Roll No. .... Total Pages : 3

8493

BT-3/D07

DISCRETE STRUCTURES

(Common With C.O. & I.T.)

Paper–CSE-205E

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *five* questions, selecting at least one question from each Unit.

UNIT–I

1. (a) Explain with example "Principle of Inclusion and Exclusion".

(b) Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(c) If sets S and T have n elements in common, show that $S \times T$ and $T \times S$ have n^2 elements in common. (7,7,6).

2. (a) State and prove Pigeon hole principle.

(b) Prove that $\forall\, n \in N$.

$\frac{1}{5}n^5 + \frac{1}{3}n^3 + \frac{7}{15}n$ is a natural number.

(c) Let the relation $(x, y) \in R$, if $x \geq y$ defined on set of positive integers. Is R a partial order relation? Prove or disprove it. (6,7,7)

UNIT–II

3. (a) A palindrome is a word that reads the same forward or backward. How many seven letter palindrome can be made out of English alphabets ?

(b) Prove that the following propositions are equivalent to $p \to q$:

(i) $\sim(p \wedge \sim q)$, (ii) $\sim p \vee q$, (iii) $\sim q \to \sim p$.

(c) Solve the difference equation

$$a_r + 6a_{r-1} + 9a_{r-2} = 3$$

with initial conditions $a_0 = 1$, $a_1 = 1$. (6,6,8)

4. (a) Solve the recurrence relation

$$a_{r+2} - 2a_{r+1} + a_r = 2^r$$

by method of generating functions with initial conditions $a_0 = 2$ and $a_1 = 1$.

(b) From the following formula, find out tautology, contradiction and contingency :

(i) $a \to a \wedge (a \vee b)$.

(ii) $(p \wedge \sim q) \vee (\sim p \wedge q)$.

(iii) $\sim(p \vee q) \vee (\sim p \vee \sim q)$. (10,10)

UNIT–III

5. (a) Consider the binary operation $*$ on Q, set of rational numbers, defined by $a * b = a + b - ab \;\; \forall \;\; a, b \in \mathrm{Q}$. Determine whether $*$ is associative or not.

(b) Let (A, +, 0) be a ring, such that $a_0 a = a \;\; \forall \;\; a \in \mathrm{A}$,

(i) Show that $a + a = 0 \;\; \forall \; a \in \mathrm{A}$, where 0 is additive identity.

(ii) Show that operation 0 is commutative. (8,12)

6. (a) Prove that every subgroup of a Cyclic group G is cyclic.

(b) Consider an algebraic system $(G, *)$, where G is set of all Non-zero real numbers and $*$ is binary operation defined by $a * b = \frac{ab}{4}$.

Show that $(G, *)$ is an Abelian group. (8,12)

UNIT–IV

7. (a) Define the following with examples :

(i) Spanning subgraph.

(ii) Bridges.

(iii) Homomorphic graph.

(iv) Undirected complete graph.

(b) Write short note on applications of Binary trees. (12,8)

8. (a) State and prove Euler's theorem.

(b) Draw unique binary tree for given In-order and Post-order traversal :

In-order :	4	6	10	12	8	2	1	5	7	11	13	9	3
Post-order :	12	10	8	6	4	2	13	11	9	7	5	3	1

Also, give its Pre-order traversal. (8,12)