10/01/11

Roll No. ....

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BT-3/DX

MATHEMATICS–III

Paper : Math-201(E)

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *five* questions in all, selecting at least *one* question from each part.

PART–A

1. (a) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$. 10

   (b) Express $f(x) = x$ as a half-range sine series in $0 < x < 2$. 10

2. (a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2 , & |x| \le 1 \\ 0 , & |x| > 1. \end{cases}$$

Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$. 10

   (b) Solve the integral equation

$$\int_\theta^\infty f(\theta) \cos \alpha\theta \; d\theta = \begin{cases} 1-\alpha , & 0 \le \alpha \le 1 \\ 0 , & \alpha > 1. \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2}\, dt$. 10

PART–B

3. (a) Separate $\sin^{-1}(\cos\theta + i\sin\theta)$ into real and imaginary parts, where θ is a positive acute angle. 10

(b) Show that the polar form of Cauchy-Riemann equations are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\cdot\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\cdot\frac{\partial u}{\partial \theta}.$$

Deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial u}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 u}{\partial \theta^2} = 0.$ 10

4. (a) Find the analytic function, whose real part is $\sin 2x/(\cosh 2y - \cos 2x)$. 10

(b) Expand $f(z) = 1/\{(z-1)(z-2)\}$ in the region

(i) $|z| < 1$,

(ii) $1 < |z| < 2$,

(iii) $|z| > 2$,

(iv) $0 < |z-1| < 1$. 10

5. (a) (i) Evaluate $\int_C \tan z \, dz$ where C is the circle $|z| = 2$.

(ii) Evaluate $\int_C \frac{z-3}{z^2+2z+5}\, dz$, where C is the circle $|z+1-i| = 2$. 10

(b) By integrating around a unit circle, evaluate

$$\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta.$$ 10

PART–C

6. (a) There are three bags : first containing 1 white, 2 red, 3 green balls, second 2 white, 3 red, 1 green balls, and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that balls so drawn came from second bag. 10

(b) Fit a binomial distribution to the following frequency distribution :

x	:	0	1	2	3	4	5	6
f	:	13	25	52	58	32	16	4

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7. (a) Fit a Poisson distribution to the set of observations :

x	:	0	1	2	3	4
f	:	122	60	15	2	1

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(b) Fit a normal distribution to the following data of weights of 100 students and test the goodness of fit : 10

Weights (kg) :	60-62	63-65	66-68	69-71	72-74
Frequency :	5	18	42	27	8

8. (a) Using Simplex method, solve the L.P.P.

Max. $Z = x_1 + 3x_2$.

subject to $x_1 + 2x_2 \leq 10$, $0 \leq x_1 \leq 5$, $0 \leq x_2 \leq 4$. 10

(b) Using Dual simplex method, solve the L.P.P.

Minimize $Z = 2x_1 + 2x_2 + 4x_3$,

subject to

$$2x_1 + 3x_2 + 5x_3 \geq 2;$$
$$3x_1 + x_2 + 7x_3 \leq 3;$$
$$x_1 + 4x_2 + 6x_3 \leq 5;$$
$$x_1, x_2, x_3 \geq 0.$$

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