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BT-4/M11 8459

MATHEMATICS—III

Paper—Math-201E

Time : Three Hours] [Maximum Marks : 100

Note :—Attempt any **FIVE** questions, selecting at least **ONE** question from each unit. All questions carry equal marks.

UNIT—I

1. (a) Obtain the Fourier series for

$$f(x) = \pi x,\ 0 \le x \le 1$$
$$= \pi(2 - x),\ 1 \le x \le 2$$

and hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \pi^2/8.$$

(b) Show that the Fourier series for $\sin ax$, $-\pi \le x \le \pi$ is

$$\frac{2\sin a\pi}{\pi}\left\{\frac{\sin x}{1^2 - a^2} - \frac{2\sin 2x}{2^2 - a^2} + \frac{3\sin 3x}{3^2 - a^2}\cdots\right\}.$$

2. Solve :

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$$

if $u(0, t) = 0$, $u(x, 0) = e^{-x}$ $(x > 0)$, $u(x, t)$ is bounded where $x > 0, t > 0$.

UNIT—II

3. (a) If $(a + ib)^p = m^{x+iy}$, prove that one of the value of y/x is $2\tan^{-1}(b/a) + \log(a^2 + b^2)$.

(b) An electrostatic field in the xy-plane is given by the potential function $\phi = 3x^2y - y^3$, find the stream function.

4. (a) If f(z) is an analytic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log |f(z)| = 0.$$

(b) Show that under the transformation $w = \frac{1}{z}$, the circle $x^2 + y^2 - 6x = 0$ is transformed into a straight line in w-plane.

UNIT—III

5. (a) Two persons A and B fire at a target independently and have a probability 0·6 and 0·7 respectively of hitting the target. Find the probability that the target is destroyed.

(b) Three machines M_1, M_2 and M_3 produce indentical items. Of their respective outputs 5%, 4% and 3% items are faulty. On a certain day M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with heighest output ?

6. (a) Fit the binomial distribution to the following data :—

x	f
0	2
1	14
2	20
3	34
4	22
5	8

(b) Show that the standard deviation for a normal distribution is approximately by 25% more than the mean deviation.

UNIT—IV

7. (a) Using graphical method, solve :

Min. : $Z = 20x_1 + 10x_2$

subject to :

$$x_1 + 2x_2 \le 40$$
$$3x_1 + x_2 \ge 30$$
$$4x_1 + 3x_2 \ge 60$$
$$x_1, x_2 \ge 0.$$

(b) Find all the basic solutions to :

Max.: $Z = x_1 + 3x_2 + 3x_3$

subject to :

$$x_1 + 2x_2 + 3x_3 = 4,$$
$$2x_1 + 3x_2 + 5x_3 = 7 \text{ and}$$
$$x_1, x_2, x_3 \ge 0$$

Which of the basic solutions are :

(i) Non-degenerate basic feasible

(ii) Optimal basic feasible ?

8. (a) Use Simplex method, to solve :

Max.: $Z = x_1 + x_2 + 3x_3$

subject to :

$$3x_1 + 2x_2 + x_3 \le 3$$
$$2x_1 + x_2 + 2x_3 \le 2$$
$$x_1, x_2, x_3 \ge 0.$$

(b) Using dual simplex method, solve :

Min. : $Z = 2x_1 + x_2$

subject to :

$$3x_1 + x_2 \geq 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \geq 0.$$