Roll No. 1606173 Total No. of Pages : 3

BT-2/J07 8247

Mathematics-II (2005-06)

Paper : MATH-102 E

Option : II

Time : Three Hours] [Maximum Marks : 100

Note :– Attempt **FIVE** questions in all, selecting at least **ONE** question from each unit. **Each** question carries **equal** marks.

UNIT–I

1. (i) Use Gauss- Jordan method to find the inverse of the following matrix :

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$$

10

(ii) Show that the equations :

$3x + 4y + 5z = a,$

$4x + 5y + 6z = b,$

$5x + 6y + 7z = c$

do not have a solution unless $a+c = 2b$. 10

2. (i) For a symmetrical square matrix, show that the eigen vectors corresponding to two unequal eigen values are orthogonal. 10

(ii) Show that the eigen values of a Skew-Hermitian matrix (and thus of a Skew-symmetric matrix) are purely imaginary or zero. 10

UNIT–II

3. (i) Solve : $\left(x^4 e^x - 2mxy^2\right)dx + 2mx^2 y\,dy = 0.$ 10

(ii) Two friends A and B order coffee and recieve cups of equal temperature at the same time. A adds a small amount of cool cream immediately but does not drink his coffee until 10 minutes later, B waits for 10 minutes and adds the same amount of cool cream and begins to drink. Assuming the Newton's law of cooling, decide who drinks the hotter coffee ? 10

4. (i) Solve : $\frac{d^2y}{dx^2} - 4y = x.\sinh x.$ 10

(ii) In an L-C-R circuit, the charge q on a plate of a condenser is given by :

$$L.\frac{d^2q}{dt^2} + R.\frac{dq}{dt} + \frac{q}{c} = E.\sin pt.$$

The circuit is tuned to resonance so that $p^2 = 1/LC$. If initially the current i and the charge q be zero, show that, for small values of R/L, the current in the circuit at time t is given by $(Et / 2L)\sin pt$. 10

UNIT–III

5. (i) Solve : $px\left(z - 2y^2\right) = (z - qy)\left(z - y^2 - 2x^3\right)$ 10

(ii) Solve : $2z + p^2 + qy + 2y^2 = 0.$ 10

6. (i) By the method of separation of variables, solve :

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, u(x,0) = 4e^{-x}.$$ 10

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(ii) Solve completely the equation : $\frac{\partial^2 y}{\partial t^2} = C^2 . \frac{\partial^2 y}{\partial x^2}$, representing the vibrations of a string of length ℓ, fixed at both ends, given that $y(0, t) = 0$; $y(\ell, t) = 0$; $y(x, 0) = f(x)$

and $\frac{\partial y}{\partial t}(x,0) = 0$, $0 < x < \ell$. 10

UNIT–IV

7. (i) Evaluate : $L\left[\int_0^t e^{-t} \cos t \, dt\right]$. 10

(ii) State and prove Convolution theorem. 10

8. (i) Solve : $t y'' + 2y' + ty = \cos t$, given that $y(0) = 1$. 10

(ii) Calculate the maximum deflection of an encastre beam 1 ft. long carrying a uniformly distributed load w lb/ft. on its central half length. 10