

Roll No. ....

Total No. of Pages

**BT-3/J05**

**85**

**DISCRETE STRUCTURES**

**(Common with CO., IT)**

**Paper : CSE-205 E**

**(According to Syllabus Dec. 2004)**

Time : Three Hours]

[Maximum Marks :

**Note :— Attempt any FIVE questions.**

1. (a) By mathematical induction, prove

$$1 + a + a^2 + \dots + a^n = \frac{1 - a^{n+1}}{1 - a}.$$

- (b) Prove principle of Inclusion and Exclusion for n sets.

2. (a) Construct truth table for :

(i)  $(p \rightarrow p) \vee (p \rightarrow \bar{p})$

(ii)  $(p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow q) \rightarrow (q \rightarrow r)$

(iii)  $p \leftrightarrow (\bar{p} \vee \bar{q})$ .

p, q, r are propositions.

$$10\frac{1}{2} = 3\frac{1}{2}$$

- (b) Formulate the Tower of Hanoi problem as a Recurrence relation and then solve this recurrence relation.

$$4 + 5\frac{1}{2} = 9\frac{1}{2}$$

3. (a) Clearly define Lattice, Antichain.

- (b) Let R be a binary relation on set of all strings of 0s and such that  $R = \{(a, b) \mid a, b \text{ are strings that have same number of 0s}\}$ .

Is R reflexive ? Symmetric/Antisymmetric ? transitive ?  
it equivalence relation or p.o. relation ?

- (c) From integers 1.....200, 101 of them are chosen arbitrarily

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(Contd.)



How many minimum pairs are found to exist such that one divides another in that pair ? 6

4. (a) Using generating functions, solve

$$a_r - 2a_{r-1} + 2a_{r-2} - a_{r-3} = 0$$

given that  $a_0 = 2, a_1 = 1, a_2 = 1$ .

12

- (b) Three integers are selected from integers 1, 2, ..., 1000. In how many ways can these integers be selected such that their sum is divisible by 4 ? 8

5. (a) Let  $(A, *)$  be a semigroup where

$a * a = b$ . Show that :

(i)  $a * b = b * a$

(ii)  $b * b = b$ .

4x2=8

- (b) Show that an integral domain that has a finite number of elements is a field. 11

6. (a) Let  $(F, +, \cdot)$  be the field of integers modulo 2 &

$(F[x], \boxed{+}, \boxed{\cdot})$  be corresponding ring of polynomials.

Construct the ring of polynomials modulo  $1 + x + x^2$  12

- (b) State Lagrange's theorem and give some example of it. 8

7. (a) Prove clearly necessary and sufficient condition for Eulerian path. 11

- (b)  $n$  cities are connected by a network of  $K$  highways.

Show that if  $K > \frac{1}{2}(n-1)(n-2)$ , then one can always travel between any two cities through connecting highways. 9

8. (a) Prove that, by coalescing of vertices, a minimum spanning tree can be obtained from a graph. 8

- (b) A tree has  $n_2$  vertices of degree 2,  $n_3$  vertices of degree 3 & ...  $n_k$  vertices of  $K$  degree. How many vertices does it have ?

- (c) How many edges are common in every circuit cut-set ? Justify your answer.