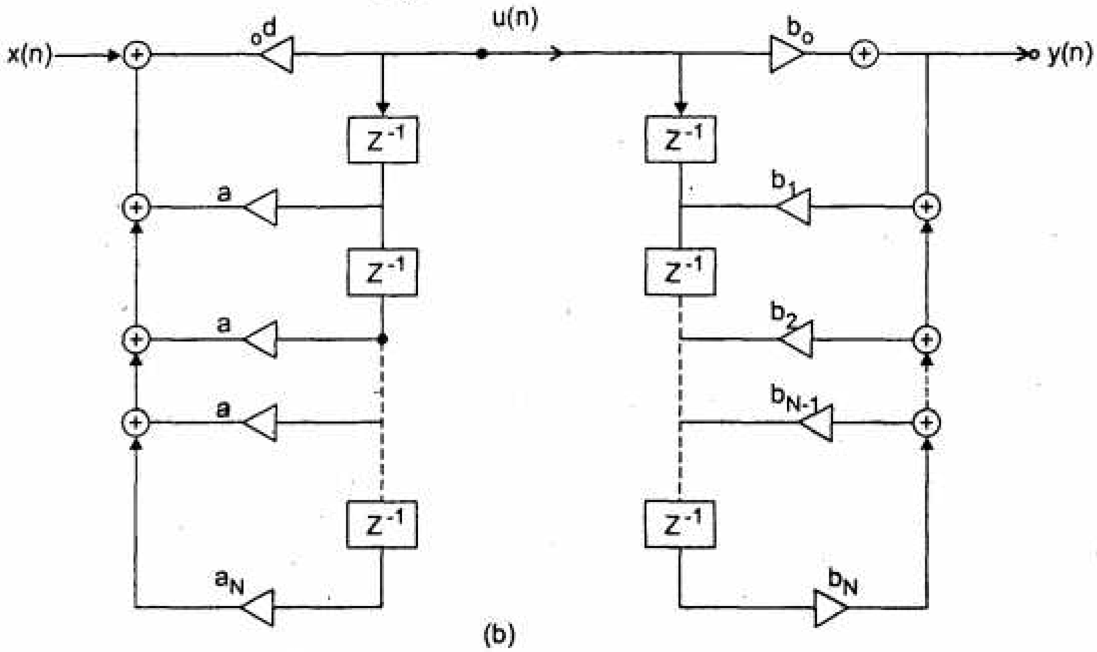
(ii) Direct Form-II : In direct form II, the poles of $H(z)$ are realised first and the zeros second. Here, the transfer function $H(z)$ is broken into a product of two transfer functions $H_1(z)$ and $H_2(z)$ where $H_1(z)$ has only poles and $H_2(z)$ contains only the zeros.

$$\therefore \qquad H(z) = H_1(z).H_2(z)$$

Where, $H_1(z) = \dfrac{1}{\left(1 - \sum_{K=1}^{N} a_K z^{-K}\right)}$ & $H_2(z) = \sum_{K=0}^{N} b_K z^{-K}$



(d_0) Direct form II Realisation



Direct form II-Realization Network

Q. 7. What is multirate signal processing? Explain the up sampling in detail with necessary derivations.

Ans. Multirate Signal Processing : Multirate digital signal processing is required in digital systems where more than one sampling rate is required. For example, in digital audio, the different sampling rates used are 32 kHz for broadcasting, 44.1 kHz for compact disc and 48 kHz for audio As, a matter of fact, different sampling rates can be obtained using up-sampler and down-sampler. The basic operations . multirate processing to achieve this are decimation and interpolation. Decimation is used for reducing the sampling rate and interpolation is used for increasing the sampling rate. Also, in digital transmission systems like teletype, facsimile, low bit rate speech where data has to be handled in different rates, multirate signal processing is used. There are various areas in which multirate signal processing is used, few are

(i) Speech and audio processing system (ii) Antenna system

(iii) Communication system (iv) Radar system

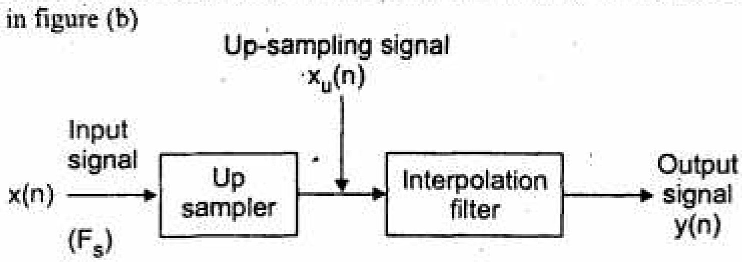
Application of MDSP :

(i) Sub band coding (ii) Voice privacy using analog phone lines.

(iii) Signal compression by sub sampling.

Up Sampling : The process of increasing the sampling rate by an integer factor (I) is called interpolation of the sampling rate. It is also called up-sampling by factor (I). Now figure (a), shows the block diagram of the interpolator. The interpolator comprises of two blocks such as up-sampler and interpolation filter. Here, up sampler is used to increase the sampling rate by the integer (I) and the interpolation filter removes the unwanted images that are yielded by up sampling.

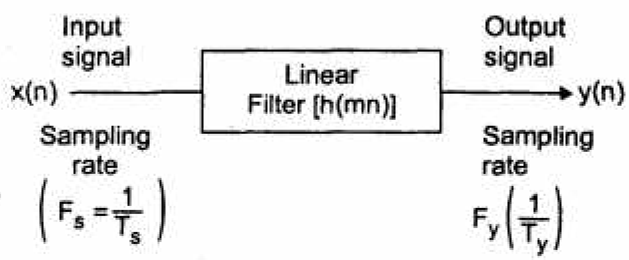
The unwanted images are removed by using low pass filter (LPF), $H(z)$ called the interpolation filter. The process of the sampling rate conversion in the digital-domain may by viewed as a linear filtering operation, as shown in figure (b)



(a)

Block diagram of a Interpolator

The input signal $x(n)$ is characterised by sampling rates $F_S = \frac{1}{T_S}$ and the output signal $y(m)$ is characterised by the sampling rate $F_y = \frac{1}{T_y}$. Where T_s and T_y are corresponding sampling intervals.



Block diagram of Linear Filter

$$\therefore \quad \frac{F_y}{F_s} = \frac{\text{Sampling frequency of output signal}}{\text{Sampling frequency of input signal}}$$

$$= \frac{I}{D} = \frac{\text{Prime integer}}{\text{Prime integer}} = \frac{(I)}{(D)}$$

Here, I is the integer factor by which interpolation of sampling rate is carried out and the D is the integer factor by which decimation of the sampling rate is carried out. However, for the case of the ratio, $\frac{I}{D}$, both I and D must be prime integer.

Q. 8. Write short notes on the following : **20**

(a) Applications of Z-transform **(b) Digital filter banks**

(c) Super position principles and linearity

Ans. (a) Applications of Z-transform :

(i) Causality of discrete-time LTI system

(ii) Stability of discrete-time LTI system

(iii) Characterization of discrete-time LTI systems by linear constant-coefficient difference equations.

(iv) Determination of poles and zeros of a rational Z-transform.

(v) System behaviour related to the system function of a discrete-time LTI system.

(i) Causality of Discrete-time LTI System : The condition for causality of a discrete-time LTI system is that the impulse response of a casual discrete-time LTI system is

$$\therefore \quad h(n) = 0 \text{ for } n < 0$$

A discrete time LTI system which has a rational transfer function $H(z)$ will be causal if and only if :

(a) The ROC is the exterior of a circle outside the outermost pole.

(b) $H(z)$ expressed as a ratio of polynomials in z, the order of the numerator should be smaller than order of denominator.

(ii) Stability Criteria for a Discrete-time LTI Systems : The stability of a discrete-time LTI system is equivalent to its impulse response $h(n)$ being absolutely summable, i.e.,

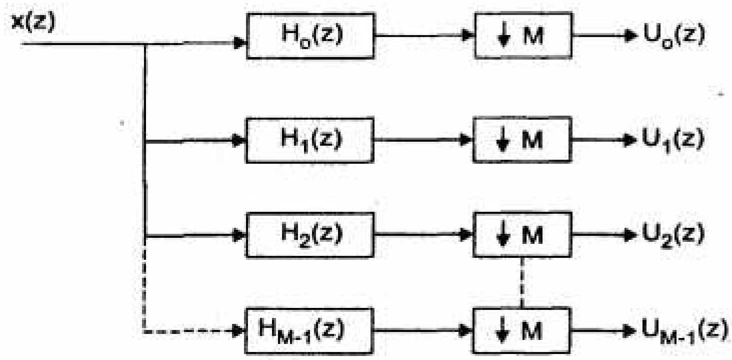
$$\therefore \quad S = \sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

A discrete-time LTI system is stable if and only if the ROC of its transfer function $H(z)$ includes the unit circle $|z|=1$

(iii) Determination of Poles and Zero of Rational Z-transform : Rational transfer function is a ratio of two polynomials of z i.e., $X(z) = \frac{P(z)}{Q(z)}$. The zeros of a z-transform $X(z)$ are the values of z for which $X(z) = 0$. The poles of a z-transform $X(z)$ are the values of z for which $X(z) = \infty$. We can represent $X(z)$ graphically by pole-zero plot in the complete z-plane.

(b) Digital Filter Bank : Filter banks are of two types, analysis filter bank and synthesis filter bank.

(i) Analysis Filter Bank : The M-channel analysis filter bank is shown in figure (a). It contains of M subfilters. The individual subfilter $H_K(z)$ is known as analysis filter. All the subfilters are equally special in frequency and each have the same bandwidth. The spectrum of the input signal $X(e^{j\omega})$ lies in the range $0 \le \omega \le \pi$. The filter bank splits the signal into number of sub bands each having a bandwidth $\frac{\pi}{M}$. The filter $H_0(z)$ is lowpass. $H_1(z)$ to $H_{M-2}(z)$ are bandpass and $H_{M-1}(z)$ is highpass. As the spectrum of the signal is band limited to $\frac{\pi}{M}$, the sampling rate can be reduced by a factor M. The down sampling moves all the subband signals into the base band $0 \le \omega \le \frac{\pi}{M}$.



Analysis Filter Bank

(iii) Synthesis Filter Bank : The M-channel synthesis filter bank is dual of M-channel analysis filter bank. In this case each $U_m(z)$ is feel to an up sampler. The up sampling process produces the signal $U_m(z^M)$. These signals are applied to filters $G_M(z)$ and finally added to get the output signal $X(z)$. The filter $G_0(z)$ to $G_{M-1}(z)$ have the same characteristics as the analysis filters $H_0(z)$ to $H_{M-1}(z)$.

(c) Super Position Principles and Linearity : A linear system is one that satisfies the superposition principle. The principle of superposition requires that the response of the system to a weighted sum of signals be equal to the corresponding weighted sum of the responses (outputs) of the system to each of the individual input signals. A system is linear if any only if.

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)] \quad \text{.... (i)}$$

For any arbitrary input sequences $x_1(n)$ and $x_2(n)$, and any arbitrary constants a_1 and a_2. Figure (a) gives a pictorial illustration of the superposition principle. The superposition principle embodied in the relation (i) can be separated in two parts. First, suppose that $a_2 = 0$, then equation (i) reduces to

$$T[a_1 x_1(n)] = a_1 T[x_1(n)] = a_1 y_1(n)$$

Where, $\quad y_1(n) = T[x_1(n)]$

This demonstrates the scaling property of linear system.

Suppose, that $a_1 = a_2 = 1$ (put in equation (i)) then,

$$T[x_1(n) + x_2(n)] = T[x_1(n)] + T[x_1(n)]$$
$$= y_1(n) + y_2(n)$$

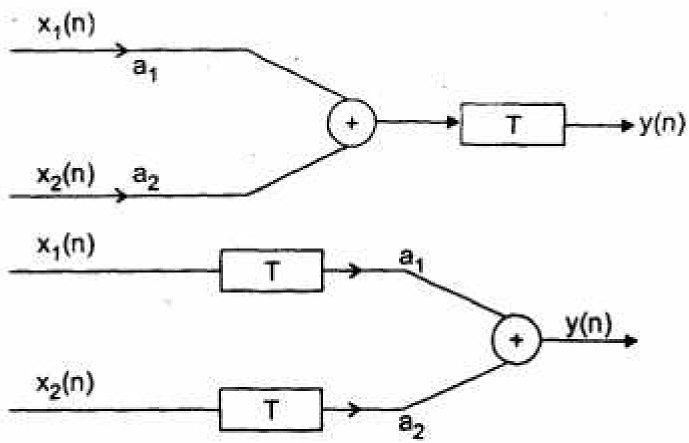


Fig. (a) Superposition Principle

This demonstrates the additive property of a linear system. So, any system which satisfy superposition principle is called a linear system. Hence, all linear systems posses the property of superposition. If a relaxed system does not satisfy the superposition principle as given by the definition above, it is called non-linear.

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