B.E.

Sixth Semester Examination, May-2008

Digital Signal Processing (EE-407-E)

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. (a) Explain in detail with suitable examples the various properties of Fourier transform.

Ans. Properties of Fourier Transform :

(i) Linearity : The four transform is a linear operation.

Therefore if $$f_1(t) \leftrightarrow F_1(j\omega)$$

$$f_2(t) \leftrightarrow F_2(j\omega)$$

Then, $$af_1(t) + bf_2(t) \leftrightarrow aF_1(j\omega) + bF_2(j\omega)$$

Where a and b are arbitrary constants.

(ii) Symmetry :

If $$f(t) \leftrightarrow F(jw)$$

Then $$F(jt) \leftrightarrow 2\pi f(-\omega)$$

Proof : Since $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}\,d\omega$$

$$2\pi f(-t) = \int_{-\infty}^{0} F(j\omega')e^{-j\omega' t}\,d\omega'$$

Where the dummy variable ω is replaced by ω'.

Now, if t is replaced by ω,

$$2Af(-\omega) = \int_{-\infty}^{\infty} F(j\omega')^{-j\omega'\omega}\,d\omega'$$

Finally ω' is replaced by t to obtain a more recognisable form and we have

$$2\pi f(-\omega) = \int_{-\infty}^{\infty} F(jt)e^{-j\omega t}\,dt = F[F(jt)]$$

Therefore $$F(jt) \leftrightarrow 2\pi f(-\omega)$$

If $f(t)$ is an even function $$f(t) \leftrightarrow f(t)$$

Hence $$F[F(jt)] = 2\pi f(\omega)$$

(iii) Scaling :

If $$f(t) \leftrightarrow F(j\omega)$$

Then $$f(at) \leftrightarrow \frac{1}{|a|}F\left(\frac{j\omega}{a}\right)$$

Proof : If $a > 0$, then the transform of $f(at)$ is

$$F[f(at)] = \int_{-\infty}^{\infty} f(at)e^{-j\omega t}\,dt$$

Putting $x = at$,

We have $dx = a\,dt$

Substituting in the above equation, we get

$$F[f(at)] = F[f(x)] = \int_{-\infty}^{\infty} f(x)e^{\frac{-j\omega x}{a}} \frac{dx}{a}$$

$$= \frac{1}{a}F\left(\frac{j\omega}{a}\right)$$

If $a < 0$, then $F[f(at)] = \frac{-1}{a}F\left(\frac{j\omega}{a}\right)$

Combining these 2 results we get

$$f(at) = \frac{1}{|a|}F\left(\frac{j\omega}{a}\right)$$

(iv) Convolution :

If $x(t) \leftrightarrow X(j\omega)$ and $h(t) \leftrightarrow H(j\omega)$

Then $y(t) \leftrightarrow x(t) \times h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$Y(j\omega) = X(j\omega)H(j\omega)$$

Proof : $F[y(t)] = Y(j\omega) = \int_{-\infty}^{\infty} e^{-j\omega t}\left[\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau\right]dt$

$$= \int_{-\infty}^{\infty} x(\tau)\left[\int_{-\infty}^{\infty} h(t-\tau)e^{-j\omega t}\,dt\right]d\tau$$

Putting $a = t-\tau$, then $t = a+\tau$ and $da = dt$

Therefore $Y(j\omega) = \int_{-\infty}^{\infty} x(\tau)\left[\int_{-\infty}^{\infty} h(a)e^{-j\omega(a+\tau)}\,da\right]d\tau$

$$= \int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}\,d\tau \int_{-\infty}^{\infty} h(a)e^{-j\omega a}\,da$$

$= X(j\omega)H(j\omega)$ **Hence proved.**

(v) Time Shifting (Delay) :

If $f(t) \leftrightarrow F(j\omega)$

Then $f(t-t_0) \leftrightarrow F(j\omega)e^{-j\omega t_0}$

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Proof : The fourier transform of $f(t-t_0)$ is given by

$$F[f(t-t_0)] = \int_{-\infty}^{\infty} f(t-t_0)e^{-j\omega t}\,dt$$

Putting $x = t - t_0$, then $t = t_0 + x$ and $dx = dt$

$$F[f(t-t_0)] = \int_{-\infty}^{\infty} f(x)e^{-j\omega(t_0+x)}\,dx$$

$$= e^{-j\omega t_0}F(j\omega)$$

Therefore, $\quad F(t-t_0) \leftrightarrow F(j\omega)e^{-j\omega t_0}$

(vi) Frequency Shifting (Modulation) :

If $\quad f(t) \leftrightarrow F(j\omega)$

Then $\quad f(t)e^{j\omega_0 t} \leftrightarrow F(j\omega - j\omega_0)$

Proof : The transform of $f(t)e^{j\omega_0 t}$ is by definition

$$F\left[f(t)e^{j\omega_0 t}\right] = \int_{-\infty}^{\infty} f(t)e^{j\omega_0 t}\,e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t}\,dt$$

$$= F(j\omega - j\omega_0)$$

Hence $\quad f(t)e^{j\omega_0 t} \leftrightarrow F(j\omega - j\omega_0)$ **Hence proved**

Q. 1. (b) Show that a time shift in time domain is equal to a phase shift in the frequency domain. 10

Ans. If $\quad f(t) \leftrightarrow F(j\omega)$

Then $\quad f(t-t_0) \leftrightarrow F(j\omega)e^{-j\omega t_0}$

Proof : The fourier transform of $f(t-t_0)$ is given by

$$F[f(t-t_0)] = \int_{-\infty}^{\infty} f(t-t_0)e^{-j\omega t}\,dt$$

Putting $x = t - t_0$, then $t = x + t_0$ and $dx = dt$

$$F[f(t-t_0)] = \int_{-\infty}^{\infty} f(x)e^{-j\omega(t_0+x)}\,dx$$

$$= e^{-j\omega t_0}F(j\omega)$$

Therefore, $\quad f(t-t_0) \leftrightarrow F(j\omega)e^{-j\omega t_0}$

When the function $f(t)$ is delayed by to the original spectrum is multiplied by $e^{-j\omega t_0}$.

Here there is no change in the amplitude spectrum and each frequency component is shifted in phase by an amount $-\omega_0 t_0$

Q. 2. (a) A system is characterised by : **10**

$$y(n)-6y(n-1)=x(n)$$

Where $x(n)=1$ for $n \geq 0$

$= 0$ for $n < 0$ and $y(0)=1$

Obtain the expression for step response of the system.

Ans. $H(z)\left[1-3z^{-1}-4z^{-2}\right]=1+2z^{-1}$

$$H(z)=\frac{1+2z^{-1}}{1-3z^{-1}-4z^{-2}}$$

$$=\frac{z(z+2)}{z^2-3z-4}=\frac{z(z+2)}{(z+1)(z-4)}$$

$$F(z)=\frac{H(z)}{z}=\frac{(z+2)}{(z+1)(z-4)}=\frac{A_1}{(z+1)}+\frac{A_2}{(z-4)}$$

$$A_1=(z+1)F(z)\Big|_{z=-1}=\frac{z+2}{z-4}\Big|_{z=-1}=\frac{-1}{5}$$

$$A_2=-(z-4)F(z)\Big|_{z=4}=\frac{(z+2)}{(z+1)}\Big|_{z=4}=\frac{6}{5}$$

$$\frac{H(z)}{z}=\frac{-1/5}{z+1}+\frac{6/5}{z-4}$$

$$H(z)=\frac{-1/5z}{(z+1)}+\frac{6z/5}{(z-4)}$$

$$h(n)=\left[\frac{-1}{5}(-1)^n+\frac{6}{5}(4)^n\right]4(n)$$

Q. 2. (b) Distinguish between IIR and FIR systems. **10**

Ans. A discrete time filter produces a discrete time output sequence $y(n)$ for the discrete time input sequence $x(n)$.

A filter may be required to have a given frequency response, or a specific response to an impulse, step or ramp, or simulate an analog system.

Digital filters are classified as either as finite duration unit pulse response (FIR) filters or infinite duration unit pulse response (IIR) filters, depending upon the form of unit pulse response of the system.

In FIR system, the impulse response is of finite duration i.e., it has finite number of non-zero terms. The IIR system has an infinite number of non-zero terms, i.e., its impulse response sequence is of infinite duration. The system with the impulse response :

$$h(n)=\begin{cases}2 & |n|\leq 4\\ 0 & \text{otherwise}\end{cases}$$

Has only a finite number of non-zero terms.

Thus, the system is FIR system.

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Suppose a system has the following difference equation represents with input $x(n)$ and output $y(n)$

$$\sum_{k=0}^{N} a_k\, y(n-k) = \sum_{k=0}^{M} b_k\, x(n-k)$$

An FIR filter of length M is described as

$$y(n) = b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2) + b_3 x(n-3) \ldots\ldots\ldots + b_{M-1} x(n-M+1)$$

$$= \sum_{K=0}^{M-1} b_K\, x(n-K)$$

FIR filters have following advantages over IIR filters :

(i) They can have an exact linear phase.

(ii) They are always stable.

(iii) The design methods are generally linear.

(iv) They can be realised efficiently in hardware.

(v) The filter start-up transients have finite duration.

FIR filters are employed in filtering problems where linear phase characteristics within the passband of the filters is required.

If this is not required, either an IIR or an FIR filter may be employed.

An IIR filter has less number of side lobes in the stop band than an FIR filter with the same number of parameters.

Q. 3. (a) Explain how sampling can be done with an impulse functions. **10**

Ans. The samples in digital systems are in the form of a number and the magnitude of these numbers represent the value of the signal $x(t)$ at the sampling instants.

In this case, the pulse width of the sampling function is infinitely small and an infinite train of impulse functions of period T can be considered for the sampling function.

i.e.,
$$g(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

The sampling function given above, when this sampling function is used, the weight of the impulse carries the sample value.

The sampling function $g(t)$ is periodic and can be represented by a Fourier series

$$g(t) = \sum_{n=-\infty}^{\infty} C_n\, e^{jn2\pi f_s t}$$

Where, $C_n = \dfrac{1}{T} \displaystyle\int_{-T/2}^{T/2} \delta(t)\, e^{-jn2\pi f_s t}\, dt$

Since $\delta(t)$ has its maximum energy concentrated at $t = 0$, a more formal mathematical definition of the unit impulse function may be defined as functional

$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$$

Where, $x(t)$ is continuous at $t = 0$

Hence, $$C_n = \frac{1}{T} e^o = \frac{1}{T} = f_s$$

Thus, C_n is same as the sampling frequency f_s for all n. The spectrum of impulse sampled signal, $x_s(t)$ is given by

$$X_S(f) = f_s\left(\sum_{n=-\infty}^{\infty} X(f - nf_s)\right)$$

To prevent aliasing, the sampling frequency f_s should be greater than two times the frequency f_h of the sinusoidal signal being sampled.

The condition to be satisfied by the sampling frequency to prevent aliasing is called the sampling theorem.

Q. 3. (b) Explain the process of changing the sampling using discrete time processing. 10

Ans. Sampling rate conversion is the process of converting the sequence $x(n)$ which is got from sampling the continuous time signal $x(t)$ with a period T, to another sequence $y(k)$ obtained from sampling $x(t)$ with a period T, to another sequence $y(k)$ obtained from sampling $x(t)$ with a period T.

Conversion of a sequence $x(n)$ to another sequence $y(k)$.

(i) Decimation : The process of reducing the sampling rate of a signal is called decimation. Let M be the integer sampling rate reduction factor for the signal $x(n)$,

$$\frac{T'}{T} = M$$

The new sampling rate F' becomes

$$F' = \frac{1}{T'}$$
$$= \frac{1}{MT}$$
$$= \frac{F}{M}$$

Let the signal $x(n)$ be a full band signal, with non-zero values in the frequency range $-F/2 \le f \le F/2$

Where $$\omega = 2\pi fT$$

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| \le 2\pi F'T/2 = \pi/M \\ 0 & \text{otherwise} \end{cases}$$

$$\omega(n) = \sum_{K=-\infty}^{\infty} h(K)x(n-K)$$

$$g(m) = \omega(Mm)$$

$$Y(m)=\sum_{K=-\infty}^{\infty} h(K)x(Mm-K)$$

Or
$$Y(m)=\sum_{n=-\infty}^{\infty} h(Mm-n)x(n)$$

The decimator is also known as sub-sampler, down-sampler or under-sampler.

$$\omega'(n)=\begin{cases}\omega(n) & n=0,\pm M,\pm 2M \\ 0 & \text{otherwise}\end{cases}$$

$$\xrightarrow[\substack{x(n)\\F}]{} \boxed{h(n)} \xrightarrow[\substack{\omega(n)\\F}]{} \boxed{\downarrow M} \longrightarrow \boxed{y(m)}\ \ F'=F/M$$

(ii) Interpolation : The process of increasing the sampling rate of a signal if interpolation. Let L be an integer interpolating factor, of the signal $x(n)$, then

$$\frac{T'}{T}=\frac{1}{L}$$

The sampling rate is given by

$$F'=\frac{1}{T'}$$
$$=\frac{L}{T}$$
$$=LF$$

Interpolation of a signal $x(n)$ by a factor L refers to the process of interpolating $L-1$ samples between each pair of samples of $x(n)$.

The signal $\omega(m)$ is got by interpolating $L-1$ samples between each pair of the samples of $x(n)$.

$$\omega(m)=\begin{cases}\omega(m/L) & m=0,\pm L,\pm 2L\ldots\ldots \\ 0 & \text{otherwise}\end{cases}$$

The z-transform of the signal $\omega(m)$ is given by

$$\omega(z)=\sum_{M=-\infty}^{\infty}\omega(m)z^{-m}$$
$$=\sum_{M=-\infty}^{\infty}x(m)z^{-m}$$
$$=X(z^{L})$$

When considered over the unit circle, $z=e^{j\omega'}$

$$\omega(e^{j\omega'})=X(e^{j\omega' L})$$

Where, $\omega'=2\pi fT'$

$$H(e^{j\omega'})=\begin{cases}G & |\omega'|<2\pi fT'/2=\pi/L \\ 0\end{cases}$$

$$y(m) = \sum_{K=-\infty}^{\infty} h(m-K)\omega(K)$$

$$= \sum_{K=-\infty}^{\infty} h(m-K)x(K/L), \quad K/L \text{ an integer}$$

$$K=-\infty$$

Q. 4. (a) Determine : **10**

$$x(n) \text{ for } x(z) = \frac{z^{-1}}{1-3z^{-1}} \text{ with ROC } |z|<3.$$

Ans.

$$x(z) = \frac{z^{-1}}{1-3z^{-1}} = z^{-1}x_1(z)$$

$$x_1(z) = \frac{1}{1-3z^{-1}}$$

Here, from the time shifting property, we have $K=1$ and $x(n) = (3)^n 4(n)$

Hence, $x(n) = (3)^{n-1} 4(n-1)$

Q. 4. (b) Find the two sided z-transform of : **10**

$$x(n) = \left(\frac{1}{3}\right)^n \qquad n \geq 0$$

$$= (-2)^n \qquad n \leq -1$$

Ans.

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n}$$

$$x(z) = x_1(z) + x_2(z)$$

$$= \frac{z^2}{z^2 \frac{-1}{4}} + \frac{z/3}{z^2 \frac{-1}{9}}$$

(i) ROC : $|z| \geq \frac{i}{2}$

(ii) ROC : $|z| > \frac{1}{3}$

Final $ROC = \frac{1}{2}|z| < 2$

Ans.

Q. 5. (a) What is a Kaiser window ? In what way is it superior to other window functions ? **10**

Ans. As the length of filter is increased, the width of the main lobe becomes narrower and narrower, and the transition band is reduced considerably.

The attenuation in the side lobes is, however independent of the length and is a function of the type of the window. Therefore, a proper window function is to be selected in order to achieve a desired stop band attenuation.

A window function with minimum stop band attenuation has the maximum main lobe width.

Therefore, the length of the filter has the maximum main lobe width, must be increased considerably to reduce the main lobe width and to achieve the desired transition band.

A desirable property of the window function is that the function is of finite duration in the time domain and that the fourier transform has maximum energy in the main lobe or a given peak side lobe amplitude.

The prolate spheroidal functions have this desirable property.

However these functions are complicated and difficult to compute. A simple approximation to these functions have been developed by Kaiser in terms of zeroth order modified Bessel functions of the first kind.

$$\omega_K(n) = \begin{cases} \dfrac{I_o(\beta)}{I_o(\alpha)} & \text{for } |n| \le \dfrac{M-1}{L} \\ 0 & \text{otherwise} \end{cases}$$

$$\beta = \alpha\left[1 - \left(\frac{24}{M-1}\right)^2\right]^{0.5}$$

$$I_0(x) = 1 + \sum_{K=1}^{\infty}\left[\frac{1}{K!}\left(\frac{x}{2}\right)^K\right]^2$$

$$= 1 + \frac{0.25x^2}{(1!)^2} + \frac{(0.25x^2)^2}{(2!)^2} + \frac{(0.25x^2)^3}{(3!)^2} \ldots\ldots\ldots$$

A_P = Actual passband ripple

A_S = Minimum stop band attenuation

A_p, A_S given by

$$A_P = 20\log_{10}\frac{1+\delta_P}{1-\delta_P}\,\text{dB}$$

$$A_S = -20\log_{10}\delta_S\,\text{dB}$$

$$A_P \le A_P \quad \text{and} \quad A_S \le A_S$$

Where, A_p and A_S are the actual pass band peak-to-peak ripple and minimum stop band attenuation, respectively.

Q. 5. (b) An analog filter has the following system function. Convert this filter into a digital filter using bilinear transformation : **10**

$$H(s) = \frac{1}{(s+0.2)^2 + 16}$$

Ans. From the system function, we note that

$$\Omega_C = 3$$

$$\Omega_C = \frac{2}{T}\tan\frac{\omega_r}{2}$$

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The sampling period is obtained from the above equation using

$$T = \frac{2}{\Omega_C} \tan \frac{\omega_r}{2} \frac{2}{T} \frac{2}{3} \tan \frac{\pi}{8} = 0.2765$$

Using bilinear transformation

$$H(z) = H(s)\Big|_{S=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

$$H(z) = \frac{\frac{2}{T}\left(\frac{z-1}{z+1}\right) + 0.1}{\left\{\frac{2}{T}\left[\frac{z-1}{z+1}\right] + 0.1\right\}^2 + 9}$$

$$= \frac{(2/T)(z-1)(z+1) + 0.1(z+1)^2}{[(2/T)(z-1) + 0.1(z+1)]^2 + 9(z+1)^2}$$

Substituting, $T = 0.2765$

$$H(z) = \frac{1 + 0.027z^{-1} - 0.973z^{-2}}{8.572 - 11.84z^{-1} + 8.177z^{-2}}$$

Q. 6. (a) Explain the importance of DSP in various fields of engineering and technology. **10**

Ans. DSP techniques are used in a variety of areas which include speech, radar, sonar images etc. These techniques are applied in spectral analysis, channel vo-coders, homomorphic processing system, speech synthesisers, linear prediction system, analysing the signals in radar tracking etc.

(i) A speech signal consists of periodic sounds, interspersed with bursts of wide band noise and sometimes short silences. The local system can be modelled with a periodic signal excitor, a variable filter representing the vocal tract, switch to pass the signal.

(ii) A filter bank is used to separate the band in an channel vocoder. At Rx side, a matching filter bank is available, So that the output level matches the encoded value. The individual outputs are combined to produce the speech signal.

(iii) In digital signal transmission, the analog signal is converted into a digital signal and after processing, it may be transmitted over a limited bandwidth channel or the digital signal can be stored. If we employ signal compression, then the efficiency of transmission or storage can be improved.

(iv) In telephone communications, privacy is required for protection against evesdropping. The long speech time segment is divided into B contiguous blocks. Then each time segment block is split into F contiguous frequencies.

(v) The important components of a radar system are the antenna, the tracking computer and the signal processor. The tracking computer is the brain of the system.

(vi) 2D signal processing is helpful in processing the images. Like image enhancement, image restoration and image coding.

Q. 6. (b) What are the effects of finite word length in digital filters ? **10**

Ans. When digital systems are implemented either in hardware or in software, the filter coefficients are stored in binary registers.

These registers can accomodate only a finite number of bits and hence, the filter coefficients have to be truncated or rounded-off in order to fit into these registers.

Truncation or rounding-off in the data results in degradation of system performance. Also, in digital processing systems, a continuous, time input signal is sampled and quantised in order to get the digital signal.

The process of quantisation introduces an error in the signal. The various effects of quantisation that arise in digital signal processing. Following are some of the issues connected with finite word length effects :

(i) Quantisation effects in analog-to-digital conversation.

(ii) Product quantisation and coefficient quantisation errors in digital filters.

(iii) Limit cycles in IIR filters and

(iv) Finite word length effects in fast fourier transform.

Rounding & Truncation Errors : Rounding or truncation introduces an error whose magnitude depends on the number of bits truncated or rounded-off. Also, the characteristics of the errors depends on the form of binary number representation. The sign magnitude and the two's complement representation of fixed point binary numbers.

$$Q_T(x) = xt\,\varepsilon_T$$

Truncation error for sign magnitude representation

$$-(2^{-B} - 2^{-L}) \le \varepsilon_T \le 0$$

i.e., for two's complement $\Rightarrow -(2^{-B} - 2^{-L}) \le \varepsilon_T \le 0$

i.e., for sign magnitude and two's complement representation

$$-\left(\frac{2^{-B} - 2^{-L}}{2}\right) \le \varepsilon_R \le \left(\frac{2^{-B} - 2^{-L}}{2}\right)$$

Ans.

Q. 7. (a) Discuss filter design and implementation for sampling rate conversion. **10**

Ans. Sampling rate conversion is the process of converting the sequence $x(n)$, which is got from sampling the continuous time signal $x(t)$ with a period T_1 *to* another sequence $y(k)$ obtained from sampling $x(t)$ with a period T'.

Decimation :

$$\frac{T'}{T} = M$$

$$F' = \frac{1}{T'}, \quad F\frac{1}{MT} = \frac{R}{M}$$

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| \le 2\pi F'\,T/2 = \pi/M \\ 0 & \text{otherwise} \end{cases}$$

$$\omega(n) = \sum_{K=-\infty}^{\infty} h(K)x(n-K)$$

$$y(m) = \omega(Mm)$$

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)x(Mm-K)$$

$$y(m) = \sum_{n=-\infty}^{\infty} h(Mm-n)x(n)$$

$$\omega'(n) = \begin{cases} \omega(n) & n = 0, \pm M, \pm 2M \ldots.. \\ 0 & \text{otherwise} \end{cases}$$

$$y(m) = \omega'(Mm) = \omega(Mm)$$

Interpolation :

$$\frac{T'}{T} = \frac{1}{L}$$

$$F' = \frac{1}{T'} = \frac{L}{T} = LF$$

$$\omega(m) = \begin{cases} x(M/L) & M = 0, \pm L, \pm 2L \\ 0 & \end{cases}$$

$$\omega(z) = \sum_{M=-\infty}^{\infty} \omega(m) z^{-m}$$

$$= \sum_{M=-\infty}^{\infty} x(m) z^{-mL}$$

$$= X(z^L)$$

$$X(e^{j\omega'}) = X(e^{j\omega' L})$$

$$H(e^{j\omega'}) = \begin{cases} 4 & |\omega'| \le 2\pi f T'/2 = \pi/2 \\ 0 & \text{otherwise} \end{cases}$$

$$Y(e^{j\omega'}) = H(e^{j\omega'}) \times (e^{j\omega' L})$$

$$= \begin{cases} 4 \times (e^{j\omega' L}) & |\omega'| \le A/L \\ 0 & \text{otherwise} \end{cases}$$

$$y(m) = \sum_{K=-\infty}^{\infty} h(M-K)\omega(K)$$

$$= \sum_{K=-\infty}^{\infty} h(m-K)x(K/L),\ K/L \text{ an integer.}$$

Q. 7. (b) Obtain the polyphase structure of the filter with filter transfer function : **10**

$$H(z) = \frac{1+4z^{-1}}{1+5z^{-1}}$$

Ans.

$$H(z) = \frac{1-4z^{-1}}{1+5z^{-1}}$$

$$= \frac{(1-4z^{-1})(1-5z^{-1})}{(1+5z^{-1})(1-5z^{-1})}$$

$$= \frac{1-9z^{-1}+20z^{-2}}{1-25z^{-2}}$$

$$= \frac{1+20z^{-2}}{1-25z^{-2}} + z^{-1} - \frac{-9}{1-25z^{-2}}$$

The polyphase components are :

$$E_0(z) = \left(\frac{1+20z^{-2}}{1-25z^{-2}}\right) \text{ and } E_1(z) = \frac{-9}{1-25z^{-2}}$$

Q. 8. Write short notes on any two of the following : **20**

(i) Parseval's Theorem

(ii) Multistage decimator & interpolators

(iii) Application of Z-Transform

Ans. (i) Parseval's Theorem : This theorem is similar to Parseval's theorem for energy signals. The theorem defines the power of a singal in terms of its Fourier series coefficients i.e., in terms of the amplitudes of the harmonic components present in the signal.

Let us consider a function $f(t)$, we know that,

$$|f(t)^2| = f(t)\,f^*(t)$$

Where, $f^*(t)$ is the complex conjugate of the function $f(t)$. The power of the signal $f(t)$ over a cycle is given by

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2\,dt = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,f^*(t)\,dt$$

Replacing $f(t)$ by its exponential Fourier series

$$P = \frac{1}{T}\int_{-T/2}^{T/2} f^*(t)\left\{\sum_{n=-\infty}^{\infty} F_n\, e^{j\omega_0 nt}\right\} dt$$

Where, $\omega_0 = \frac{2\pi}{T}$

Interchanging the order of integration and summation,

$$P = \frac{1}{T}\sum_{n=-\infty}^{\infty} F_n \int_{-T/2}^{T/2} f^*(t)\, e^{jn\omega_0 t}\,dt$$

The integral in the above expression is equal to TF_n^*.

Hence we may write,

$$P = \sum_{n=-\infty}^{\infty} F_n F_n^* = \sum_{n=-\infty}^{\infty} |F_n|^2$$

The above expression is known as Parseval's power then the equation defines that the power of the signal is equal to the sum of square of magnitudes of various harmonics present in a discrete spectrum.

(ii) Multistage Decimator & Interpolators : Sampling rate conversion is the process of converting the sequence $x(n)$ which is got from sampling the continuous time signal $x(t)$ with a period T, to another sequence

$y(k)$ obtained from sampling $x(t)$ with a period T, to another sequence $y(k)$ obtained from sampling $x(t)$ with a period T.

Conversion of a sequence $x(n)$ to another sequence $y(k)$.

Multistage Decimator : The process of reducing the sampling rate of a signal is called decimation. Let M be the integer sampling rate reduction factor for the signal $x(n)$,

$$\frac{T'}{T} = M$$

The new sampling rate F' becomes

$$F' = \frac{1}{T'} = \frac{1}{MT} = \frac{F}{M}$$

Let the signal $x(n)$ be a full band signal, with non-zero values in the frequency range $-F/2 \le f \le F/2$

Where $\omega = 2\pi fT$

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| \le 2\pi F'T/2 = \pi/M \\ 0 & \text{otherwise} \end{cases}$$

$$\omega(n) = \sum_{K=-\infty}^{\infty} h(K)x(n-K)$$

$$g(m) = \omega(Mm)$$

$$Y(m) = \sum_{K=-\infty}^{\infty} h(K)x(Mm-K)$$

Or
$$Y(m) = \sum_{n=-\infty}^{\infty} h(Mm-n)x(n)$$

The decimator is also known as sub-sampler, down-sampler or under-sampler.

$$\omega'(n) = \begin{cases} \omega(n) & n = 0, \pm M, \pm 2M \\ 0 & \text{otherwise} \end{cases}$$

Interpolators : The process of increasing the sampling rate of a signal if interpolation. Let L be an integer interpolating factor, of the signal $x(n)$ then

$$\frac{T'}{T} = \frac{1}{L}$$

The sampling rate is given by

$$F' = \frac{1}{T'}$$

$$= \frac{L}{T}$$

$$= LF$$

Interpolation of a signal $x(n)$ by a factor L refers to the process of interpolating $L-1$ samples between each pair of samples of $x(n)$.

The signal $\omega(m)$ is got by interpolating $L-1$ samples between each pair of the samples of $x(n)$.

$$\omega(m) = \begin{cases} \omega(m/L) & m = 0, \pm L, \pm 2L \ldots\ldots \\ 0 & \text{otherwise} \end{cases}$$

The z-transform of the signal $\omega(m)$ is given by

$$\omega(z) = \sum_{M=-\infty}^{\infty} \omega(m) z^{-m}$$

$$= \sum_{M=-\infty}^{\infty} x(m) z^{-m}$$

$$= X(z^{L})$$

When considered over the unit circle, $z = e^{j\omega'}$

$$\omega(e^{j\omega'}) = X(e^{j\omega' L})$$

Where, $\omega' = 2\pi f T'$

$$H(e^{j\omega'}) = \begin{cases} G & |\omega'| < 2\pi f T'/2 = \pi/L \\ 0 & \end{cases}$$

$$y(m) = \sum_{K=-\infty}^{\infty} h(m-K)\omega(K)$$

$$= \sum_{K=-\infty}^{\infty} h(m-K)x(K/L), \quad K/L \text{ an integer}$$

$K = -\infty$

(iii) Application of Z-Transform : Step and impulse responses of series R-L circuit :

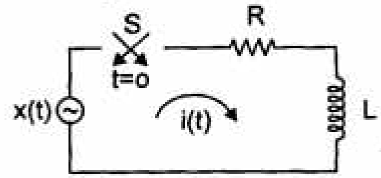
(a) Step Response : In the series RL circuit, shown in figure, let the switch S be closed at time $t = 0$. For step response, the input excitation is $x(t) = V_o u(t)$

Applying Kirchoff's voltage law to the circuit, we get the following differential equation

$$\frac{Ldi(t)}{dt} + R_i(t) = V_o u(t)$$

Taking Laplace transform, the above equation becomes.

$$L\{SI(s)-i(o^{+})\}+RI(s)=\frac{V_o}{S}$$



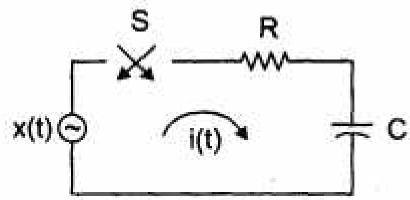
Series RL circuit

$$I(s)=\frac{V_o}{L}\cdot\frac{1}{S\left(S+\frac{R}{L}\right)}$$

$$=\frac{V_o}{L}\cdot\frac{L}{R}\left(\frac{1}{S}-\frac{1}{S+R/L}\right)=\frac{V_o}{R}\left[\frac{1}{S}-\frac{1}{S+R/L}\right]$$

$$i(t)=\frac{V_o}{R}\left[1-e^{-Rt/L}\right]$$

(b) Step Response of Series R-C Circuit :



$$\frac{1}{C}\int_{-\infty}^{t} i(t)\,dt+Ri(t)=V_o u(t)$$

$$\frac{1}{C}\int_{0}^{t} i(t)\,dt+\frac{1}{C}\int_{-\infty}^{0} i(t)\,dt+R_i(t)=V_o u(t)$$

$$\frac{1}{C}\left[\frac{I(s)}{S}\right]+\frac{1}{C}L[q(o^{+})]+RI(s)=\frac{V_o}{S}$$

$$I(s)\left[\frac{1}{CS}+R\right]=\frac{V_o}{S}$$

$$I(s)=\frac{V_o/R}{S+\frac{1}{R_C}}$$

Therefore, $$i(t)=\frac{V_o}{R}e^{-t/R_C}$$